Some Recent Developments in the Transmission-Line Modelling (TLM) Method

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Summary

This paper is based on a Lecture delivered at the University of Niš on 19th September 1994. It surveys the broad development and application of the TLM method as a general tool for simulating electromagnetic phenomena, transient and other wave and diffusion problems.

Introduction

Modelling and simulation of physical systems by numerical means using powerful computers has become an alternative to "hard-wired" experiments. In a "numerical experiment" all major interactions are modelled in software without the need to construct a physical experiment. The benefits of this approach are financial savings, shorter experimentation time, ability to isolate and control individual parameters, full and unobtrusive diagnostics and ability to test scenarios which would be too dangerous or costly to perform in reality.

The disadvantages are rooted to the fact that no model and therefore no simulation is ever perfect and hence caution is required when transferring the results of simulation to the real world. The benefits of simulation are, however, so great and the possibilities offered by modern computer workstations are so extensive, that simulation is set to dominate the analysis and design of all kinds of engineering systems. In order to exploitfully these benefits, it is desirable that the modelling methods used are well suited to the manner of operation of modern digital computers and also to the way of thinking and conceptual level of understanding of the applications engineer who is most likely to be the user of such tools.

Transmission-Line Modelling (TLM) is a technique for studying a very wide range of wave and diffusion phenomena. It belongs to the class of time-domain differential methods, although recently a frequency-domain version has become available. TLM operates by establishing a correspondence between electromagnetic field components, and voltages and currents in a network of transmission lines. Thus, electromagnetic field concepts are related to the more familiar circuit concepts. This provides for intellectual economy, a physical and intuitive feel to the modelling process and efficient computation. A general introduction to TLM may be found in [1]. The fundamental aspects of TLM and some of the more recent developments are presented in the following sections.

TLM - Basic Concepts

There are three ways of looking at TLM as a modelling method.

First, one can show that the equations describing wave propagation in suitable lumped component circuits and those describing EM field propagation have the same form. This isomorphism can be used to establish equivalence and thus obtain the solution to EM field problems by analogy to the solutions to circuit problems. In TLM, combinations of lumped circuits are used to form transmission line segments, which, suitable interconnected at nodes, form the basic building blocks of the model. The basic node used in three-dimensional calculations is shown in Fig. 1 and was developed by Peter Johns [2]. It is known as the symmetrical condensed node (SCN).

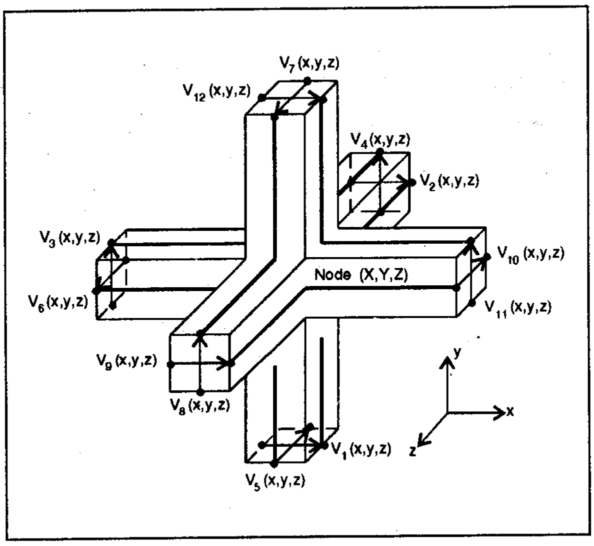

Fig. 1. Schematic of the symmetrical condensed node (SCN)

The node, which describes a volume of space $(\Delta l)^3$, is the junction between six transmission lines. At each time step, twelve voltage pulses V^i are incident on the ports of the node and are reflected according to transmission line theory, to produce twelve reflected pulses V^r. Incident and reflected pulses are related by the scattering matrix **[S]**.

$$V^r = [\mathbf{S}]\, V^i \tag{1}$$

These reflected pulses, say at time step k, become the new incident pulses at adjacent nodes at time step k+1. Thus, as an example, the new incident voltage on port 11 of node (x,y,z) is equal to the voltage pulse reflected at the previous time step into port 3 of node (x+Δl,y,z)

$$_{k+1}V^i_{11}(x,y,z) = {}_kV^r_3(x+\Delta l,y,z) \tag{2}$$

This process, repeated for all ports and node in the system, is described as connection. The TLM algorithm is essentially a repetition of scattering and connection at each time step with modifications to account for excitation and boundaries.

A second, alternative way, of looking of TLM is to observe the propagation of waves following excitation at a single node. This is shown, for simplicity, for the case of a two-dimensional mesh in Fig. 2. It depicts an initial isotropic excitation (a) followed by the situation after Δt (b) and 2Δt (c). It can be seen that each reflected pulse sets up a secondary spherical radiator in accordance with Huygens principle [3,4]. TLM may thus be viewed as the discrete equivalent of Huygens principle.

A third alternative, is to establish a formal mathematical framework to derive TLM directly from Maxwell's equations using the Method of Moments. This has been described in [5].

Whichever approach one wishes to adopt to account for the modelling philosophy, TLM offers a flexible, versatile and efficient modelling medium for a very wide class of problems in electromagnetics and physical sciences in general.

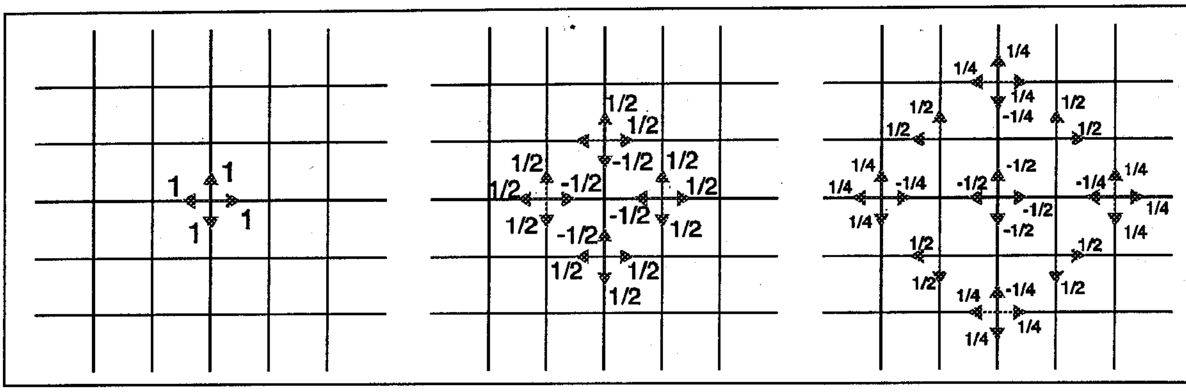

Fig. 2. Scattering on a two-dimensional mesh **(a)** isotropic impulse excitation, **(b)** pulses after one timestep and **(c)** pulses after two timesteps.

A typical simulation in TLM, proceeds as follows:

i) The geometrical features and material properties of the problem are mapped on a TLM mesh. This may be a regular mesh (each node representing a cube of space of side (Δl) or in one of the more general irregular meshes as explained in the next session.

The selection of (Δl) is based on the dimensions of the finest geometrical feature to be modelled and the shortest wavelength of interest. As an approximate rule, it is recommended that four segments of length Δl are used to model a fine feature and ten segments of length Δl are used to describe the shortest wavelength of interest. Violation of these rules results in coarseness and velocity errors respectively. The presence of physical conducting boundaries is indicated in the mesh by terminating adjacent transmission line segments by zero (or very low resistance). Different materials can also be described by introducing stubs or by modifying transmission line properties as explained in the next session.

ii) Excitation and initial conditions are then imposed on the mesh. This determines the value of all incident pulses V^i at the first timestep everywhere in the mesh. Various forms of excitation are possible, namely, pulse, Gaussian, sinusoidal, without the risk of compromising stability. Plane-wave, point and antenna excitations have been routinely used in TLM.

iii) Following the initial value of V^i, scattering on all nodes is implemented to obtain all reflected voltages using equation [1].

iv) Connection is then performed to obtain the incident voltages at the next timestep using equations analogous to (2). This process is modified at boundaries to account for reflections.

v) At any stage during this calculation, field components may be supplied to the user in the time domain, or if required, in the frequency domain (after a DFT). Following a small amount of post-processing, data such as current densities, impedances and radiation patterns may be provided in the form required by the user. In addition, electrical parameters may be changed to account for non-linear behaviour.

vi) The process may then be repeated from step (iii) for as long as required. This is typically a few thousand steps, to allow for a sufficient number of wave transits across the structure being modelled.

The largest problem that may be simulated depends on the computations resources available. With a typical workstation, it is possible to model regions not exceeding 10λ in each dimension, where λ is the shortest wavelength of interest. Fields outside this region may be obtained by using near-to-far field transformations. Dispersion errors can, depending on the particular problem, be kept below 2%. As expected from a time domain, differential method, full visualisation and animation of fields can be obtained, thus offering the user an unparalleled insight into the evolution of fields and currents in complex electromagnetic structures.

TLM - Advanced Formulations

The solution of practical problems in electromagnetics requires maximum flexibility in the structure of the mesh and material properties. An irregular mesh (where each node describes a non-cubical volume) is referred to in TLM as a graded mesh. This is shown schematically in Fig. 3(a). An alternative, is the subdivision of parts of the mesh into a finer structure to produce a multigrid or multiple grid mesh as shown in Fig. 3(b).

Non-uniform material properties may be introduced by placing additional inductance and capacitance to those parts of space which have higher m and e compared to the background medium (normally air). In modifying the basic TLM node described in the previous section to accommodate grading, multigriding and non-uniformities two requirements need to be addressed.

First, all pulses must arrive at node boundaries at the same fixed time interval (the timestep Δt of the simulation), or if this is not possible, some form of time averaging must be found to permit exchange of pulse in an orderly fashion occurs the interface between regions described with different timestep. This is the "synchronism" requirement and its violation cannot be attempted without accepting some loss of rigour.

Second, if synchronism is maintained across interfaces between different materials, this will inevitably lead to a different space discretisation length Δl hence loss of connectivity. This means that each port on one side of the interface has to communicate directly with more than one port on the other side. These difficulties have been resolved for a general graded mesh in a non-uniform region (mesh shown in Fig. 3(a)) in several ways.

i) By adding three capacitive and three inductive stubs to the basic SCN to produce the stubbed SCN [2]. In this formulation all link lines forming the basic node have parameters corresponding to the background medium.

ii) By adding either capacitive stubs only [6], or inductive stubs only 7 and allowing for three different values of link line impedances. This results in a hybrid node (HSCN).

iii) By keeping link lines only in the basic node, but allowing for different impedances. This is described as the symmetrical super-condensed node (SSCN) [8].

iv) By using six stubs and different link line impedances to form a general SCN [9].

All these formulations may be obtained from the general TLM constitutive equations which are [10]:

$$\text{a)}\quad C_{ik}\Delta i + C_{jk}\Delta j + C_o^k = \varepsilon_k \frac{\Delta i \Delta j}{\Delta k}$$

$$L_{ij}\Delta i + L_{ji}\Delta j + L_s^k = \mu_k \frac{\Delta i \Delta j}{\Delta k} \qquad (3)$$

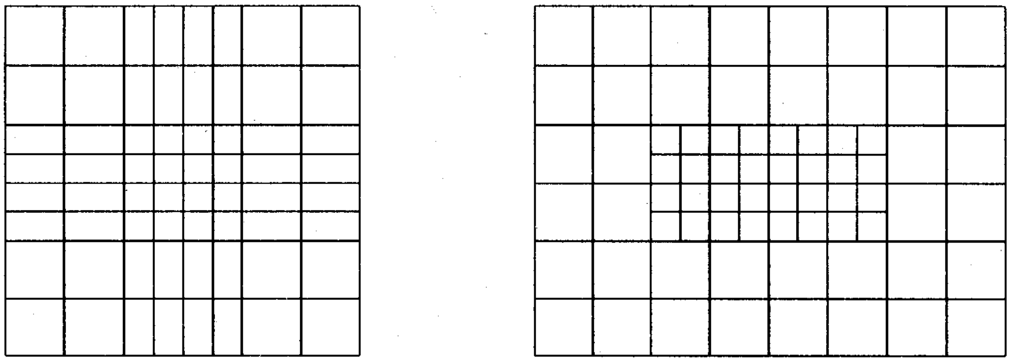

Fig. 3. Schematic of a graded **(a)** and multigrid **(b)** mesh

where i, j and k correspond to all possible combinations of x, y and z. C_o^k, L_s^k are stub parameters in the k-direction and C_{ij}, L_{ij} are the capacitance and inductance per unit length of the i-directed, j-polarised link line. Equations (3) ensure that the combination of link and stub lines represents the correct medium parameters.

b) $$\Delta t = \Delta i \sqrt{C_{ij} L_{ij}} \qquad (4)$$

for all i, j enforcing a common time-step t throughout the problem (synchronism). Equations (3) and (4) for all permutations of i, j and k give twelve equations. There are however eighteen parameters to be determined Thus six degrees of freedom remain, allowing considerable scope for developing different nodal structures. As an illustration, if six additional conditions are imposed requiring that all link line impedances are the same the SCN stubbed node is obtained. In contrast, if $C_o^i = L_s^i = 0$ is demanded, the SSCN is obtained. All known TLM node may be derived from the general constitutive equations (3) and (4) as shown in [10]. More importantly, other more powerful formulations such as the GSCN may be formulated to obtain optimum performance.

There are cases where a full time-domain solution is not required or it is not the most effective approach. Examples are narrow-band problems in microwave circuits, or problems involving very long diffusion time-constants. A frequency-domain version of TLM is now available to deal with such problems. [11-13].

The developments described above deal with the most general formulations for graded meshes. A feature of grading, evident from Fig. 3(a) is that mesh refinements are not localised but spread unevenly throughout the mesh. The local refinement shown in Fig. 3(b) can be more advantageous, but it clearly violates synchronism and connectivity. The rules allowing the conversion of pulses across a fine-coarse mesh interface are described in [14-15]. It turns out that some loss of rigour must be accepted so that an efficient conversion scheme may be produced. Conversion ratios as high as 8 (ratio of coarse to fine mesh discretisation length) have been used without loss of stability.

Even with the flexibility offered by grading and multigriding there are many practical problems where many fine features need to be modelled and where is impractical to reduce the mesh, even locally, to offer the necessary resolution. Three particular problems will be addressed here, to illustrate the TLM modelling approach.

In many situations, it is necessary to model this panels made out of conducting material. It is not always the case that infinite electrical conductivity may be assumed, hence, it is necessary to model in detail the diffusion of EM fields through the thin panel. It has been shown that one-dimensional model of propagation through the panel may be interposed between TLM nodes modelling the three-dimensional field on either side of the panel. Normally, a ten-section one-dimensional model is sufficient. The development and efficient implementation of these models has been described in [16,17].

Another class of problems requiring very fine spacial resolution is that of modelling very fine wires and narrow slots. Fine wires may be modelled by placing short-circuits at the boundaries of the nodes corresponding to the outline of the wire. The smallest wire diameter that may be modelled is 1 and in this limiting case, where one node only is used to describe the wire cross-section, some velocity error must be accepted. This results in wire resonances at frequencies lower than expected by up to 5%. This error may be reduced by increasing the number of nodes describing the wire cross-section but this becomes impractical for thin wires. The cause of this errors, which also appear at the edges of thin sheets, has been identified and ways of reducing it, without excessively increasing computational demands, have been developed [18,19]. An altogether different approach to the modelling of thin wires is to develop special nodes or interfaces between nodes to account for the presence of wires. In the former case this involves the derivation of a new scattering matrix [20[and in the latter an interface network which effects the connection between TLM nodes adjacent to the wire [21].

Narrow slots are similarly treated by exploiting the principle of duality [21].

Undoubtedly further developments in this area are possible to improve the accuracy and efficiency of modelling. One case envisage the availability of customised nodes which have embedded in them particular features (e.g. thin wire, slots etc.) useful to the user.

A problem that often arises in differential schemes, is related to the description of boundaries (flat, curved etc.) by the mesh. With the exception of the case where boundaries are parallel to coordinate planes, a stepped or staircase approximation is necessary. Cylindrical and spherical TLM nodes are available as described in [1] but in the vast majority of problems a Cartesian mesh is used. It is possible to reduce these errors by placing boundaries at any desired position (not necessarily a multiple of Δl) by using the node described in [22].

An issue to considered in differential schemes such as TLM is the modelling of open-boundary problems. Since a termination of the numerical scheme is necessary, a numerical boundary with appropriate absorbing boundary conditions must be inserted. An effective and simple way of doing this in three-dimensional TLM is to introduce matching (zero reflection coefficient) on lines adjacent to the numerical boundary. For some problems, such as those involving the calculation of very low values of the radar cross-section or S-parameters of microwave junctions, a better absorbing boundary condition may be necessary. A comparison of the performance of some common absorbing boundary conditions applied to a TLM mesh may be found in [23]. Further details of the theoretical development of TLM and its application to the whole range of electromagnetic problems including microwave circuits, antennas, electromagnetic compatibility (emc), radar cross-section (rcs) and electromagnetic heating may be found in [1,24].

Conclusions

TLM since its beginnings more than twenty years ago has developed into a powerful modelling technique at the forefront of electromagnetic simulation of complex systems. With an excess of thirty groups working on TLM and over three hundred research publications in the scientific literature a vigorous development of TLM is pursued in order to increase its efficiency, accuracy and range of applicability. Several large international companies now use TLM based software for analysis and design thus providing useful feedback which stimulates further work.

Amongst the most useful features of TLM, unconditional stability, inherent parallelism, generality and a physical feel, must be regarded as the most pronounced. Particular attention in the years to come will focus on the development of customised nodes with built-in extra features and the hybridisation of the technique with other methods to give more powerful and efficient formulations.

Acknowledgement

I wish to thank the University of Niš and the Yugoslav Section of the IEEE MTT for their invitation to give this lecture and the warmth and hospitality they extended to me during my visit.

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