Lattice Wave Digital Filters with a Reduced Number of Multipliers

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Abstract

In this paper, a new design method for lattice wave digital filters (WDF) which provides the implementation of a half of multiplication constants with few shifters and adders is proposed. The lattice WDF can be designed to have in all second order sections of the lattice branches, one common constant independent of the filter order and transition bandwidth. The value of the common constant depends only on the frequency for which the filter attenuation is 3dB and may be adjusted according to the predetermined number of shift-and-add operations. A very efficient filter is also obtained for the separation of 1/3 of the used band.

I Introduction

Wave digital filters (WDFs) are derived from real lossless reference analog filters and, if properly designed, behave completely like passive circuits [1], [2], [3], [4]. The advantages of WDFs are: excellent stability properties even under nonlinear operating conditions resulting from overflow and roundoff effects, low coefficients wordlength requirements, inherently good dynamic range, etc., [4]. For a proper design of WDFs a solid mathematical basis developed for classical synthesis techniques (including those for microwave filters) is at disposal. Moreover, explicit formulas for Butterworth, Chebyshev, inverse Chebyshev and elliptic filters are derived [4].

For WDFs there exists a great number of different structures according to the realization possibilities of reference filters [2]. The design method for lattice WDFs where both lattice branches are realized by cascaded first- and second-degree allpass sections is presented in [4]. The number of multipliers in resulting lattice WDF is equal to the filter order what is a minimum in comparison with the number of multipliers required for other possible realizations of a given transfer function. Therefore, any reduction in the number of multipliers in a lattice WDF contributes to the computational efficiency of IIR filters.

The purpose of this paper is to present a new design method which provides the replacement of a half of multipliers in a lattice WDF with a few shifters and adders (or with shifters only). The investigations are restricted to elliptic function digital filters including Butterworth filter as a boundary case. The reason for choosing elliptic filters is twofold. It is well known that an elliptic IIR filter can achieve a sharper transition between band edges than any other filter with the same number of coefficients. The second reason is the possibility to design an elliptic filter transfer function with z-plane poles located on the circle in the z plane [5]. Consequently, the obtained disposition of the poles can be used to adjust the values of a part of multiplication constants. As a boundary case, the Butterworth filter also belongs here.

The design method which provides the reduction of the number of multipliers in lattice WDFs exists only for half-band (bi-reciprocal filters) [4], [6], [7] and can be achieved for elliptic and Butterworth filters. The Butterworth filter and elliptic filter with equal tolerances in pass- and stop-bands can fulfill half-band filter requirements. It is shown in [5] that a half-band filter is only a boundary case of an IIR filter derived by the bilinear transformation from an elliptic minimal Q-factors analog prototype [8] or from a Butterworth prototype. Generally, those filters have their z-plane poles on the circle which in the half-band filter case degenerates to the imaginary axis. Thus, one should examine the possibilities to apply an elliptic minimal Q-factors transfer function for an arbitrary lattice WDF design in order to reduce the number of multiplication constants.

It is shown in this paper that a lattice WDF, if derived from an elliptic minimal Q-factors analog prototype or from a reference Butterworth filter, can be designed to have in all second order sections of the lattice branches, one common constant independent of the filter order and transition bandwidth. The value of the common constant depends only on the frequency for which the filter attenuation is 3dB and may be adjusted according to the predetermined number of shift-and-add operations. This way, in the majority of practical filters, the half of multiplications has to be evaluated only by two shifts and one addition or even by only one shift.

This paper is divided into six major sections. In the next section, some basic definitions concerning WDFs are briefly recapitulated. In the third section, the synthesis using first- and second-degree allpass sections is described. The WDFs design using an elliptic minimal Q-factors analog prototype is presented in the fourth section. Several examples illustrating simplicity and efficiency of the proposed design are given in the fifth section.

II Basic definitions for lattice WDFs

The principles of WDFs have been described in [3]. In this section, only basic definitions, as given in [4], will be repeated.

The WDFs are derived from a real lossless reference filter using the voltage wave quantities [1]. The lattice WDF is derived from a real symmetric two-port equally resistively terminated [3], [9]. The reference filter is an analog prototype designed in *s* variable. The transfer function of a lattice WDF is obtained by the bilinear transformation:

$$s = \frac{z-1}{z+1} \qquad (1)$$

and replacing $s=j\Omega$ and $z=e^{j\omega T}$, the relation between the frequencies of reference filter and the frequencies of WDF is obtained:

$$\Omega = \tan\left(\frac{\omega T}{2}\right) \qquad (2)$$

where $T=1/f_s$ and f_s is the sampling frequency.

In both lattice branches of the lattice WDF (Fig. 1) $S_1(s)$ and $S_2(s)$ are reflectances of reactances, i.e., allpass functions. Therefore, they may be written in the following form:

$$S_1(s) = \frac{g_1(-s)}{g_1(s)} \qquad (3)$$

and

$$S_2(s) = \frac{g_2(-s)}{g_2(s)} \qquad (4)$$

where $g_1(s)$ and $g_2(s)$ are Hurwitz polynomials of degree n_1 and n_2, respectively.

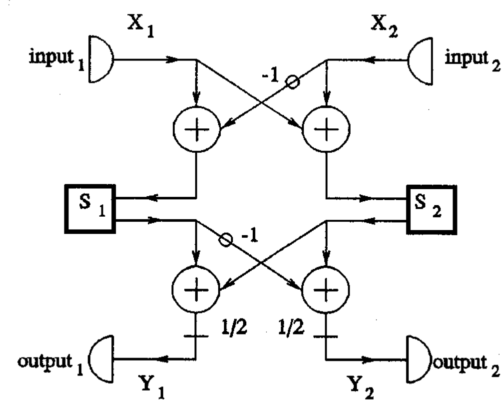


Fig. 1. Wave-flow diagram of a lattice WDF

It is well known that the transfer functions which are realized by these WDFs are given by:

$$S_{11}(s) = S_{22}(s) = \frac{S_1(s) + S_2(s)}{2} = \frac{h(s)}{g(s)} \qquad (5)$$

$$S_{21}(s) = S_{12}(s) = \frac{S_2(s) - S_1(s)}{2} = \frac{f(s)}{g(s)} \qquad (6)$$

where h(s), f(s) and g(s) are the so-called canonic polynomials.

We consider in the following the low-pass or high-pass case, and we will suppose that n_1 is odd and n_2 is even. The opposite choice would simply amount to changing the sign in [6] and this possibility can be ignored. Interchanging $g_1(s)$ and $g_2(s)$ in (3) and (4) would simply lead to the dual realization, therefore this possibility will also be ignored.

From (3), (4), (5) and (6) we can see that

$$g(s) = g_1(s) \times g_2(s) \qquad (7)$$

i.e., g(s) is a Hurwitz polynomial of degree n where $n=n_1+n_2$. Consequently, n must be always an odd number for the low-pass (high-pass) filters. The degree of the lattice WDF [3] is the sum of the degrees of the two reflectances $S_1(s)$ and $S_2(s)$.

Further, from (3), (4), (5) and (6) it is clear that

$$h(s) = \frac{g_1(-s)g_2(s) + g_1(s)g_2(-s)}{2} \tag{8}$$

and

$$f(s) = \frac{g_1(s)g_2(-s) + g_1(-s)g_2(s)}{2} \tag{9}$$

i.e., h(s) and f(s) are even and odd polynomials, respectively.

It is known, that the transfer functions are related at real frequencies $s=j\omega$ by the Feldkeller equation:

$$\left|S_{11}(j\Omega)\right|^2 + \left|S_{21}(j\Omega)\right|^2 = 1 \tag{10}$$

The attenuation (loss) is defined by

$$a(\Omega) = -20 \log |S_{21}(j\Omega)| \tag{11}$$

The filter characteristic function is defined by

$$C(s) = \frac{S_{11}(s)}{S_{21}(s)} = \frac{h(s)}{f(s)} \tag{12}$$

III Synthesis using cascaded allpass functions

In this paper we will consider the realization as a cascade of elementary sections by means of circulators [1]. The elementary sections are the first- and second-degree allpass sections. A section of degree one has a reflectance of the following form:

$$S(s) = \frac{-s + B_0}{s + B_0} \tag{13}$$

and a corresponding signal-flow diagram of a wave digital realization using the so-called two-port adaptor is given in Fig. 2 where R is the port resistance and the multiplier coefficient γ_0 is given by [1], [4]

$$\gamma_0 = \frac{1 - B_0}{1 + B_0} \tag{14}$$

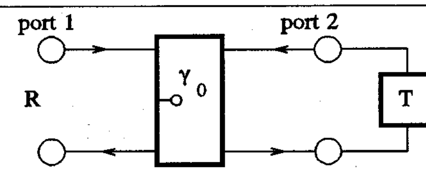
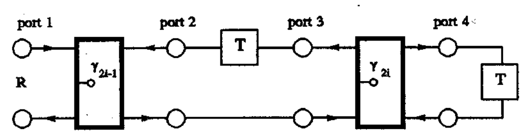


Fig. 2. Wave-flow diagram of an allpass section of degree one



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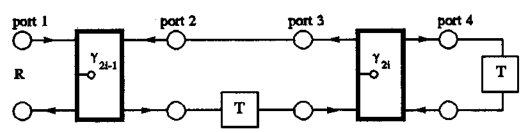


Fig. 3. Equivalent wave-flow diagrams of the *i*th second-order allpass section

A second degree allpass section has a reflectance of the form

$$S(s) = \frac{s^2 - A_i(s) + B_i}{s^2 + A_i(s) + B_i} \tag{15}$$

and using the two-port adaptors the corresponding wave digital realization has equivalent wave-flow diagrams given by Fig. 3, where the coefficient values are given by [1], [4]

$$\gamma_{2i-1} = \frac{A_i - B_i - 1}{A_i + B_i + 1} \tag{16}$$

and

$$\gamma_{2i} = \frac{1 - B_i}{1 + B_i} \tag{17}$$

Hurwitz polynomial g(s) can be presented in a product form

$$g(s) = (s + B_0) \prod_{i=1}^{(n-1)/2} (s^2 + A_i s + B_i) \tag{18}$$

where A_i and B_i are determined by the transfer function pole s_i:

$$A_i = -2 \text{ Re } (s_i) \tag{19}$$

and

$$B_i = | s_i |^2 \tag{20}$$

In Fig. 4, the signal-flow diagrams of two-port adaptors which always lead to the scaling in the best possible way are shown [4]. We can observe that a different structure can be chosen depending on the multiplier value γ. Furthermore, we can see that the multiplier coefficient α which has to be implemented is always positive and not larger than one-half.

It is proved in reference [4] that the poles are alternately distributed among $g_1(s)$ and $g_2(s)$. According to this property and using only the parameters defined in (18), all adaptor coefficients can be computed by (14), (16) and (17). Therefore, the adaptor coefficients are determined by the transfer function poles of the analog reference filter.

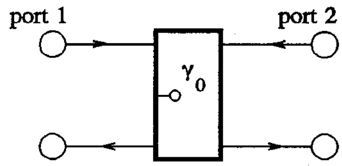
In the next section, it will be demonstrated that for one class of WDF's the filter design can be even simpler if based strictly on the z-plane pole parameters.

IV WDFs design using an elliptic minimal Q-factors analog prototype

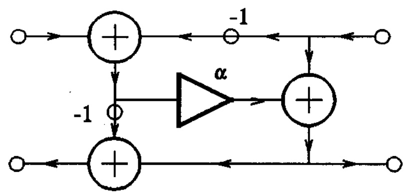
Equation (17) presents a direct relation between the radius of the reference filter pole and the adaptor coefficient γ_{2i}. It is clear from (17) and (20) that for the poles placed on the circle whose center is at the origin of the s-plane, the coefficients γ_{2i} are equal for all second-degree allpass sections, what is already obtained for Butterworth filters in [4]. In the following this property will be extended to the elliptic filters.

It is shown in [8], that the poles of an elliptic minimal Q-factors filter are placed on the circle with the center at the origin of the s plane. The radius of the circle is $\sqrt{\Omega_a}$, where Ω_a is the normalized stop band edge frequency and for $s=j\sqrt{\Omega_a}$ the filter has the attenuation of 3 dB.

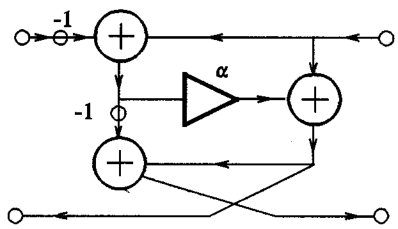
Let the required digital filter specifications be given with boundary frequencies for the pass-band F_p and F_a for the stop-band, pass-band ripple A_p and minimal stop-band attenuation A_a, expressed in dB, as shown in Fig. 5.



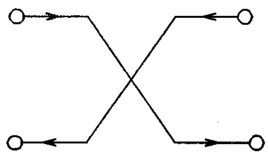
two port adaptor γ



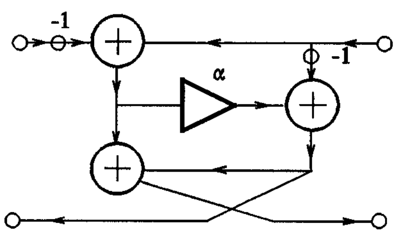
a) $\alpha = 1-\gamma$ for $1/2 < \gamma < 1$



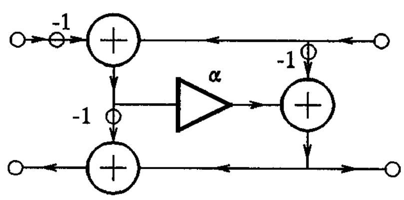
b) $\alpha = \gamma$ for $0 < \gamma < 1/2$



c) $\gamma = 0$



d) $\alpha = -\gamma$ for $-1/2 < \gamma < 0$



e) $\alpha = 1+\gamma$ for $-1 < \gamma < -1/2$

Fig. 4. Signal-flow diagrams of two-port adaptor yielding optimal scaling for sinusoidal excitation

As shown in Fig. 5, some margin in the filter performance always exists, what permits some freedom in the choice of the transfer function boundary frequencies f_p and f_a. For a proper design, it is necessary to establish the relation between f_p and f_a and the stop-band edge frequency of the analog prototype Ω_a. From (2), it follows directly that:

$$\Omega_a = \frac{\tan \pi f_a}{\tan \pi f_p} \tag{21}$$

where the pass-band edge frequency of the analog prototype is assumed to be unity.

For the elliptic filters derived from [8], the tolerances of the square magnitude function $|H(e^{j\omega})|^2$ in the pass- and stop-band are equal:

$$\delta_p = \delta_a = \frac{1}{1+L} \tag{22}$$

where L is the module of an elliptic filter characteristic function:

$$L = \sqrt{\frac{\left|H(e^{j2\pi f_a})\right|^2 - 1}{\left|H(e^{j2\pi f_p})\right|^2 - 1}} \tag{23}$$

L is uniquely determined by Ω_a from (21) and can be calculated from [10].

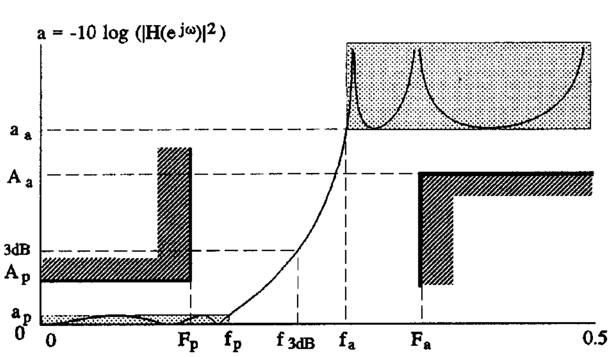


Fig. 5. A typical elliptic filter

The pass- and stop-band attenuation of the realized filter:

$$a_p \le A_p \quad \text{and} \quad a_a \ge A_a \tag{24}$$

are also uniquely determined by L, i.e. Ω_a.

$$a_p = 10\log(1+1/L), \quad a_a = 10\log(1+L) \tag{25}$$

Usually, but not necessarily, we have $f_p \ge F_p$ and $f_a \le F_a$.

The frequency where a digital filter has the attenuation of 3 dB corresponds in the analog filter domain to the frequency $\sqrt{\Omega_a}$, and can be directly determined by the relation

$$\tan^2 \pi f_{3dB} = \tan \pi f_p \tan \pi f_a \tag{26}$$

It is shown in [5], that the poles of an elliptic IIR filter, derived by the bilinear transformation from an analog minimal Q-factors prototype [8], are placed on the circle which is orthogonal with the unit circle and has the center on the real axis of the z plane at the point x_0, Fig. 6. For the corresponding frequencies $\sqrt{\Omega_a}$ (*s* plane) and f_{3dB} (*z* plane), x_0 is obtained directly from [5]:

$$x_0 = \frac{1+\tan^2 \pi f_{3dB}}{1-\tan^2 \pi f_{3dB}} = \frac{1}{\cos 2\pi f_{3dB}} \tag{27}$$

For an arbitrary complex pole $z_i = r_i\, e^{j\theta i}$ by the elimination of the vertical leg $\{x_i, z_i\}$ of right triangles $\{0, x_i, z_i\}$, $\{x_0, x_i, z_i\}$, Fig. 6, the following useful relation can be established:

$$2r_i \cos\theta_i = \frac{1+r_i^2}{x_0} = (1+r_i^2)\cos 2\pi f_{3dB} \tag{28}$$

The analysis presented in this section can be used for the computation of the adaptor coefficients γ_0, γ_{2i-1} and γ_{2i} by the filter parameters defined in the z-plane.

Let us observe equations (14), (16) and (17).

In equation (14), γ_0 is determined by the coefficient B_0 which presents the real first order pole (6), (18). For a minimum Q-factors elliptic filter, the real pole is placed at the intersection of the negative part of the real axis and the circle $(0, \sqrt{\Omega_a})$. Hence, instead of (14) we can write:

$$\gamma_0 = \frac{1-\sqrt{\Omega_a}}{1+\sqrt{\Omega_a}} \tag{29}$$

The digital filter frequency f_{3dB} corresponds in the s plane to $s=j\sqrt{\Omega_a}$, and using the equivalence from (2), equation (29) becomes:

$$\gamma_0 = \frac{1 - \tan \pi f_{3dB}}{1 + \tan \pi f_{3dB}} \qquad (30)$$

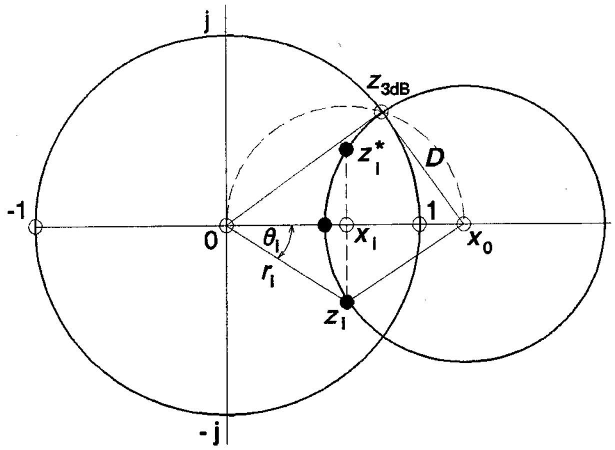


Fig. 6. Poles loci in the z plane

Complex poles s_i, $i=1,2,\ldots(n-1)$, are placed on the circle $(0,\sqrt{\Omega_a})$ in the left half of the s plane. Therefore all B_i, $i = 1, 2, \ldots(n-1)/2$, in (17) and (18) are equal and consequently, all adaptor coefficients γ_{2i} are equal for $i=1,2, \ldots(n-1)/2$. Substituting $B_i=\Omega_a=\tan^2 f_{3dB}$ in equation (17), the expression for γ_{2i} can be obtained:

$$\gamma_{2i} = \frac{1 - \tan^2(\pi f_{3dB})}{1 + \tan^2(\pi f_{3dB})} = \cos(2\pi f_{3dB}) = \frac{1}{x_0} \qquad (31)$$

$i=1,\ldots,(n-1)/2$

The same expression has been derived in [4] for Butterworth filters. Equation (31) is a generalization of the result from [4] and signifies that the equal values for the adaptor coefficients γ_{2i} are obtained for all transfer functions whose poles are placed on the circle. It is to be noticed that γ_{2i} is uniquely determined by f3dB, and f3dB depends only on the boundary frequencies f_p and f_a.

The expression for γ_{2i-1} given by (16) can be modified if the parameters A_i and B_i are represented by the corresponding z-plane parameters. Applying the already established relation $B_i=\Omega_a=\tan^2(\pi f_{3dB})$ and expressing A_i by the bilinear transformation (1) and using (28), equation (16) may be simplified to the form:

$$\gamma_{2i-1} = -r_i^2, \qquad i=1,2,\ldots, (n-1)/2 \qquad (32)$$

where r_i is the radius of the ith pole in the z plane.

Computations of γ_0, γ_{2i-1} and γ_{2i} from (30), (31) and (32) are based on the z-plane poles parameters what permits the application of the conventional computer programs for IIR digital filters. Obviously, the computer program has to be developed on the basis of bilinear transformation.

It is clear (Fig. 5), that filter specifications are fulfilled for various combinations of the elliptic filter boundary frequencies f_p and f_a. This property offers the opportunity to select a convenient value (with a minimal number of adders) for γ_{2i} and afterwards to determine the new values for f_p and f_a in order to check whether the specifications are fulfilled. By increasing the filter order, the bandwidth where f_p and f_a may be placed is extended.

One should notice some additional properties:

1. The absolute value of the constant, $|\gamma_{2i}|=|1/x_0|$ is always smaller than unity, because the center of the z-plane poles loci x_0, is always placed outside the unit circle.

2. The distance of the z-plane poles from the unit circle is nearly maximal if compared with elliptic filters having another pass-band ripple [5], what is of importance for the implementation.

3. Expression (31) is independent of the filter order and of the transition band what permits, by an appropriate choice of f_{3dB}, adjusting the value of the adaptor coefficient γ_{2i}. This way, a half of multipliers in the obtained WDF have the same value which, by a proper choice of a single frequency, can be adjusted according to the minimal number of shift-and-add operations without any influence on the other filter characteristics.

4. The choice $f_{3dB}=1/4$, gives the half-band (bireciprocal) filter described in [4], [6], [7], [11]. According to (31), x_0 becomes infinite what means that the poles loci in the z plane, the circle with the center at $z=x_0$, degenerates to the imaginary axis. Consequently, all adaptor coefficients γ_{2i}, $i=1, 2,\ldots, (n-1)/2$, are set to zero. This is to say, that a half-band WDF is derived from a minimal Q-factors analog prototype by the appropriate choice of s-plane/ z-plane equivalent frequencies.

V Design examples

From given F_p and F_a the initial guess for f_{3dB} and also the range of the permitted values for γ_{2i} are to be determined:

$$\tan^2 \pi f_{3dB} = \tan \pi F_p \tan \pi F_a . \tag{33}$$

$$\gamma_{2i} \approx -\frac{1-\tan^2 \pi f_{3dB}}{1+\tan^2 \pi f_{3dB}}$$

$$-\frac{1-\tan^2(\pi F_p)}{1+\tan^2(\pi F_p)} < \gamma_{2i} < -\frac{1-\tan^2(\pi F_a)}{1+\tan^2(\pi F_a)} \tag{34}$$

The first step is to see if anyone of γ_{2i} from Table 1 belongs to the range defined in (34), and is also close to the approximate value from (34).

Similarly to Table 1, another table can be created presenting the values which can be made by the sum or difference of two coefficients. The next step is to see if from the new table, a value for γ_{2i} can be selected such as to lie in the range (34) and to be as close as possible to the approximate value from (34). For the examples from [12] and [9], the values of the parameters obtained by this procedure are given in [13].

Table 1.

γ_{2i}	γ_0	f_{3dB}
$-1/2$	$-0.267949 \approx -1/2^2-1/2^6$	$0.166667=0.5/3$
$-1/2^2$	$-0.127949 \approx -1/2^3-1/2^8$	0.209785
$-1/2^3$	$-0.0627461 \approx -1/2^4$	0.230053
$-1/2^4$	$-0.0312806 \approx -1/2^5$	0.240046
$-1/2^5$	$-0.0156288 \approx -1/2^6$	0.245026
$-1/2^6$	$-0.00781298 \approx -1/2^7$	0.247513
$-1/2^7$	$-0.00390631 \approx -1/2^8$	0.248757
$-1/2^8$	$-0.00195313 \approx -1/2^9$	0.249378
0	0	$0.25=0.5/2$
$1/2^8$	$0.00195313 \approx 1/2^9$	0.250622
$1/2^7$	$0.00390631 \approx 1/2^8$	0.251243
$1/2^6$	$0.00781298 \approx 1/2^7$	0.252487
$1/2^5$	$0.0156288 \approx 1/2^6$	0.254974
$1/2^4$	$0.0312806 \approx 1/2^5$	0.259954
$1/2^3$	$0.0627461 \approx 1/2^4$	0.269947
$1/2^2$	$0.127949 \approx 1/2^3+1/2^8$	0.290215
$1/2$	$0.267949 \approx 1/2^2+1/2^6$	$0.333333=1/3$

Important remarks:

a) If for γ_{2i} a single power of 2 is used ($1/2^6$ for example), it practically means that there are neither multiplications nor additions, but only shifting.

b) If γ_{2i} is presented with two powers of two ($1/2^3+1/2^2$ for example), then the multiplication is replaced with two shiftings and one addition.

c) Usually the filter order is increased if compared with a classical design, but the number of multipliers is reduced, [13].

d) For $|1/x_0|<<1$, the real pole can be implemented with shifting only (Table 1), what reduces the number of multipliers by one more. As a consequence, the passband ripple is increased, but the resulting increase is usually negligible if compared with A_p.

e) a_p is usually considerably smaller than A_p what increases the pass-band margin. This consequently, means that the rounding of the other multiplication constants (γ_{2i-1}) to the convenient values affects the stop-band characteristic rather than that of the pass-band.

f) For the computation of poles, conventional computer programs for digital filters can be used if based on the bilinear transformation [4], [5], [14]. Afterwards, it is necessary to distribute the z-plane poles among two lattice branches. The following simple procedure, as proved in [13] can be applied: order the poles according to the increasing modules, one branch encloses the real pole and then every second conjugate complex pair, the second branch encloses the remaining poles.

Example 1: Figs. 7 and 8 display several magnitude characteristics of the filters realized with parameters from the first row of Table 1, γ_{2i} =1/2, and f_{3dB} =0.166667 i.e. f_{3dB} =0.5/3. Obviously, the resulting filters separate 1/3 of the used band of a digital filter regardless of the filter order, Fig. 7, or the transition bandwidth, Fig. 8. All multiplication constants for the filters from Figs. 7 and 8 are given in Table 2. It is evident from Table 2 that all the second-order sections of the presented filters have the common constant γ_{2i} =1/2. The distinction between sections is only in the values of the parameters γ_{2i-1} which are the squares of the pole radii.

Table 2.a.

example	a)	b)	c)
f_{3dB}	0.5/3	0.5/3	0.5/3
f_a	0.22	0.20	0.18
n	9	9	9
γ_0	$1/2^2+1/2^6$	$1/2^2+1/2^6$	$1/2^2+1/2^6$
γ_1	−0.1451	−0.1683	−0.2252
γ_3	−0.3328	−0.3945	−0.5245
γ_5	−0.5753	−0.6453	−0.7697
γ_7	−0.8456	−08797	−0.9223
γ_{2i}	1/2	1/2	1/2

Table 2.b.

example	c)	d)	e)
f_{3dB}	0.5/3	0.5/3	0.5/3
f_a	0.18	0.18	0.18
n	9	11	13
γ_0	$1/2^2+1/2^6$	$1/2^2+1/2^6$	$1/2^2+1/2^6$
γ_1	−0.2252	−0.1785	−0.1500
γ_3	−0.5245	−0.4152	−0.3376
γ_5	−0.7697	−0.6478	−0.5467
γ_7	−0.9223	−08205	−0.7217
γ_9	/	−0.9448	−0.8524
γ_{11}	/	/	−0.9533
γ_{2i}	1/2	1/2	1/2

Table 2 with Figs. 7 and 8 illustrates the flexibility of the elliptic filters designed to have one common constant for all second-order sections. The second-order sections are of the type given in Fig. 3 with γ_{2i} = 1/2 and γ_{2i-1} from Table 2.

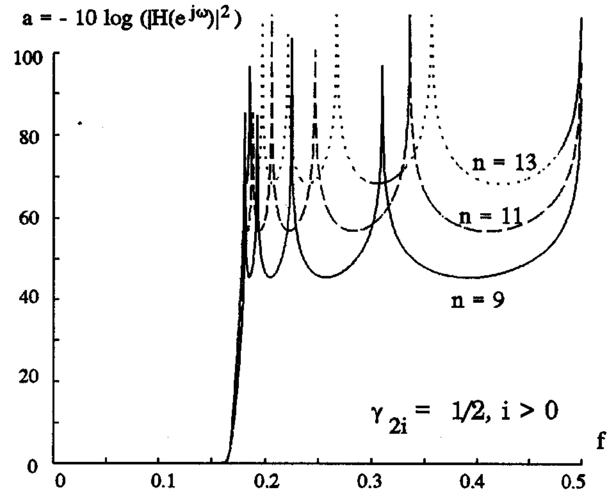


Fig. 7. Amplitude characteristics for n=9, 11 and 13 and f_{3dB} =0.5/3, f_a=0.18.

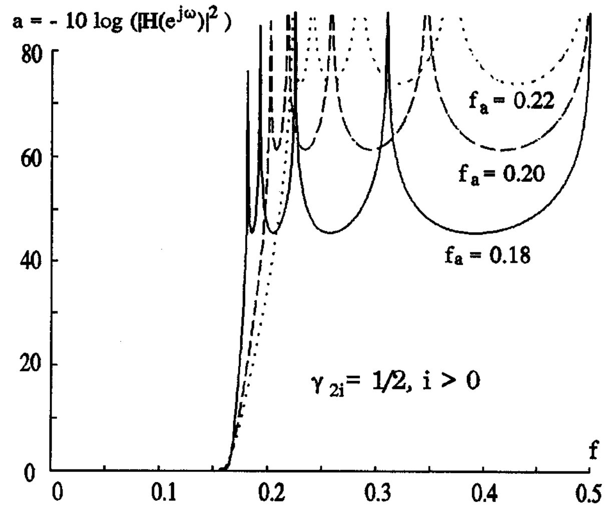


Fig. 8. Amplitude characteristics for n=9 and f_{3dB} =0.5/3, f_a = 0.18, 0.20 and 0.22.

Example 2: Requirements from [4], example No. 4: sampling frequency 16kHz, pass-band edge frequency 3.4kHz (F_p = 3.4/16 = 0.2125), maximal pass-band attenuation A_p=0.2 dB, stop-band edge frequency 4.6kHz (F_a = 4.6/16 = 0.2875), minimum stop-band attenuation A_a=65dB. The 7th-order elliptic IIR filter, realized with 7 multipliers, is used in [4] to fulfill the requirements. It will be demonstrated that by applying the approach presented in this paper, various filters can be designed which satisfy the requirements and provide realizations with the reduced number of multipliers.

Let us, from given boundary frequencies, determine the range of the permitted values of γ_{2i}. Using (34) one obtains:

$$-0.2334 < \gamma_{2i} < 0.2334$$

and consequently γ_{2i} can bechosen as a value from the set:

$$\gamma_{2i} \in \{ -1/2^2, -1/2^3, ..., 0,..., 1/2^3, 1/2^2\}$$

The choice $\gamma_{2i} = 0$ is the half-band filter which fulfills the requirements with $n=11$ giving $a_p=8 \times 10^{-8}$ dB, $a_a=77.3$dB, $f_p=F_p$, $f_a=F_a$. The filter has only five non zero multiplication constants: $\gamma_1=-0.0661$, $\gamma_3=-0.2364$, $\gamma_5=-0.4525$, $\gamma_7=-0.6711$, $\gamma_9=-0.8856$.

If we select $\gamma_{2i}=1/2^4$ and $f_a=F_a$, the specifications can be satisfied with $n=9$. For $\gamma_{2i}=1/2^4$, Table 1 gives $f_{3dB}=0.24$ and f_p can be calculated from (12):

$$fp=\frac{1}{\pi}\tan^{-1}\left(\frac{\tan^2 \pi f_{3dB}}{\tan \pi f_a}\right)=0.195$$

Obviously, $f_p=0.195$ is smaller than F_p. However, the pass-band ripple of the obtained filter is very low and the attenuation at the frequency F_p, which lies in the transition band, is substantially below A_p. Due to a very small pass-band ripple of the obtained filter, we can also quantize the constant γ_0 which represents the real pole. With approximate value $\gamma_0=1/2^5$ the pass-band attenuation reaches its maximum at F_p where it amounts to $a_p=0.005$ dB, what is substantially below the required $A_p=0.2$dB. The minimum stop-band attenuation is $a_a=67.6$dB. For the implementation only four multiplication constants are needed: $\gamma_1=-0.0840$, $\gamma_3=-0.2935$, $\gamma_5=-0.5565$ and $\gamma_7=-0.8401$. The remaining 5 constants $\gamma_0=1/2^5$, $\gamma_2=\gamma_4=\gamma_6=\gamma_8=1/2^4$ are to be implemented as shifters.

Since the obtained margin in the pass- and stop-band is large enough, we can try to decrease the margin assuming $f_a<F_a$ in order to reduce the number of multipliers by one more. In a few attempts, using the same procedure as in the previous case, the 9th order filter with $f_a=4.57424$ kHz is obtained which can be implemented with only three multipliers. The calculated values of the constants are: $\gamma_{2i}=1/2^4$, $\gamma_0=1/2^5$, $\gamma_1=-0.0858$, $\gamma_3=-0.2985$, $\gamma_7=-0.8432$ and $\gamma_5=-1/2-1/2^4$.

Obviously, γ_5 can be implemented by two shifts and one adder. The maximal attenuation in the pass-band is $a_p=0.004$dB and the minimum stop-band attenuation is $a_a=66.7$dB.

It is also of interest to notice the decrease in the radii of filter poles in comparison with the classical elliptic filter design. If the radii of the poles nearest to the unit circle are compared, we have for the example No 4 in [4], $r_{max}=0.947$ and the smaller values for the new filters $r_{max}=0.918$.

It is demonstrated through these representative designs that by the appropriate selection of the elliptic filter design parameters, the number of multipliers can be reduced from 7, as obtained in [4], to only 3 although the filter order is increased from 7 to 9. Furthermore, the radii of the transfer function poles are decreased. The attenuation curves of the four discussed realizations are displayed in Fig. 9. The flow diagram for the filter with 3 multipliers is presented in Fig. 10.

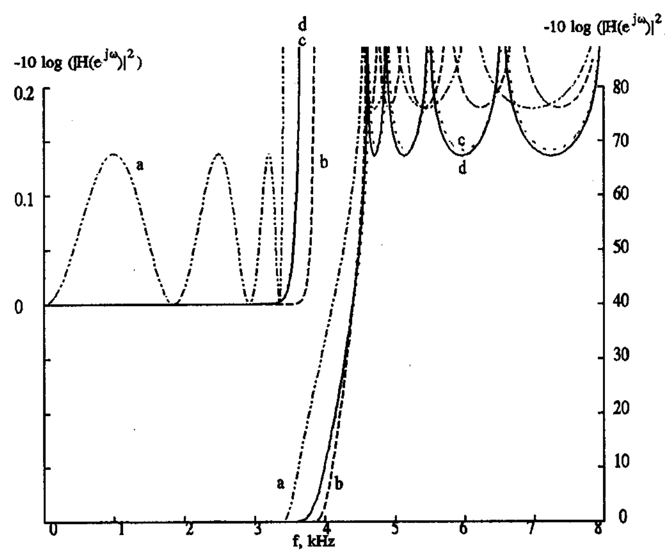


Fig. 9. Amplitude characteristics for $F_p=3.4$kHz, $A_p=0.2$dB, $F_a=4.6$kHz, $A_a=65$dB, sampling frequency 16kHz:

a - [4]

b - $\gamma_{2i}=0$, $\gamma_1=-0.0661$, $\gamma_3=-0.2364$, $\gamma_5=-0.4525$, $\gamma_7=-0.6711$, $\gamma_9=-0.8856$.

c - $\gamma_{2i}=1/2^4$, $\gamma_0=1/2^5$, $\gamma_1=-0.0840$, $\gamma_3=-0.2935$, $\gamma_5=-0.5565$, $\gamma_7=-0.8401$.

d - $\gamma_{2i}=1/2^4$, $\gamma_0=1/2^5$, $\gamma_1=-0.0858$, $\gamma_3=-0.2985$, $\gamma_7=-0.8432$, $\gamma_5=-1/2-1/2^4$, ($f_a=4.57424$ kHz).

VI Conclusion

A new approach to designing elliptic wave digital filters is presented. It is shown that a digital filter transfer function derived by the bilinear transformation from an elliptic minimal Q-factors analog prototype has the z-plane poles placed on the circle which is orthogonal with the unit circle. Accordingly, the Butterworth filter is obtained as a boundary case. This particular disposition of the poles is used for the presentation of the WDFs multiplication constants. It is shown that a half of the adaptor coefficients have the same value which can be adjusted according to the prescribed number of shift-and-add operations. The adjustment of the coefficient value is achieved only by a slight shift of the frequency for which the filter attenuation is 3 dB. It is demonstrated through examples that filter specifications are fulfilled with a reduced number of multipliers in comparison with the classical elliptic WDF design. Moreover, the transfer function poles radii are decreased. It is shown in the paper that by the appropriate choice of s -plane/ z -plane equivalent frequencies, a half-band (bi-reciprocal) WDF is obtained.

The proposed design method is very simple. The computation of the coefficients is based on the z-plane pole parameters and therefore, the conventional computer programs for digital filters can be used.

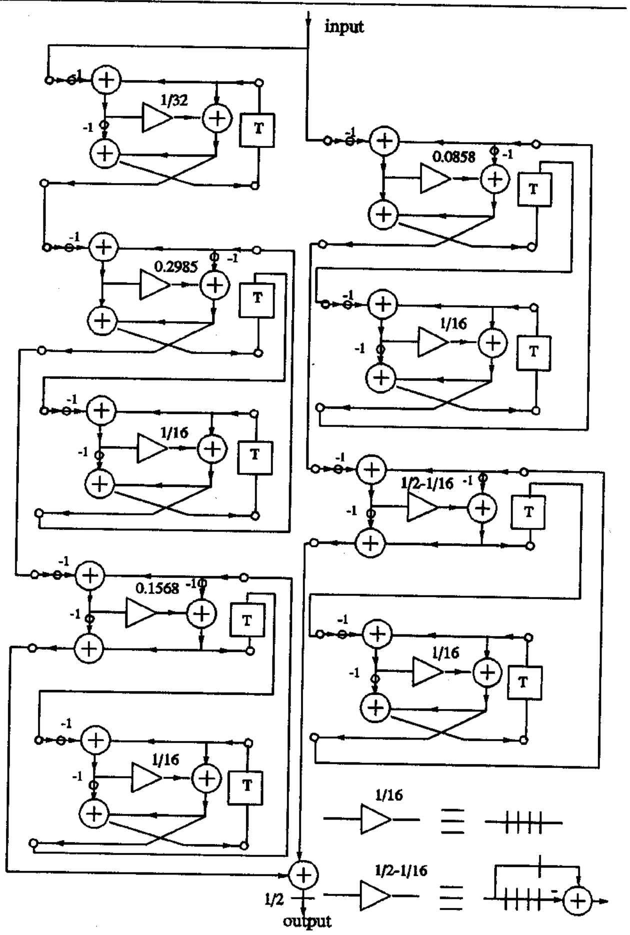


Fig. 10. Block diagram for the filter from example 2, n=9, three multipliers design.

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