### Frequency Dependence of the Equivalent Gate and Drain Noise Temperatures of Microwave Transistors

Vera Marković, Bratislav Milovanović, Olivera Pronić, Nataša Maleš-Ilić Faculty of Electronic Engineering, Beogradska 14, 18000 Niš, Yugoslavia

#### **Abstract**

The equivalent gate and drain noise temperatures of microwave FET transistors in terms of equivalent circuit elements, noise parameters and frequency are considered in this paper. The corresponding equations are derived and a calculation procedure using circuit simulator Libra [1] is developed. The frequency dependent gate and drain temperatures are used for the transistor noise parameters modelling.

### 1. Introduction

It is well known that any linear noisy twoport network can be represented by linear noiseless two-port and two separated noise sources [2]. There are several representations of noise in linear twoports, depending on type and position of noise sources.

In paper [3] Pospieszalski derived a set of equations for noise parameters calculation of MESFET intrinsic circuit. For this calculation, it is necessary to know equivalent circuit elements and two frequency independent temperatures called equivalent gate and drain temperatures. Starting from this approach, some authors have developed procedures for noise parameters modelling using standard programs for CAD of microwave circuits.

A simple CAD prediction of noise parameters behaviour for MESFET and HEMT transistors in wide frequency range is proposed in a previous paper, [4], where it was assumed that gate temperature is equal to the room temperature and drain temperature is obtained in "tuning" mode of program package Libra.

The concept of equivalent noise temperatures is also used in [5], where the expressions for the wave noise parameters depending on elements of linear transistor model and two equivalent temperatures, are obtained.

The approach of Pospieszalski is applied in [6] for the analysis of semidistributed HEMT model. Using the optimisation, the equivalent circuit elements and equivalent gate and drain temperatures for each region slice are extracted.

So, in all these papers linear models and two equivalent constant temperatures are used for simultaneous signal and noise parameters modelling. The purpose of this paper is to derive a direct relation between noise temperatures, on one hand, and the noise parameters and intrinsic circuit model elements, on the other hand. The obtained relations can be used for efficient noise parameters modelling using commercially available circuit simulator software.

# 2. Derivation of Equivalent Noise Temperature Expressions

The intrinsic equivalent circuit of MESFET is shown in Fig.1.a). The noise properties of this circuit are treated by assigning equivalent temperatures  $T_g$  to gate-to-source resistance  $r_{gs}$  and  $T_d$  to drain-to-source conductance  $g_{ds}$ , [2].

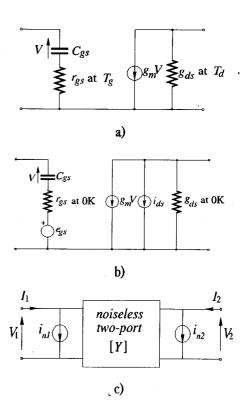


Fig. 1. a) Intrinsic circuit of MESFET, b) Intrinsic circuit with separated noise sources, c) "Y" representation of noisy two-port

In Fig.1.b) an equivalent circuit is shown, involving  $r_{gS}$  and  $g_{dS}$  at the temperature 0 K and corresponding voltage noise source  $e_{gS}$  and current noise source  $i_{dS}$ . These sources can be expressed as

$$\langle |e_{gS}|^2 \rangle = 4kT_g r_{gS} B,$$
 (1)

$$\langle |i_{dS}|^2 \rangle = 4kT_d g_{dS} B , \qquad (2)$$

where k is Boltzmann's constant, B is incremental bandwidth, and brackets  $\langle \rangle$  represent time average.

One commonly used noise representation is shown in Fig.1.c). Two noise sources are separated from the intrinsic two-port, which is treated as noiseless two-port defined by Y-parameters. The admittance form of noisy two-port representation is

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + i_{n1} \tag{3}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + i_{n2} \tag{4}$$

The equations for  $i_{n1}$  and  $i_{n2}$  in terms of noise parameters and Y-parameters of intrinsic circuit are, [2]:

$$\langle |i_{n1}|^2 \rangle = 4kT_0BR_n \left[ |Y_{11}|^2 + |Y_{opt}|^2 + 2\operatorname{Re} \left\{ Y_{11} \left( Y_{opt}^* - F' \right) \right\} \right]$$
(5)

$$\langle |i_{n2}|^2 \rangle = 4kT_0BR_n |Y_{21}|^2, \qquad (6)$$

$$\langle i_{n1}i_{n2}^* \rangle = 4kT_0BR_n Y_{21}^* \left( Y_{11} + Y_{opt} - F' \right),$$
 (7)

where  $T_0$  is standard reference temperature of 290 K,  $F_{min}$  is minimum noise figure,  $F'=(F_{min}-1)/2R_n$ ,  $R_n$  is equivalent noise resistance,  $Y_{opt}=G_{opt}+jB_{opt}$  is optimal source admittance and \* indicates complex conjugation.

Straightforward comparison of the circuits shown in Fig.1.b) and Fig.1.c) give:

$$i_{n1} = -Y_{11}e_{gg} , (8)$$

$$i_{n2} = i_{ds} - Y_{21}e_{gs}. (9)$$

From equation (8) follows

$$\langle |i_{n1}|^2 \rangle = \langle |e_{gs}|^2 \rangle |Y_{11}|^2,$$
 (10)

thus,

$$\langle |e_{gs}|^2 \rangle = 4kT_0BR_n \left[ 1 + \frac{|Y_{opt}|^2 + 2\operatorname{Re}\left\{Y_{11}\left(Y_{opt}^* - F'\right)\right\}}{|Y_{11}|^2} \right]$$
(11)

From relations (8) and (9) follow

$$i_{ds} = i_{n2} - \frac{Y_{21}}{Y_{11}} i_{n1},$$
 (12)

$$\langle |i_{ds}|^2 \rangle = \langle \left(i_{n2} - \frac{Y_{21}}{Y_{11}} i_{n1}\right) \left(i_{n2}^* - \frac{Y_{21}^*}{Y_{11}^*} i_{n1}^*\right) \rangle$$
(13)

$$\langle |i_{ik}|^2 \rangle = \langle |i_{n2}|^2 \rangle + \frac{|Y_{21}|^2}{|Y_{11}|^2} \langle |i_{n1}|^2 \rangle - 2 \operatorname{Re} \left\{ \frac{Y_{21}}{Y_{11}} \langle i_{n1} i_{n2}^* \rangle \right\}. \tag{14}$$

By substituting equations (5), (6) and (7) to (14), it follows:

$$\langle |i_{ds}|^2 \rangle = 4kT_0 BR_n |Y_{opt}|^2 \frac{|Y_{21}|^2}{|Y_{11}|^2} .$$
 (15)

Finally, using (1) and (2) the equivalent gate and drain noise temperatures are

$$T_g = T_0 \frac{R_n}{r_{gs}} \left[ 1 + \frac{\left| Y_{opt} \right|^2 + 2 \operatorname{Re} \left\{ Y_{11} \left( Y_{opt}^* - F' \right) \right\}}{\left| Y_{11} \right|^2} \right],$$
(16)

$$T_d = T_0 |Y_{opt}|^2 \frac{R_n}{g_{ds}} \frac{|Y_{21}|^2}{|Y_{11}|^2}$$
 (17)

Parameters  $Y_{11}$  and  $Y_{21}$  for the intrinsic circuit shown in Fig.1. a) are

$$Y_{11} = \frac{j\omega C_{gS}}{1 + j\omega r_{gS}C_{gS}}, \qquad (18)$$

$$Y_{21} = \frac{g_m}{1 + j\omega r_{gs} C_{gs}},$$
 (19)

so, the expressions for  $T_g$  and  $T_d$  are got in terms of frequency, intrinsic circuit elements and noise parameters,

$$T_g = T_O \frac{R_n}{r_{gs}} \left\{ 1 + 2r_{gs} \left( G_{opt} - F' \right) + \frac{2B_{opt}}{\omega C_{gs}} + \frac{2B_{opt}}{\omega C$$

$$+\frac{\left|Y_{opt}\right|^{2}\left(1+\omega^{2}r_{gs}^{2}C_{gs}^{2}\right)}{\omega^{2}C_{gs}^{2}}\right\},$$
 (20)

$$T_{d} = T_{o} \frac{R_{n} g_{m}^{2} |Y_{opt}|^{2}}{g_{ds} \omega^{2} C_{gs}^{2}}.$$
 (21)

## 3. Determination of Intrinsic Circuit Noise Parameters

Equivalent circuit of MESFET chip is shown in Fig.2, where the intrinsic part is denoted by the broken line.

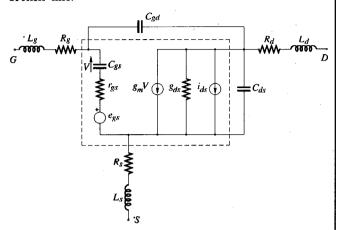
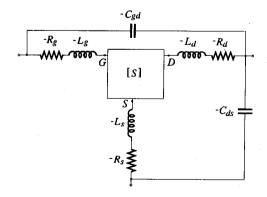


Fig. 2. Equivalent circuit of MESFET chip

Noise parameters from the manufacturer data books are given for the complete chip. In order to get noise parameters of intrinsic circuit, needed for  $T_{\mathcal{G}}$  and  $T_{\mathcal{d}}$  calculation, the following procedure was done using circuit simulator Libra:

- First, the topology of equivalent circuit is described within Libra CKT file. The extraction of circuit element values is performed by optimisation process, configured to minimise the difference between simulated and measured  $\mathcal{S}$ -parameters.
- The aim of the next step is de-embedding of device parasitics (gate, drain and source resistances and inductances, as well as gate-to-drain and drain-to-source capacitances). The de-embedding process is done by adding elements with negative values and in reverse order to the measured two-port data. This process is illustrated in Fig.3. In this way we got the noise parameters of the intrinsic circuit.



### 4. Numerical Results

The proposed procedure is applied to various MESFET and HEMT transistors using S-parameters and noise data from manufacturer data books. As illustration, the results for modelling of NEC MESFET, type N71000A, will be presented. The circuit element values resulting from the optimisation are shown in Table 1.

Table 1

element	value		
$R_{\varrho}$ $(\Omega)$	0.28		
$L_{\varphi}$ (nH)	0.158		
$R_d(\Omega)$	2.2		
$L_d^{(nH)}$	0.131		
$R_{\mathcal{S}}(\Omega)$	4.13		
$L_{\mathcal{S}}(nH)$	0.0368		
$C_{pd}$ (pF)	0.0455		
$C_{ds}$ (pF)	0.0974		
$r_{g_S}$ $(\Omega)$	2.91		
$C_{QS}$ (pF)	0.311		
$C_{ds}(pF)$	0.0974		
$r_{ds}(\Omega)$	245.00		
$g_{m}(S)$	0.0479		

The values of noise parameters for *complete* chip ("C") and for *intrinsic* chip ("I") obtained by deembedding process are given in Table 2.

The equivalent gate and drain noise temperatures are calculated by using expressions (20) and (21). Their frequency dependences are shown in Fig.4. and Fig.5, respectively. For these calculations  $Y_{opt}$  is expressed by  $\Gamma_{opt}$  using reference impedance  $Z_0 = 50\Omega$ .

Table 2

f(GHz)	NF <sub>min</sub>		$\left \Gamma_{opt} ight $		∠Γ <sub>opt</sub> (°)		$r_n$	
	C	I	C	I	С	I	C	I
2	0.55	0.35	0.85	0.92	21	22	0.51	0.43
6	0.8	0.26	0.69	0.91	55	53	0.38	0.34
10	1.3	0.70	0.56	0.82	85	74	0.28	0.31
14	1.9	1.32	0.49	0.79	114	90	0.20	0.32
18	2.5	2.16	0.45	0.69	140	99	0.16	0.38

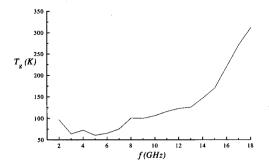


Fig. 4. Frequency dependence of  $T_{\rm g}$ 

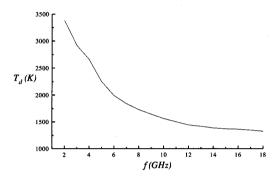


Fig. 5. Frequency dependence of T<sub>d</sub>

Frequency dependent equivalent temperatures are used for noise parameters modelling. It should be emphasised that noise modelling is done directly, without optimisation or fitting in "tuning" mode. Noise parameters of the transistor N71000A are calculated by using obtained equivalent circuit element values and temperatures, and by applying equations derived [3]. This is illustrated Figs. 6-9 and corresponding characteristics are denoted "NEW". At the same figures measured noise parameters (denoted by "MEAS") are also shown. Modelling is also done by standard approach [4], i.e. using optimisation for constant equivalent Obtained temperatures determination. values  $T_{o}=350$  K and  $T_{d}=1400$  K for the whole frequency range (these results are denoted by "CONS"). Very good agreement of measured optimum reflection coefficient and noise resistance to those modelled by here suggested procedure can be observed. Concerning optimum noise figure, quite a good agreement with measured one is observed especially at the medium of the frequency range.

### 5. Conclusions

Equivalent gate and drain noise temperatures are often used in microwave transistor noise modelling. The expressions for these temperatures in terms of frequency, intrinsic circuit elements and noise parameters are for the first time derived in this

paper. The temperature computation procedure is implemented using circuit simulator Libra. Frequency dependent noise temperatures are used for efficient transistor noise parameters modelling.

As example, the results for N71000A MESFET modelling are presented. The comparison is done among noise parameters obtained by here proposed modelling procedure, those obtained by standard constant-temperature modelling and measured values. Generally, a better agreement is observed between measured values and those modelled by proposed approach. In addition, the determination of the equivalent temperatures is done without optimization or tuning. The proposed modelling procedure enables extrapolation of noise parameters.

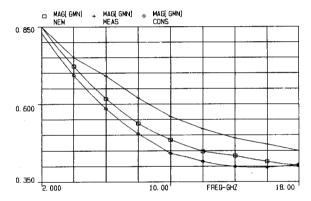


Fig. 6. Magnitude of optimum source reflection coefficient

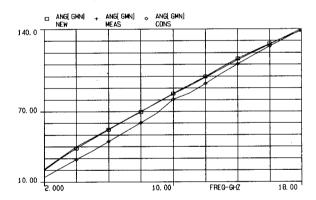


Fig. 7. Angle of optimum source reflection coefficient

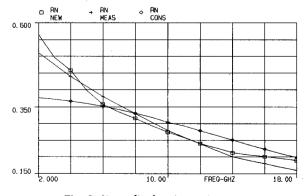


Fig. 8. Normalised noise resistance

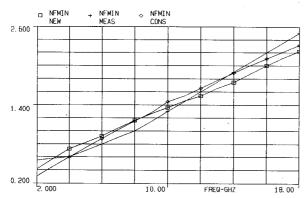


Fig. 9. Minimum noise figure

### References

- [1] Touchstone and Libra Users Manual, EEsof, Inc. 1990.
- [2] J. A.Dobrowolski, Introduction to Computer Methods for Microwave Circuit Analysis and Design, London, Artech House, 1991.

- [3] M.W. Pospieszalski, "Modelling of noise parameters of MESFET's and MODFET's and their frequency and temperature dependence", *IEEE Trans. Microwave Theory Tech.*, vol.MTT-37, pp. 1340-1350, September 1989.
- [4] V. Marković, B. Milovanović, N. Maleš, "CAD prediction of noise parameters behaviour for microwave FET devices", *CAS '94*., pp.375-378., Sinaia, Romania, October 1994.
- [5] S.W.Wedge and D.B.Rutledge, "Wave Techniques for noise modelling and measurement", *IEEE Trans. Microwave Theory Tech.*, vol.MTT-40, pp.2004-2012, November 1992.
- [6] M.T.Hickson, P.Gardner, D.K.Paul, "A semidistributed HEMT model for accurate fitting and extrapolation of S-parameters and noise parameters", *IEEE Trans. Microwave Theory Tech.*, vol.MTT-40, pp.1709-1712, August 1992.