Application of the Spectral Domain Method to Analysis of Single and Coupled Microstrip Lines on Two-Layer Substrate

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Abstract - The paper presents the application of the spectral domain method to analysis of effective dielectric constant and characteristic impedance of a single and coupled microstrip lines with two substrate layers. Analysis was preformed for two combinations of most frequently used hibrid microwave integrated circuits materials, i.e. allumina and teflon-fiberglass. It is shown that a directional coupler with high directivity can be realized in the very wide frequency range using two-layer microstrip tehnology.

1. Introduction

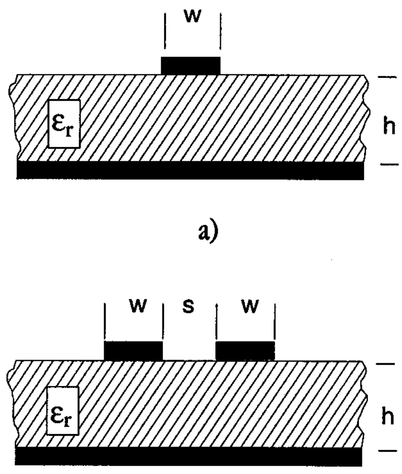
Cross-section of a microstrip line is shown in Fig. 1. a). The wave that propagates along the microstrip line is quasi-TEM at low frequencies. Longitudinal field components increase with frequency, so the fullwave analysis has to be applied.

Frequency limit up to which microstrip is used is about 60 GHz. It is usual to use microstrip lines with characteristic impedances from 25 Ω to 125.Ω [1]. Lines with lower characteristic impedances are not used because of the presence of higher order modes and lines with higher characteristic impedances are avoided- because the total loss per unit length increses with characteristic impedance.

Microstrip is used in almost all planar microstrip circuits (passive and active), such as matching networks, couplers, power deviders, hybrid couplers, filters, feeding networks for printed antennas and arrays, detectors, mixers, amplifiers and oscillators.

In planar microwave technique, it is quite common to use coupled lines as elements of printed microwave circuits, e.g. couplers and filters [1]. Cross-section of coupled microstrip lines is shown in Fig. 1. b).

In monolite microwave integrated circuits fabricated on GaAs or some other similar semiconductor monocrystals, the monocrystal ($\varepsilon_r=12.95$) is coated by a thin passivizing layer having considerably lower dielectric costant ($\varepsilon_r=3.5$) [2]. Even a very thin passivizing layer changes the characteristics of the microstrip line. Due to this fact, it is necessary to treat microstrip as a structure on two-layer dielectric substrate, as shown in Fig. 2.



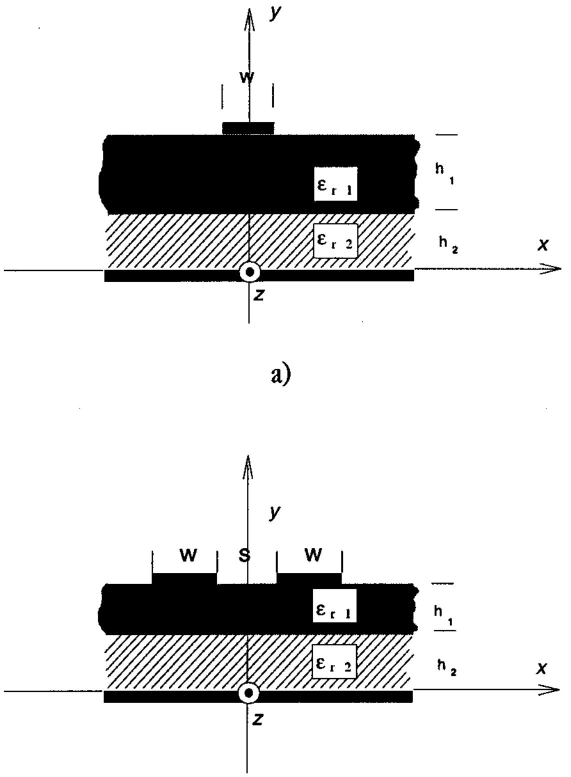
a)

b)

Fig. 1. a) Cross-section of the single microstrip line; b) cross-section of the coupled microstrip lines

Use of a passivizing layer is imposed by the monolite circuits realization technology. The aim of this paper is to show that in some applications of microstrip lines in hybrid technology of microwave integrated circuits fabrication, two-layer dielectric substrate has considerable advantages over the single-layer one. Adding of a dielectric layer can result in improving of some characteristics of microstrip.

Up to now, the microstrip line on two-layer substrate has been analyzed in [2] where the closed form expressions for the effective dielectric constant have been derived based on fullwave analysis. These formulas can be applied directly in case of a two-layer substrate. This paper does not take into account dispersion. In [3], the model which enables analysis of the dispersion in a more general microstrip structure (four-layer dielectric) is described, but its accuracy is acceptable in the frequency range of 2-18 GHz. None of these models calculate characteristic impedance and it will be shown that it can be either monotonously decreasing or monotonously increasing function of the frequency. Generally, it is most reliable to use one of the fullwave analysis methods.



a)

b)

Fig. 2. a) Cross-section of the single microstrip line on two-layer substrate; b) cross-section of the coupled microstrip lines on two-layer substrate.

This paper summarizes the results of the analysis of two-layer substrate influence on dispersion of effective dielectric constant and characteristic impedance of microstrip lines, published in [4] and [5]. In the analysis, simple and reliable Galerkin's method in spectral domain is used.

2. Galerkin's Method in FTD

Galerkin's method is described in detail in the literature, so we will give here only the basics. The procedure solves inhomogenious matrix equation in spectral domain, i.e. in Fourier transform domain (x-coordinate is transformed into spectral variable α).

$$\begin{bmatrix} \tilde{E}_x \\ \tilde{E}_z \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{xx} & \tilde{Z}_{xz} \\ \tilde{Z}_{zx} & \tilde{Z}_{zz} \end{bmatrix} \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} , \qquad (1)$$

In this matrix equation, current and electric field components are put into relation through spectral Green's functions. The equation is solved by expanding the unknown components of current distribution into series in terms of known basis functions Fp and Fs, where Fp is expansion of Jz and Fs is expansion of Jx. After the transformation into spectral domain, these expansions are:

$$\tilde{F}_p(\alpha) = \sum_{m=0}^{N} c_m \tilde{F}_{pm}(\alpha) \quad ,$$
$$\tilde{F}_s(\alpha) = \sum_{m=0}^{M} d_m \tilde{F}_{sm}(\alpha) \quad . \qquad (2)$$

By substitution of these expansions into an equation and by calculation of the inner product of each equation with corresponding basis function [6] one obtains homogenious matrix equation written as follows:

$$\sum_{m=0}^{N} K_{km}^{(1,1)} c_m + \sum_{m=1}^{M} K_{km}^{(1,2)} d_m = 0, \quad k = 0,1,\dots,N,$$
$$\sum_{m=0}^{N} K_{lm}^{(2,1)} c_m + \sum_{m=1}^{M} K_{lm}^{(2,2)} d_m = 0, \quad l = 1,2,\dots,M, \qquad (3)$$

where coefficients K are written as:

$$K_{km}^{(1,1)} = \int_{-\infty}^{+\infty} \tilde{F}_{pk}(\alpha) \tilde{Z}_{xx}(\alpha,\beta) \tilde{F}_{pm}(\alpha) d\alpha \quad ,$$
$$\dots \text{ etc.} \qquad (4)$$

Wave coefficient and consequently effective dielectric constant are obtained from the condition of non-triviality of the matrix equation solution.

$$\det([K]) = 0 \qquad (5)$$

Characteristic impedance, in case of a single mikrostrip line, is defined by the following expression:

$$Z_c = \frac{I_0^2}{P_0} \quad , \qquad (6)$$

where the total current in *z* direction is:

$$I_0 = \int_w J_z(x) dx \quad , \qquad (7)$$

and average power transmitted along the microstrip line is:

$$P_0 = Re\left(\iint_S \left(\boldsymbol{E} \times \boldsymbol{H}^* \right) \cdot \boldsymbol{n}_z dx dy \right) \quad . \qquad (8)$$

In the above expressions S denotes a cross section area of the microstrip line and w is a strip width. More detailed description of spectral domain analysis methods for planar lines is given in [7].

Basis functions have to satisfy the condition that they have to be equal to zero on the whole interface surface, except on the strips. Otherwise, inhomogenious matrix equaton occurs. Beside this basic condition, basis functions have also to fulfill some additional conditions [8]. In this paper, set of basis functions proposed by T. Itoh [7] are used:

$i=0$

$$F_{p0}(x)=\frac{1}{\sqrt{1-X^2}},\quad F_{s0}(x)\equiv 0\ , \tag{9}$$

$i=2,4,6,\ldots,\infty$

$$F_{pi}(x)=\frac{\cos\frac{i\pi}{2}X}{\sqrt{1-X^2}},\quad F_{si}(x)=\frac{\sin\frac{i\pi}{2}X}{\sqrt{1-X^2}}\ , \tag{10}$$

$i=1,3,5,\ldots,\infty$

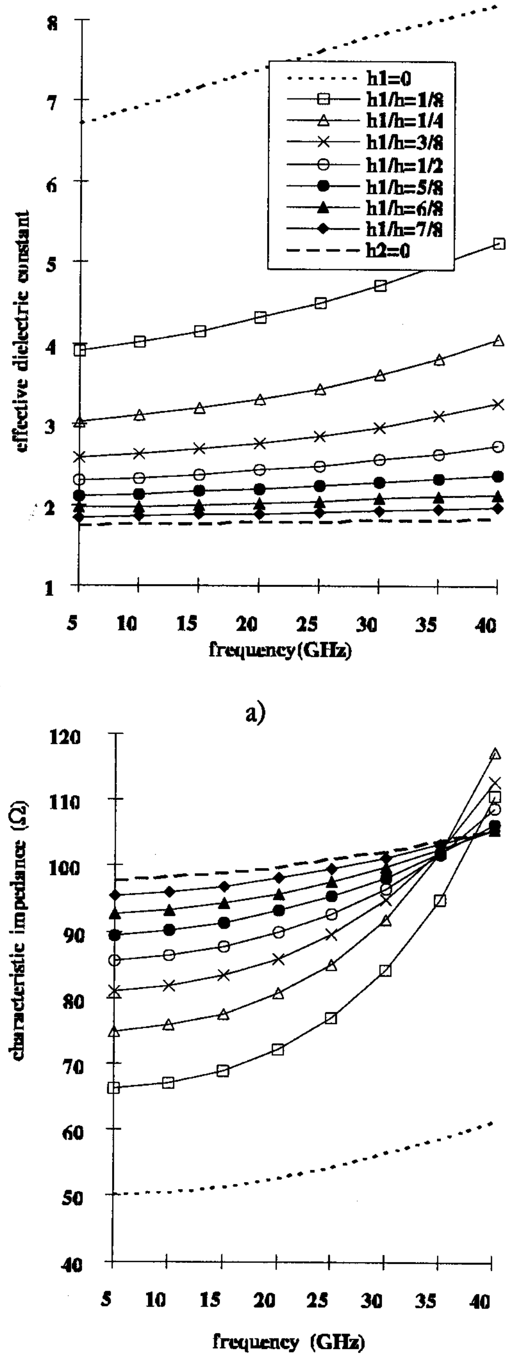
$$F_{pi}(x)=\frac{\sin\frac{i\pi}{2}X}{\sqrt{1-X^2}},\quad F_{si}(x)=\frac{\cos\frac{i\pi}{2}X}{\sqrt{1-X^2}}\ . \tag{11}$$

In previous expressions, X designates normalized coordinate X=2x/w.

3.1. Results of a Single Microstrip Line Analysis

The influence of a two-layer dielectric substrate is investigated here in cases of two most commonly used substrates in hybrid technology - alumina and teflon-fiberglass. Two cases of a single microstrip line on two-layer substrate are analyzed.

In the first case, 50Ω microstrip line on alumina substrate ($\varepsilon_r=9.9$ and h2=0.635 mm) is investigated. Under the strip of constant width w=0.6 mm, a layer of teflon-fiberglass of height h_1 and $\varepsilon_r=2.17$ is inserted in steps of 1/8 of substrate thickness, so that the total substrate height remains constant. Analyzing the diagrams from Fig. 3, one can conclude that even a very thin layer of height h_2 considerably influences the dispersion (relative change of characteristics with frequency). This influence results in decrease of dielectric costant dispersion. At first, dispersion of characteristic impedance increases drastically, and then decreases with height of the layer of lower dielectric constant.



a)

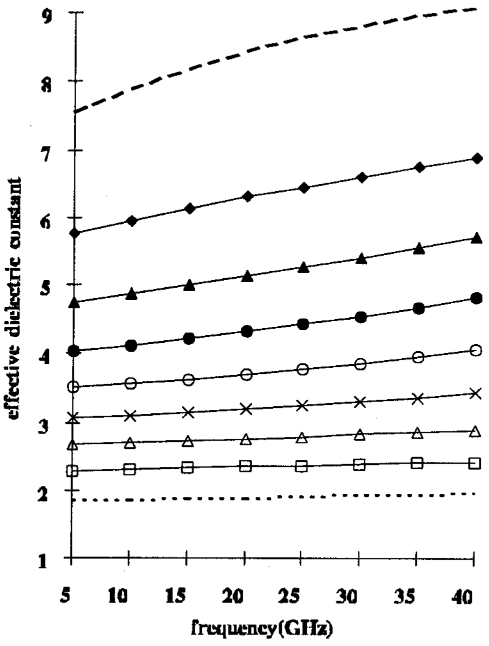
b)

Fig 3. Single microstrip line on a two-layer substrate;
a) dispersion of the effective dielectric constant;
b) dispersion of the characteristic impedance;
$\varepsilon_{r1}=2.17$, $\varepsilon_{r2}=9.9$, $h_1+h_2=0.635$ mm, w=0.6 mm.

In the second case, 50Ω microstrip line on teflon-fiberglass ($\varepsilon_r=2.17$ and $h_2=$ 0.635 mm) is investigated. Under the strip of constant width w=1.75 mm, in steps of 1/8 of substrate thickness the alumina layer (ε_r=9.9) of height h_1 is inserted, so that the total height of substrate remains constant. Dispersion diagrams are shown in Fig. 4.

Variation of effective dielectric constant with respect to thickness of inserted layer is remarkably slighter than in previous case. Characteristic impedance vary almost linearly with change of height h_2 at the frequency of 5 GHz. In Fig. 4. b) we can also see that characteristic impedance decreases with frequency.

The course of change of effective dielectric constant and of characteristic impedance with respect to thickness of inserted layer is expected. For both combinations of dielectric layers in two-layer substrate, values of effective dielectric constant and of characteristic impedance gradually approache the characteristics of the single-layer substrate, of either higher or lower dielectric constant. Examined case is hypothetical to a certain extant, because the alumina substrate is fabricated only in two standard thicknesses. So, featured results of analysis are intended to illustrate only qualitative change of characteristics. The teflon-fiberglass substrate is fabricated in a wide range of thicknesses, so that necessary h_1/h_2 ratio can easily be obtained.



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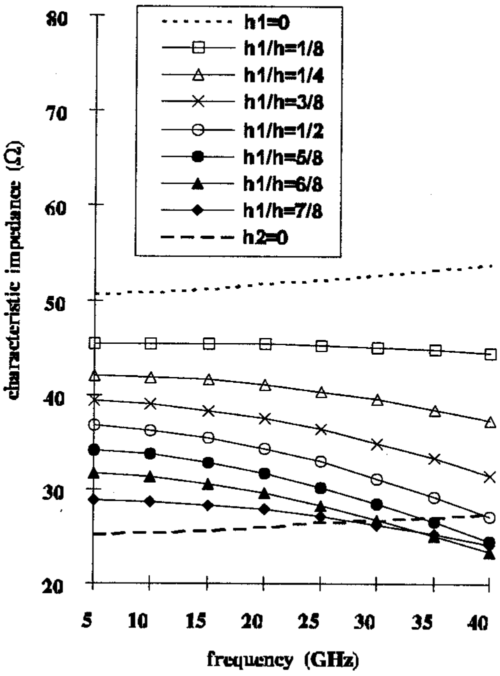


b)

Fig.4. Single microstrip line on two-layer substrate; a) dispersion of the effective dielectric constant; b) dispersion of the characteristic impedance; ε_{r1}=9.9, ε_{r2}=2.17, h_1+h_2=0.635 mm, w=1.75 mm

3.2. Results of the Coupled Microstrip Lines Analysis

The influence of two-layer substrate on characteristics of coupled microstrip lines has been investigated under same conditions as in the case of a single microstrip line, i.e. with two most frequently used substrates in hybrid technology - alumina and teflon-fiberglass.

Effective dielectric constant is evaluated like in the case of a single microstrip line and characteristic impedance is defined as:

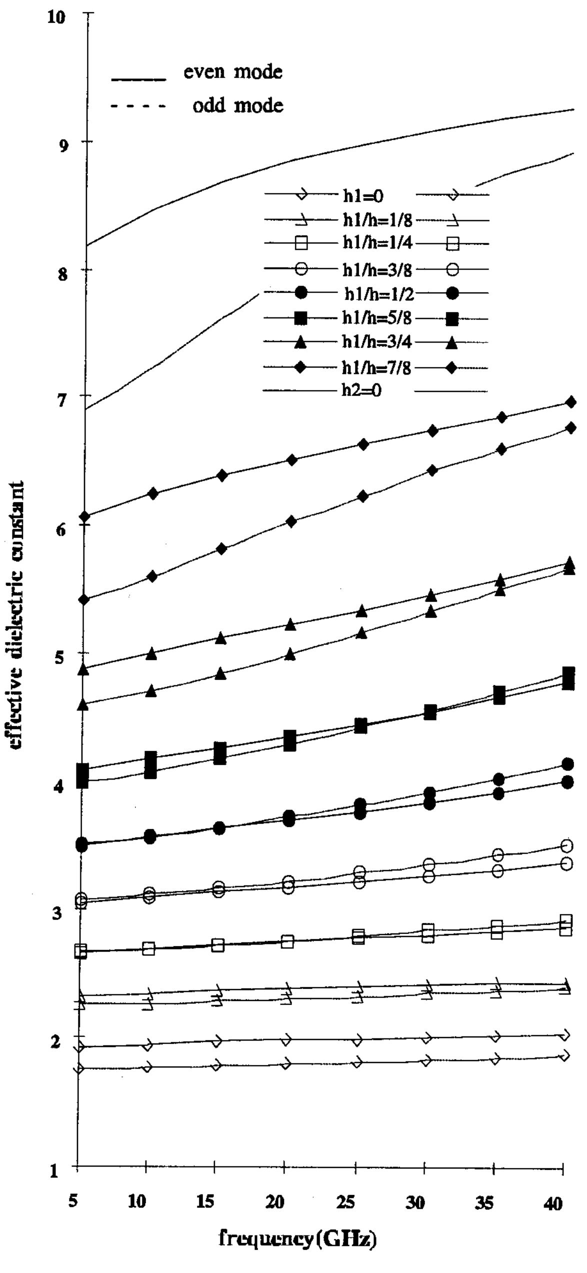
$$Z_{0\,o,e} = \frac{I^2_{0\,o,e}}{P_{0\,o,e}} \ , \qquad (12)$$

where the total currents in z direction for both modes are determined from the following expression:

$$I_{0\,o,e} = \int_w J_{z\,o,e}(x)dx \ , \qquad (13)$$

In the first case, 50Ω coupled microstrip lines on alumina substrate of ε_r=9.9 and height

$h_2=0.635$ mm have been investigated. Teflon-fiberglass layer of $\varepsilon_r=2.17$ and height h_1 is inserted under the strips of constant width w=0.6 mm and separation width s=0.4 mm, in steps of 1/8 of substrate thickness, so that the total substrate thickness remains unchanged. Dispersion diagrams are given in [5]. Similarly to the dispersion characteristics of a single microstrip line given in Fig. 3, even a thin layer influences considerably the characteristics of coupled microstrip lines. This is particularly observable with characteristic impedances in both modes.



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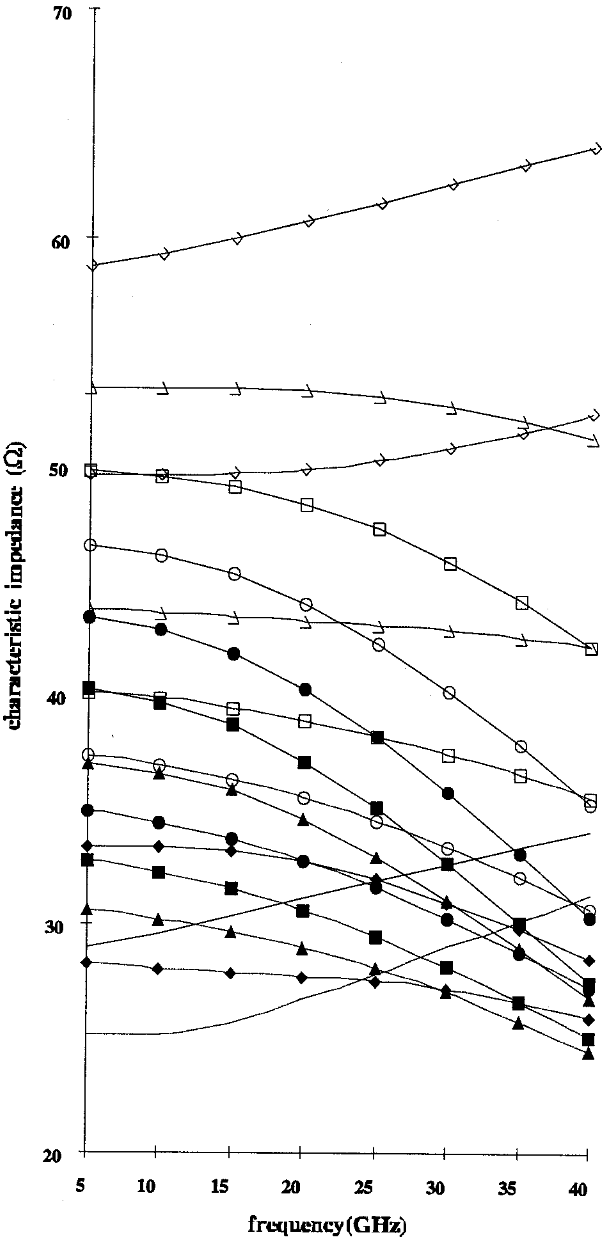

b)

Fig. 5. Coupled microstrip lines on two-layer substrate; a) dispersion of the effective dielctric constant, b) dispersion of the characteristic impedance $h_1+h_2=0.635$ mm, w=1.75 mm, s=1 mm; ___ even mode, - - - odd mode.

In the second case, we have investigated 50Ω coupled microstrip lines on a teflon-fiberglass dielectric substrate ($\varepsilon_r=2.17$, $h_2=0.635$ mm) with inserted layer of alumina substrate ($\varepsilon_r=9.9$, height h_1) under the strips of constant width w=1.75 mm and separation width s=1.0 mm. The layer is inserted in steps of 1/8 of substrate thickness and total thickness of the substrate remains unchanged. Dispersion curves are given in Fig. 5.

In this case, characteristic impedances of both even and odd modes decrease with frequency as in the case of a single microstrip line. It is interesting to note that effective dielectric constants of even and odd modes are of almost the same value in the relatively wide frequency range for several ratios of layers' thicknesses ($h_1/h=1/4$, 3/8, 1/2 and 5/8). This is very important for realization of directional couplers with high directivity. Calculated directivities and couplings diagrams of the directional coupler with one $\lambda/4$ section are shown in Fig. 6. for four mentioned cases. It is obvious that two-layer microstrip technology is suitable for realization of directional couplers with couplings of about -20 dB and directivities better than -40 dB in wide frequency range, i.e. up to 25 GHz. In the frequency range from 25 GHz to 40 GHz, couplings become looser and their variation with frequency is somewhat greater, the directivity still remaining better than -25 dB, which is most desirable. Directivities and couplings are calculated according to the expressions from [1].

Up to now, directional couplers in standard microstrip technology have been realized by laying of a dielectric plate over the microstrip line (shown in Fig. 1 b)), by means of which phase velocities of even and odd modes are equalized, which is the main condition for high directivity. Practically, this is the use of two-layer technology. The advantage of the solution suggested in this paper is an easiser access to the strips, which considerably simplifies additional adjustments.

4. Conclusion

This papers features the analysis of coupled microstrip lines dispersion on two-layer substrate for two combinations of dielectric substrates most commonly used in hybrid MICs technology. It is shown that two-layer dielectric substrate significantly changes the dispersion. Observed examples show that directional couplers of high directivity can be realized on two-layer substrate in wide frequency range, because effective dielectric constants of both modes for some ratios of layers' thicknesses are almost equal.

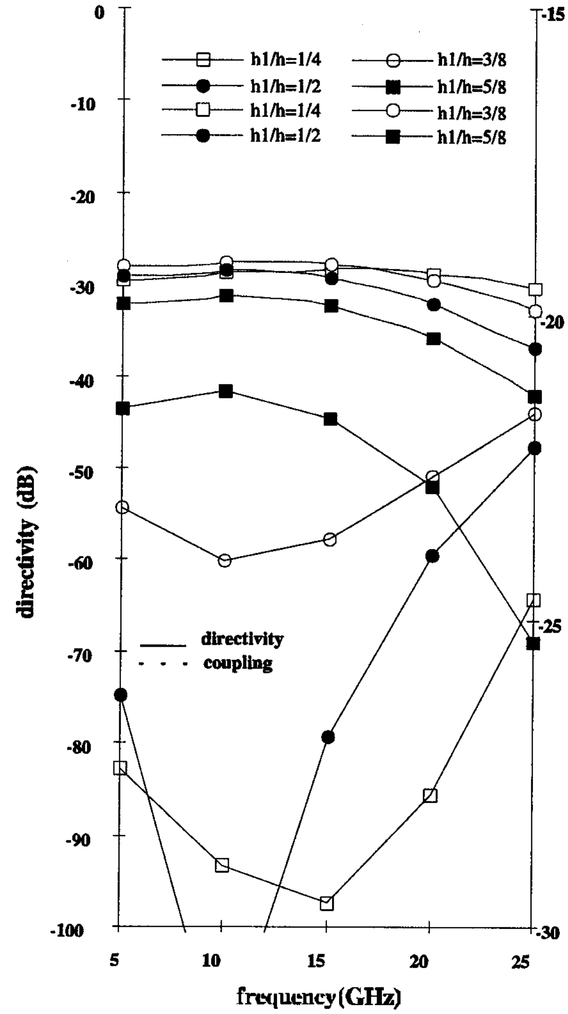

Fig. 6. Calculated directivity and coupling of one-section directional coupler realized with coupled microstrip lines on two-layer substrate; $\varepsilon_{r1}=9.9$, $\varepsilon_{r2}=2.17$, $h_1+h_2=0.635$ mm, w=1.75 mm, s=1mm.

5. References

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