Complex Resonant Frequencies Determination of Circular Cylindrical Cavity Loaded by One Lossy Dielectric Slab

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Abstract

A cylindrical metallic cavity of circular cross-section loaded by one lossy dielectric slab is investigated. The effect of the load in form of a homogeneous lossy dielectric slab on the resonant frequencies is analysed. The characteristic equation with unknown complex frequency is obtained from the transverse resonance condition. The complex resonant frequencies are determined directly, by numerically solving this transcendental equation. Besides, the complex resonant frequencies are determined indirectly, by using the method presented in reference [1]. Calculated results are analysed and compared. The resonant frequencies and damping factor curves versus filling factor are shown in this paper. Also, the influence of the losses in dielectric slab on the resonant frequencies is investigated.

Introduction

Modelling of a cylindrical cavity with a homogeneous dielectric load is interesting for experimental and theoretical analysis. In practice, this load form is used in the processes of microwave heating of dielectric. Tuning mode behaviour under loading condition of the cylindrical cavity has been the research subject of a number of authors. In general, these results have been addressed to the rectangular cylindrical cavity [1,2,3,4]. In references [2,3,4] the loading effect of lossy dielectric ($\varepsilon_r = \varepsilon' - j\varepsilon''$) is analysed using impedance/admittance matching condition. The resonant frequencies have been treated as real and they have been determined by solving only the imaginary part of characteristic equation. But, when the cavity is loaded with lossy dielectric its resonant frequencies become complex. The complex angular resonant frequency ($\underline{\omega} = \omega_r + j\omega_i$, where the real part, ω_r, denotes the angular resonant frequency, and the imaginary part, ω_i, denotes the damping factor of the cavity) may be determined from corresponding characteristic equation. It can be formulated using the transverse resonance method as in reference [1,2,3,4]. The complex resonant frequency may be determined directly, but this procedure demands the complex transcendental equation solving by appropriate numerical technique, supposing that the starting values are properly chosen. In reference [1] the complex resonant frequencies for a rectangular cavity are determined indirectly, by using the method of simplified calculation.

In this paper the mode tuning effects of circular cylindrical cavity loaded by one homogeneous lossy dielectric slab located on the cavity floor (Fig.1) are investigated. The complex frequencies have been determined directly, by numerically solving the corresponding characteristic equation and indirectly, by using the method presented in [1]. Calculated results are analysed and compared. The resonant frequency curves versus filling factor are shown for TM_{11p} and TE_{11p} modes, where p denotes axial mode number of the cavity. The damping factor curves of several modes versus filling factor are shown, too. Besides, the influence of the losses in dielectric slab to the resonant frequencies is investigated.

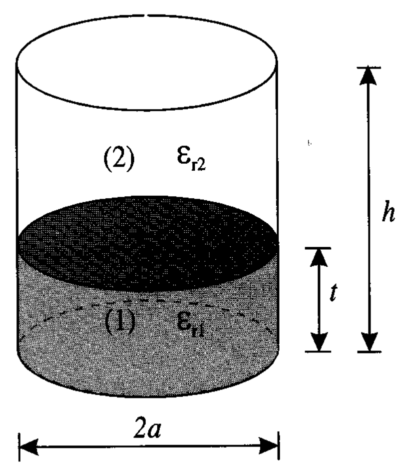


Fig. 1. A circular cylindrical cavity loaded by one lossy dielectric slab of the thickness t

Theoretical Analysis

The analysed cylindrical cavity loaded by one lossy homogeneous dielectric slab may be represented by equivalent circuit model (Fig.2.), which consists of two equivalent transmission lines connected in cascade. These sections are the homogeneous transmission lines with characteristic impedances $\underline{Zc}_a$ and $\underline{Zc}_b$, and propagation coefficients $\underline{\gamma}_a$ and $\underline{\gamma}_b$, corresponding to the wave impedance and propagation coefficients of a waveguide of circular cross-section, respectively.

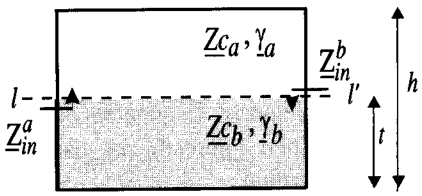


Fig. 2. Equivalent transmission line model of a loaded cavity (l - l' is the reference plane).

The characteristic equations of considered cavities may be obtained from the transverse resonance method. For the reference plane *l-l'* transverse resonance condition can be written in the form:

$$\underline{Z}_{in}^{b} + \underline{Z}_{in}^{a} = 0. \qquad (1)$$

Index *b* refers to the input impedance Z_{in} of the lower part of the cavity below the reference plane, and index *a* refers to the input impedance Z_{in} above the reference plane. The input impedances $\underline{Z}_{in}^{b}$ and $\underline{Z}_{in}^{a}$ can be easily found by using the well known line impedance transformation starting from the two short circuits and finishing at the selected interface at which the resonance condition is applied. So, equation (1) becomes:

$$\underline{Zc}_b\, th(\underline{\gamma}_b t) + \underline{Zc}_a\, th(\underline{\gamma}_a (b - t)) = 0, \qquad (2)$$

and the complex resonant frequencies may be calculated from follow equations:
for TM_{mn} modes

$$\frac{\underline{\gamma}_b}{j\underline{\omega}\varepsilon_0\underline{\varepsilon}_{r1}}\, th(\underline{\gamma}_b t) + \frac{\underline{\gamma}_a}{j\underline{\omega}\varepsilon_0\underline{\varepsilon}_{r2}}\, th(\underline{\gamma}_a (b - t)) = 0, \qquad (3)$$

and for TE_{mn} modes

$$\frac{j\underline{\omega}\mu}{\underline{\gamma}_b}\, th(\underline{\gamma}_b t) + \frac{j\underline{\omega}\mu}{\underline{\gamma}_a}\, th(\underline{\gamma}_a (b - t)) = 0. \qquad (4)$$

The propagation coefficient γ_i (for lower ($i = b$) and upper ($i = a$) dielectric slab) is determined by the expression:

$$\underline{\gamma}_i^2 = \underline{\omega}^2\varepsilon_0\mu_0\underline{\varepsilon}_{ri} - k_c^2 \qquad (5)$$

The constant k_c is governed by the dimensions of the cylindrical cavity and is given by the expression [5]:

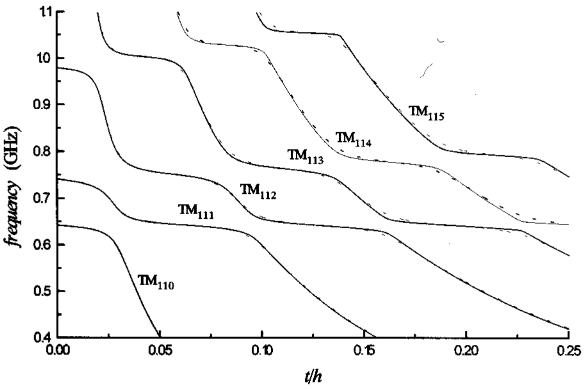
$$k_c = x_{mn} / a , \qquad (6)$$

where x_{mn} represents the *n*-th zero of the first kind Bessel function of order *m* for the TM_{mn} modes, and the *n*-th zero of the first derivation of the same Bessel function for the TE_{mn} modes. The obtained equations (3) (for TM_{mn} modes) and (4) (for TE_{mn} modes) are complex transcendental equations and could be solved by an appropriate numerical technique.

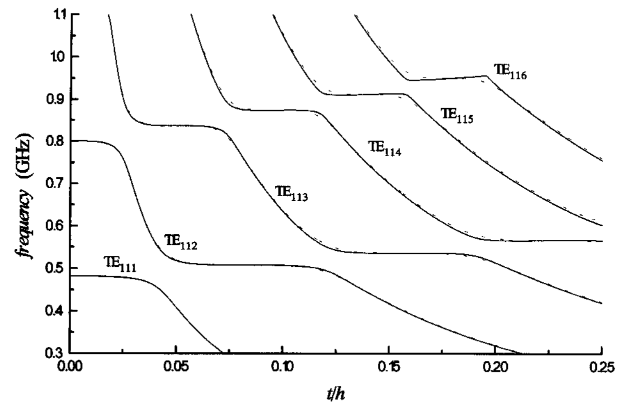
Xu and Bosisio are suggested in reference [1] a simplified calculation method. They expand the complex angular resonant frequency into a Taylor series, when the imaginary part of the dielectric constant ε" is small compared to its real part ε':

$$\omega \approx \omega\Big|_{\varepsilon''=0} - j\frac{\delta\omega}{\delta\varepsilon'}\Big|_{\varepsilon''=0}\varepsilon'' + \frac{1}{2}\frac{\delta^2\omega}{\delta\varepsilon'^2}\Big|_{\varepsilon''=0}(-j\varepsilon'')^2 + \frac{1}{6}\frac{\delta^3\omega}{\delta\varepsilon'^3}\Big|_{\varepsilon''=0}(-j\varepsilon'')^3 + \ldots \qquad (7)$$

From equation (7) it is clear that resonant frequency could be calculated by solving only the real transcendental equation (by putting ε" = ω" = 0 in equation (3 or 4) and (5)), and calculating the differentials of its solution with respect to ε'. Putting ω complex is to make the formulation strict and the use of a Taylor series expansion is a method of simplified calculation.



a)



b)

Fig. 3. The resonant frequencies of circular cylindrical cavity versus filling factor t/h for a) TM_{11p} (p = 0, ...,5) and b) TE_{11p} (p = 1, ...,6) modes:
—— lossy dielectric (ε_{r1} = 77-j10),
- - - - lossless dielectric (ε_{r1} = 77).

Numerical Results

Dimensions of the analysed circular cavity (Fig.1.) are a = 28.5 cm and b = 40.6 cm; the dielectric constants of the lower and upper dielectric slabs are ε_{r1} = 77-j10 and ε_{r2} = 1, respectively. The thickness of lossy dielectric slab is t.

A computer program has been developed for the complex frequencies calculation. The transcendental equations (3) (for TM_{mn} modes) and (4) (for TE_{mn} modes) are solved numerically using Newton-Raphson method. To investigate the influence of the losses in dielectric slab on the resonant frequencies, the case of the lossless dielectric slab (ε_{r1} = 77) is considered, too. The behaviour of the resonant frequencies versus filling factor, t/h, are shown in Fig.3a) and 3b) for TE_{11p} and TM_{11p} modes, respectively, where p denotes axial mode number of the cavity. The curves for lossless dielectric slab (ε_{r1} =77) are marked as dotted and the curves for lossy dielectric slab (ε_{r1} = 77-j10) are marked as solid. The resonant frequency curves for both modes have the similar behaviour: they are continuous and decrease gradually with the filling factor increasing. It can be seen that the presence of losses in dielectric in some parts of the curves leads to the increasing and in the other parts leads to the decreasing of the resonant frequencies regarding to the curves for the case of lossless dielectric. These deviations are more noticeable as axial mode number increases and when the dielectric losses are bigger, what can be seen from Fig.4, where the behaviour of the resonant frequency for TM_{111} mode is illustrated in the frequency range from 0.55 to 0.65 GHz for several values of losses (ε'' = 0,5,10).

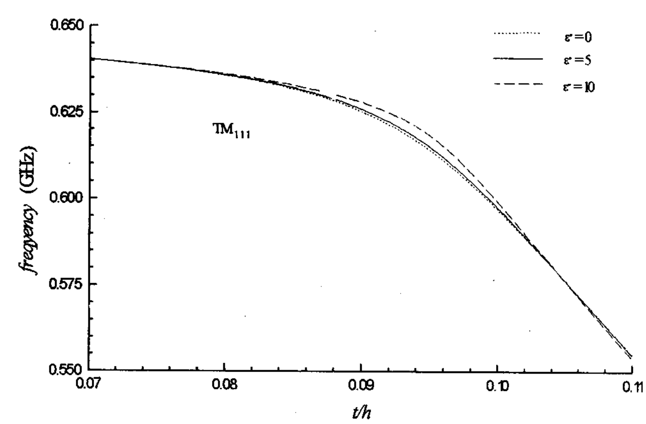


Fig. 4. The influence of losses in dielectric slab (ε'' = 0,5,10) to the resonant frequencies for TM_{111} mode

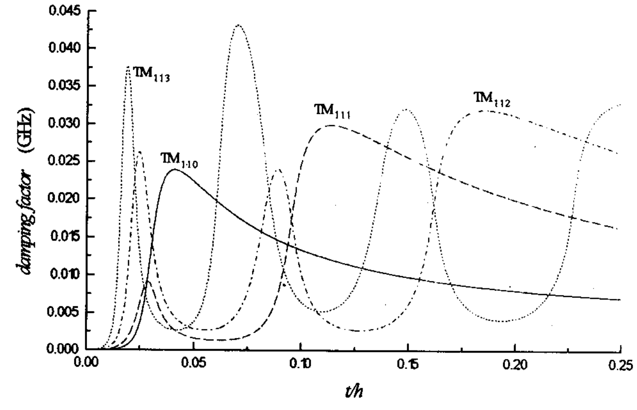


Fig. 5. The damping factor of circular cylindrical cavity loaded by lossy dielectric (ε_{r1} = 77-j10) versus filling factor t/h for several TM_{11p} (p = 0,1,2,3) modes

The damping factor curves versus filling factor are shown in Fig. 5. for several modes. It can be seen that the curves ripple depends on the axial mode number and increases as axial mode number increases.

Besides, the resonant frequencies are determined indirectly, using the method presented in [1]. In this aim a corresponding computer program has been developed, too. At first, under assumption that the dielectric slab is lossless ($\varepsilon'' = 0$) the dependence between resonant frequency and dielectric constant of the slab is determined and shown in Fig.6a) and 6b) for TE_{11p} and TM_{11p} modes, respectively. Identification of the resonant modes is done as suggested in [6]. Further, from equation (7) the complex resonant frequencies are determined, where:

a) only the first two terms,
b) the first four terms,
c) the first six terms and
d) the first eight terms of Taylor series are considered.

For these cases, the results of calculation for TM_{114} mode are shown in Table 1, for selected values of t/h. The complex resonant frequencies for same mode and same selected values of t/h determined directly, solving transcendental equation (3), are shown in Table 1, too. The analysis of these results shows that the results obtained by simplified calculation method, using more terms in Taylor series, converge to the results obtained by numerically solving the characteristic equation (3). It may be seen, that in our case the accuracy is satisfactory using only the first four terms of Taylor series. For higher dielectric losses, accuracy of calculation may be further enhanced by using more terms in equation (7).

Conclusion

Mode tuning behaviour of the circular cylindrical cavity loaded by one lossy dielectric slab located on the cavity floor is investigated. The analysis is made assuming that in the case when the cavity is loaded with lossy dielectric its resonant frequencies become complex. The complex resonant frequencies are determined directly, by numerically solving the characteristic equation. Besides, the complex resonant frequencies are determined indirectly, using Taylor series [1]. The calculated results obtained by using both methods are compared, and the influence of number terms in Taylor series to the convergence to the solution obtained by numerically solving complex characteristic equation is analysed. Accuracy of the simplified calculation method is satisfactory, and agreement with results obtained directly grows by using more terms in Taylor series.

The resonant frequency curves versus filling factor are shown for TE_{11p} and TM_{11p} modes, where p denotes axial mode number of the cavity. The analysis of these results show that the curves of resonant frequencies for both modes have the similar behaviour: they are continuous and decrease gradually with the filling factor increasing. The presence of losses in dielectric in some parts of the curves leads to the increasing and in the other parts leads to the decreasing of the resonant frequencies regarding to the curves for the case of lossless dielectric. These deviations are more noticeable as axial mode number increases and when the dielectric losses are bigger. The damping factor curves versus filling factor are shown for selected modes. It can be seen that the ripple of shown curves depends on the axial mode number and increase as axial mode number increase.

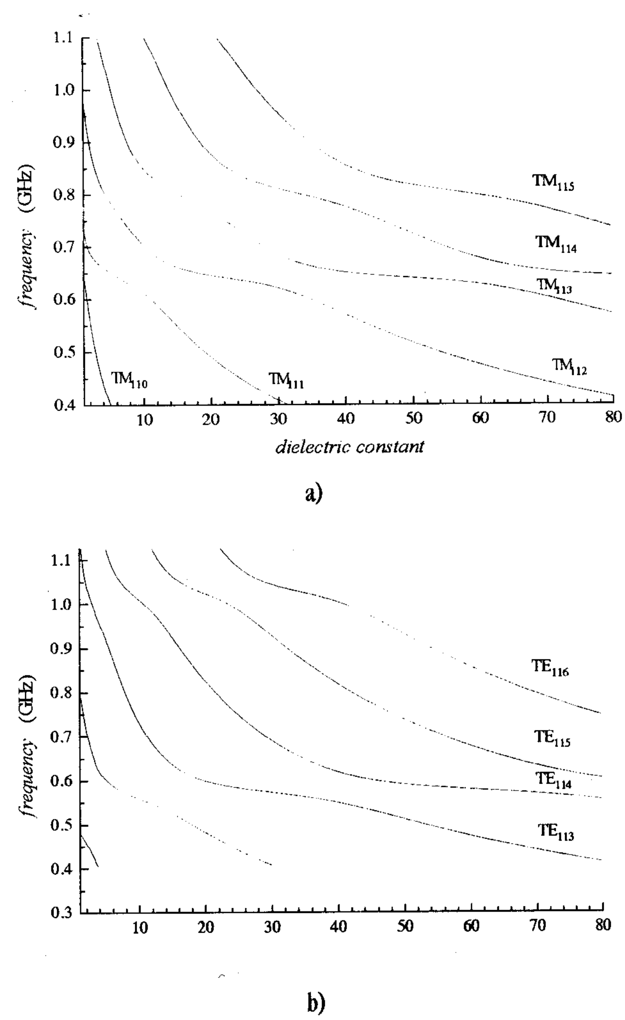


Fig. 6. Dependence between resonant frequency and dielectric constant of the slab for t/h=0.25 for a) TM_{11p} ($p = 0, ..., 5$) and b) TE_{11p} ($p = 1, ..., 6$) modes

Table 1. The results for TM_{114} mode obtained indirectly, using simplified calculation method and directly, by numerically solving the complex transcendent equation (3)

	filling factor t/b	0.05	0.10	0.15	0.20	0.25
$\frac{\omega_r}{2\pi}$ (GHz)	indirectly with 2 terms of Taylor series	1.23641	1.00538	0.78701	0.72600	0.64714
	indirectly with 4 terms of Taylor series	1.24033	1.01356	0.78334	0.72611	0.64488
	indirectly with 6 terms of Taylor series	1.23990	1.01485	0.78373	0.72564	0.64535
	indirectly with 8 terms of Taylor series	1.23966	1.01529	0.78372	0.72567	0.64538
	directly by solving the equation (3)	1.23967	1.01532	0.78373	0.72559	0.64532
$\frac{\omega_i}{2\pi}$ (GHz)	indirectly with 2 terms of Taylor series	0.05081	0.02677	0.01133	0.03449	0.00712
	indirectly with 4 terms of Taylor series	0.05398	0.02682	0.00991	0.03638	0.00579
	indirectly with 6 terms of Taylor series	0.05423	0.026745	0.00997	0.03642	0.00600
	indirectly with 8 terms of Taylor series	0.05446	0.02698	0.00999	0.03649	0.00610
	directly by solving the equation (3)	0.05419	0.02655	0.00999	0.03631	0.00597

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