Synthesis of Cascaded N-Tuplet Filters

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***Abstract*—In the theory of Darlington type filter synthesis extraction of transmission zero sections one by one, leading to a cascade of Darlington-A, B, C, D and Brune sections bears serious limitations like numerical accuracy problems, negative and/or extreme element values or impractical topologies for Darlington-C-D and Brune sections. In this paper all such problems are evaded/eased by extracting the transmission zeros as groups to form circuit sections which are readily convertible to multiple coupled resonator forms. In this approach, transmission zeros at s=0 and s=∞ are extracted as groups to form doublets of resonators, the jω-axis and σ-axis transmission zeros are extracted as groups to form triplets of resonators, pairs of jω-axis and σ-axis transmission zeros and complex transmission zeros are extracted as groups to form quadruplets of resonators, etc. The classical element extraction procedure in transformed frequency domain is revised to cover extraction of complex transmission zeros as forth order section by zero shifting from both s=0 and s=∞ leading to realisation as a quadruplet of resonators. Thus, the theory of cascade synthesis is generalised to include cascaded doublets, triplets, quadruplets and other N-tuplets of resonators as building blocks. It is found out that the limitations due to usual numerical accuracy problems are also reduced a lot and the resulting element value spreads can be controlled to avoid extreme values.**

I. Introduction

Cascade synthesis for direct design of filters by placement of transmission zeros (TZs) is a well established technique which involves no approximations and has maximum flexibility for shaping both amplitude and phase response by adjusting locations of TZs [1]-[6]. However this theory encountered difficulties in the following aspects:

1. Circuit sections corresponding to complex and σ-axis transmission zeros (Darlington-D and C sections) come out to be impossible to realise in most cases.
2. Circuit sections corresponding to s=jω axis transmission zeros may get extreme or even negative element values when the transmission zeros are very close to passband edges.
3. In general, this approach involves extreme numerical precision problems when the order of the filter gets large.
4. At microwave frequencies most filters are realized in direct coupled resonator forms as shown in Fig. 1.a where the circles denote LC resonators and the thick lines denote L or C type couplings. However such structures can't meet the extreme selectivity and phase linearity requirements of modern applications. These requirements are overcome by introducing cross couplings between non-adjacent elements, as described in the pioneering works of Rhodes [7], Atia-Williams [8], Cameron [9], Pfitzenmaier [10], [18], Bell [11] followed by a great number of publications [12]-[20] using different and mostly special approaches, leading to different topologies some which are shown in Fig. 1.b-d.

The main topic of this paper is to provide a systematic approach for synthesizing both direct coupled and cross-coupled filters of all types by modifying the classical cascade synthesis approach. The essence of the approach is that, contrary to the classical approach, the transmission zeros are not extracted one by one, but as groups leading directly to cross-coupled resonator blocks like Doublets, Triplets, Quadruplets, Quintuplets, etc. Thus the resulting filter will be cascades of N-tuplets of resonators. A novel approach is developed for the extraction of either complex TZs or a pair of jw-axis or σ-axis TZs in the form of cross coupled quadruplets, by zero shifting from both s=0 and s=∞. It is also shown that s=σ axis TZs can be realized as Triplet sections, in the same way as s=jω axis TZs. The main advantages of this approach can be summarized as follows:

i) The close correspondence between the transmission zeros and the circuit sections realising them eases tuning and adjustment of response.

ii) The designer has the control to reach at all the possible alternative equivalent solutions, enabling judicious selection of the most feasible solution.

iii) A further advantage of this approach appears in its numerical precision. The resulting element value spreads can be controlled during extraction stage of circuit blocks to avoid extreme element values and breakdown of the extraction process because of numerical precision-roundoff type problems in high degree filters.

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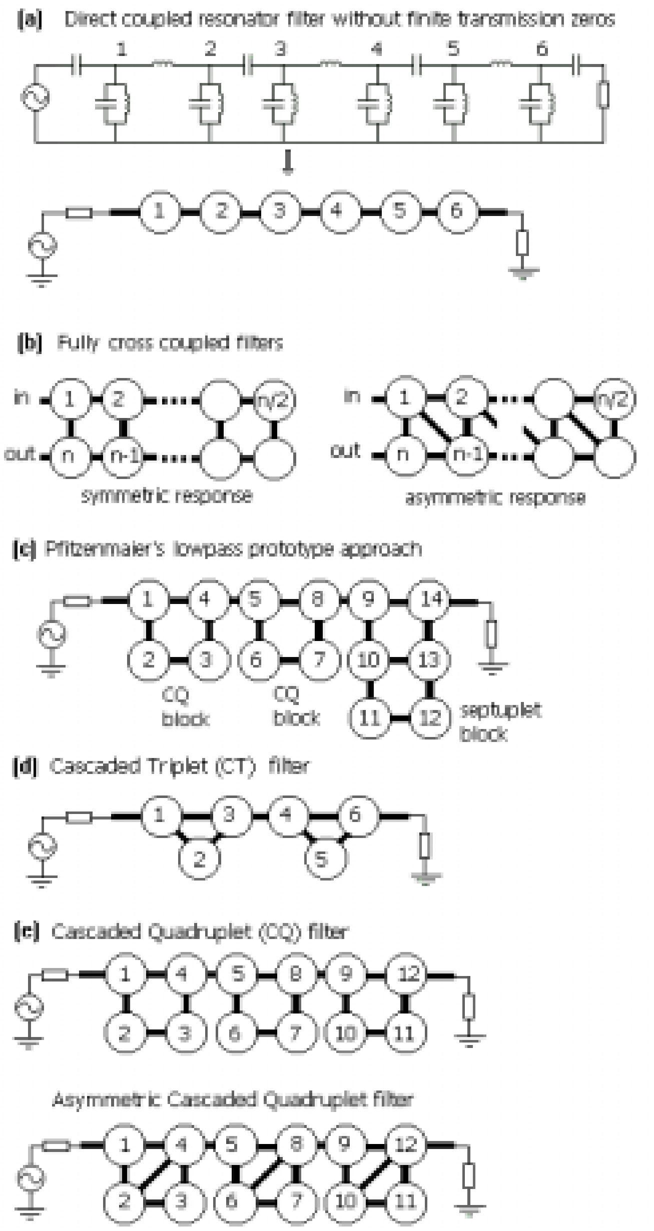

Fig. 1. Direct coupled and some cross-coupled resonator filter structures.

II. Extraction of Coupled Resonator Blocks

Two port transfer function of a filter can be formed by placing transmission zeros (TZ) at s=0, s=∞, some finite transmission zeros (FTZs) on jω–axis ($s_i=\pm j\omega_i$), some on σ–axis ($s_i=\pm\sigma_i$) and some complex TZs ($s_i=\pm\sigma_i\pm j\omega_i$), depending on the requirements on amplitude selectivity and group delay flatness. In the classical cascade (ladder) synthesis the circuit is realised by extracting these transmission zeros as proper circuit sections one by one to form the cascaded element filter. When the order of the filter is high, and if the transmission zeros are extracted in a careless order, numerical accuracy problems start to accumulate and after some steps, extreme element values start to show up which, in some cases makes the synthesis impossible. Through the experience of the authors, it was observed that the best cure for such numerical accuracy problems can be evaded to a great extent by extracting the transmission zeros not one-by-one, but as a group to form direct or cross-coupled resonator blocks. Besides overcoming numerical problems, this approach yields direct or cross-coupled topologies directly, without using the special techniques devised for cross-coupled topologies. The coupled resonator topologies which correspond to certain transmission zero groups are classified as Doublets (two resonators coupled inductively or capacitively), Triplets (three resonators with simple L or C type couplings) , Quadruplets, Quintuplets, Septuplets, etc. The filters resulting by cascading such N-tuplets are termed as Cascaded Triplet (CT), Cascaded Quadruplets (CQ), etc. filters. In the following subsections extraction of such N-tuplet sections and resulting filter types will be described.

A. Transmission Zeros at s=0 and s=∞: Doublets and Direct Coupled Resonator Filters

Direct coupled resonator filters are formed by assigning TZs only at s=0 and s=∞. The number of TZs at s=0 (Nzero) and s=∞ (Ninf) set selectivity of the filter in lower and upper stopbands. The TZs at s=0 come out to be series capacitors and shunt inductors while the TZs at s=∞ are series inductors and shunt capacitors, as shown in Fig. 2.a. These TZs can be extracted in different orders to form a variety of circuit topologies. Since our main interest is coupled resonator filters, an example with Nzero=3 and Ninf=3 is given in Fig. 2.b. In this example, the transmission zeros are extracted one by one, in the order zero-zero-zero-Inf-Inf-Inf. The resulting circuit can then be converted into coupled resonator form by application of Norton transformation on the two series elements, as described in Fig. 2.b. Norton transformation is shown in Fig. 2.e as applied on a parallel LC in series arm. This conventional approach suffers from numerical accuracy problems for high degree filters. Further, it also requires iterative application of Norton transformations to reduce the spread in the shunt element values, for example to form equal shunt inductors or equal shunt capacitors.

Fig. 2.c-d describes a shorter way to reach coupled resonator form. In this approach, the transmission zeros are extracted as four degree sections formed by either one TZ at s=0 and three TZs at s=∞ or three TZs at s=0 and one TZ at s=∞. The TZs can either be extracted one by one and then Norton transformation is applied to form coupled resonator form named as Doublet, or, through proper formulation, one may extract readily formed doublets corresponding to four TZs. In either formulation one can enforce equal shunt inductor or equal shunt capacitor conditions during extractions. Since the adjacent doublets will share common resonators, this fact should be taken into account during extraction.

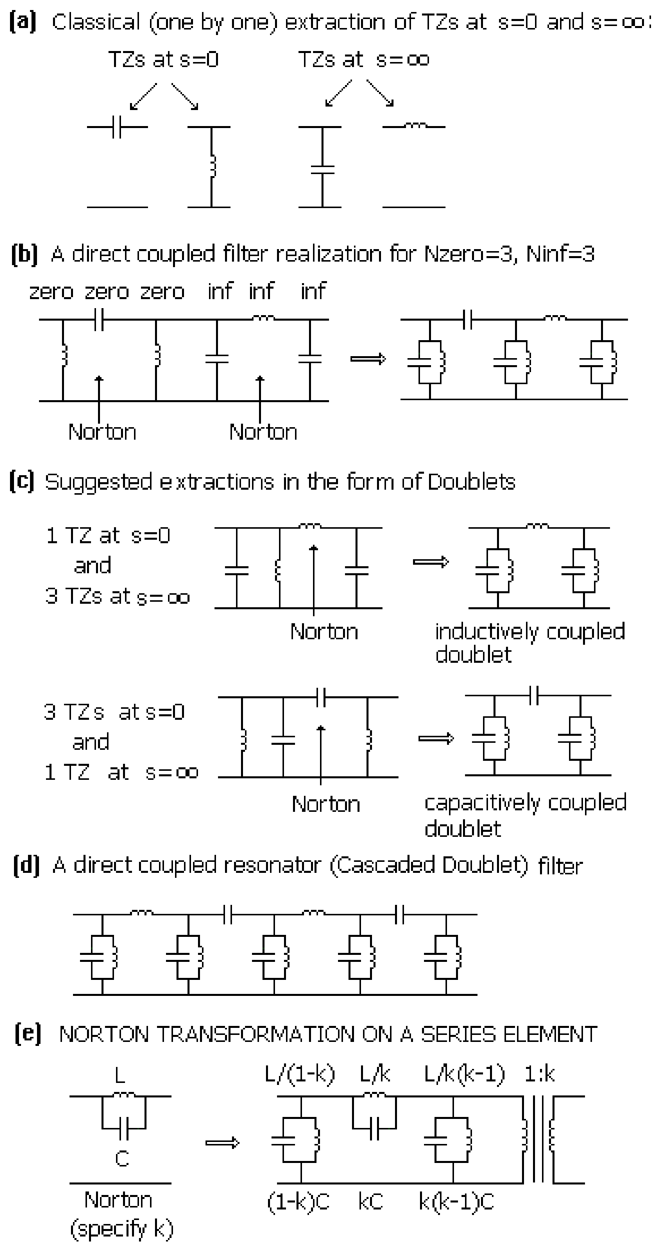

Fig. 2. Extraction of TZs at s=0 and s=∞ and formation of doublets and direct coupled resonator filters.

B. Transmission Zeros on s=jw and s=σ axis: Triplets and C Triplet Filters

TZs on s=jω axis are used to increase selectivity of filters near the passband edges while TZs on s=σ axis are used to linearize phase responses of lowpass filters. s=jw-axis transmission zeros are extracted by zero shifting from either s=0 (if the TZ is in lower stopband) or from s=∞ (if the TZ is in upper stopband), leading to the circuit sections shown in Fig. 3.a (with positive signs). s=σ axis TZs also lead to similar circuits, but with one negative element in the LC resonators (named as Darlington-C section). When converted into coupled resonator form (using Norton transformation), the LC resonators become the coupling elements between resonators, as shown in Fig. 3.a. LC type coupling elements are usually undesirable. Such couplings can be replaced by a simple L or C type coupling elements by introducing cross-couplings between nonadjacent resonators. This is possible by extracting sixth order circuit sections as shown in Fig. 3.b which involves, besides the s=jw axis or s=s axis FTZ, there are either one TZ at s=0 and three TZs at s=∞ or three TZs at s=0 and one TZ at s=∞. The TZs may be extracted one by one, in the classical way as shown in the top of Fig. 3.b and then it can be converted into coupled resonator form by applying Norton transformation to the series elements.

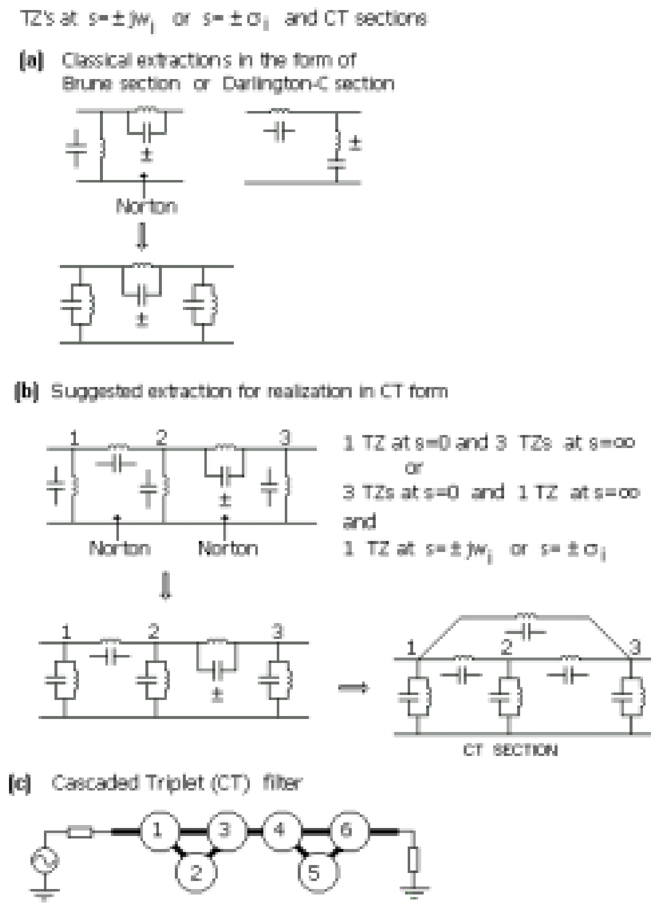

Fig. 3. Extraction of jω-axis and σ-axis TZs and realization in CT form.

It is also possible to extract the same circuit in two pieces, one as a Doublet, followed by extraction of the FTZ section. The spread in element values can be adjusted by repeated application of Norton transformations to the series elements. Thus, we get a direct coupled three resonator circuit, with one of the couplings being LC type. The LC type coupling can be converted into simple L or C type coupling by introducing cross coupling between Node-1 and Node-3, thus obtaining three mutually coupled resonator circuit named as Cascaded Triplet (CT). Elimination of one of L or C of the LC coupling and determination of element values of the resulting CT section can be done by elementary row-column operations applied on the admittance matrix of the three node circuit. This procedure is described in the Appendix qualitatively. The nature of coupling elements (L or C type) depends on the position of the realised FTZ with respect to passband, as well as on the relative number of TZs at s=0 and s=∞. Though tedious to formulate, one may as well extract the CT section directly, without passing through the two stages described in Fig. 3.b. If more than one FTZs exist, then they may all be realised as separate CT sections, thus forming a CT filter, as shown in Fig. 3.c. It should be noted that each CT section of the filter realises on FTZ. Therefore in such filters it is possible to tune a FTZ without affecting the other FTZs. It should also be noted that each CT section requires a total of four TZs at s=0 and s=∞.

C. Pairs of jω-axis or σ-axis Transmission Zeros: Quadruplets and CQ filters

A pair of jω-axis or σ-axis transmission zero can be realised in the classical way, as shown in Fig. 4.a, as cascades of Brune or Darlington-C sections.

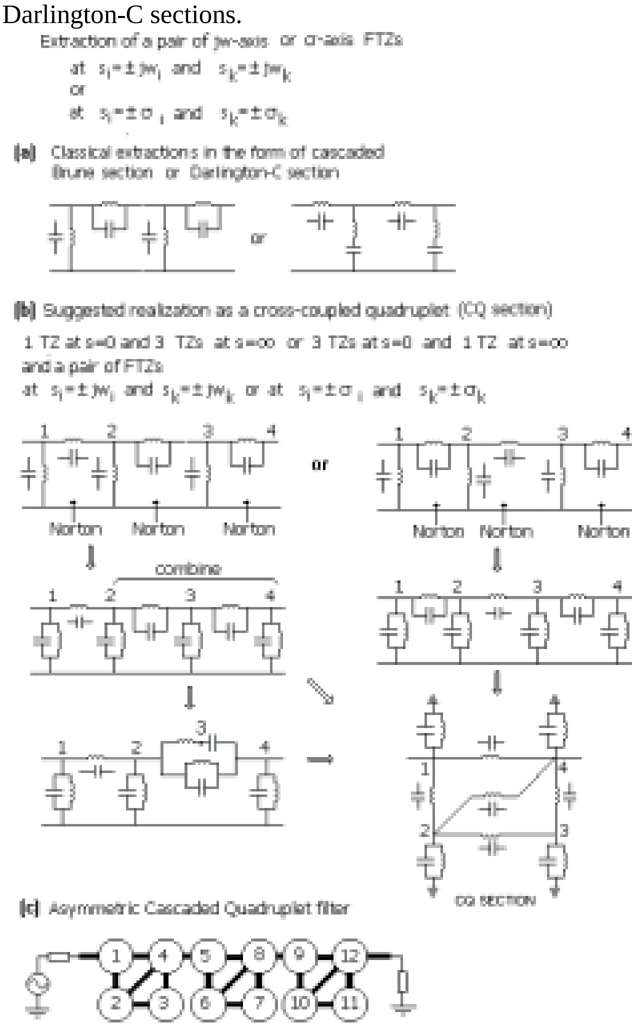

Fig. 4. Realisation of a pair of jω-axis or σ-axis FTZs as a CQ section and formation of CQ filters.

A direct coupled resonator form can be formed by including either one TZ at s=0 and three TZs at s=∞ or three TZs at s=0 and one TZ at s=∞. The FTZ's may be extracted in different orders as shown in the top of Fig. 4.b. Application of Norton transformation to the series elements converts the circuit into direct coupled resonator form with LC type couplings between the resonators 2-3 and 3-4. LC type couplings can be replaced by simple L or C type coupling elements by introducing cross-coupling elements between resonators 1-4 and 2-4 or 1-4 and 1-3 by applying row-column additions on the admittance matrices of these direct coupled resonator forms. One may also combine the two adjacent FTZ sections to form a single fourth order section (appearing as a fourth order coupling element) and then apply the row-column operations. Though tedious to formulate, instead of extracting the TZs one by one, it is also possible to realise the circuit by extraction a doublet and a fourth order section or by extracting the quadruplet directly. Such quadruplet are termed as CQ (Cascaded Quadruplet) sections. Thus, bandpass filters with even number of FTZs can be formed by cascading such quadruplets as shown in Fig. 4.c, named as CQ filters.. The diagonal cross-couplings disappear if the FTZ pairs are located symmetrically on the two sides of the passband. However symmetry condition is usually unknown and can be found by tuning the FTZ locations until the diagonal cross-couplings disappear.

D. Extraction of Complex Transmission zeros as CQ Sections

Complex TZ quadruplets, $s_i=\pm\sigma_i\pm j\omega_i$ are used to linearize phase response of bandpass filters. Conventionally a complex transmission zero quadruplet, can be extracted in the form of a Darlington-D section as shown in Fig. 5.a. This structure is useless as it is for use in coupled resonator filters. The authors of this paper had developed a simpler version which involves zero shifting from both s=0 and s=∞ [31]. The complex TZ quadruplet is extracted as a shunt LC resonator followed by a series arm fourth order section, as shown in Fig. 5.b. If a complex TZ is extracted as a block together with either one TZ at s=0 and three TZs at s=∞ or one TZ at s=0 and three TZs at s=∞, as shown in Fig. 5.c, then this circuit section can be converted into a CQ section, in the same way as in the previous (double FTZ) case. Since the four complex conjugate TZs are always symmetric with respect to both jω-axis and σ-axis, the resulting CQ section will not have diagonal cross-coupling.

By cascading such CQ sections, linear phase CQ filters can be formed as shown in Fig. 5.c.

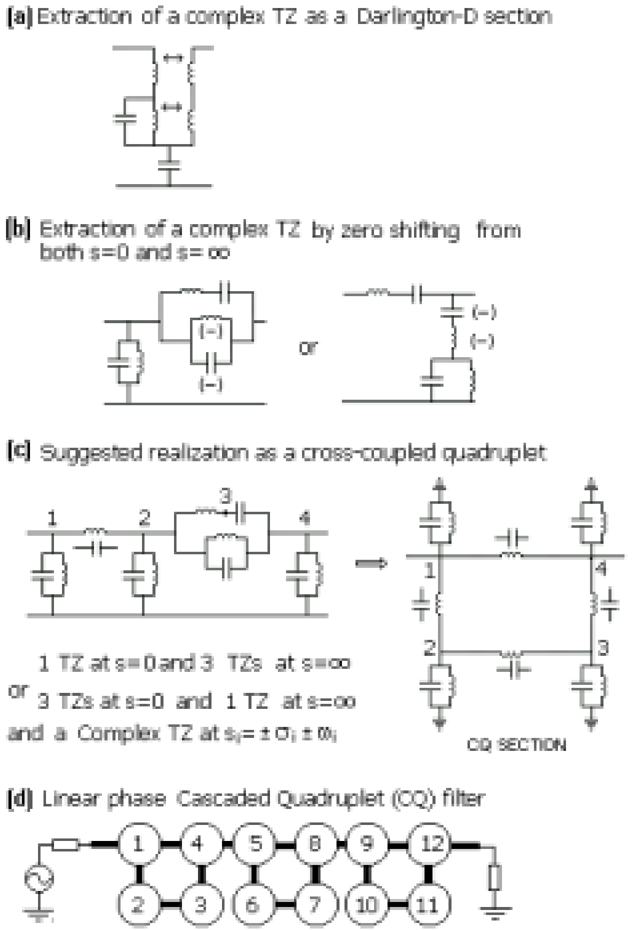

Fig. 5. Extraction of a complex TZ as a CQ section and formation of a linear phase CQ filter.

E. Quintuplets, Septuplets and N-tuplets

The same approach can be applied to form higher order cross coupled modules to realise more than two j -axis or complex TZ's as a single N-tuplet. Fig. 6.a shows formation of a five resonator block named as quintuplet. It is extracted as a degree 10 circuit section with either Nzero=1, Ninf=3 or Nzero=3, Ninf=1 and converted into direct coupled resonator form. Then the row-column operations are applied to introduce necessary cross couplings and eliminate undesired couplings. A quintuplet can realise either three jω-axis FTZ's or a complex TZ and one j -axis FTZ. It is also possible to realise these FTZs as a cascaded of a CQ section and a Doublet or three CT sections at the expense of using more TZs at s=0 and s=∞.

Fig. 6.b shows formation of a six resonator block named as septuplet. A septuplet can realise either four j -axis finite TZ's or two complex TZ's. It is extracted as a degree 12 circuit section with either Nzero=1, Ninf=3 or Nzero=3, Ninf=1 and converted into direct coupled resonator form. Then row-column operations are applied to place the necessary cross couplings and eliminate undesired couplings. These FTZs may also be realized as cascades of CT and/or CQ sections, but at the expense of using more TZs at s=0 and s=∞.

Higher order N-tuplets can be formed in similar ways. However since row-column operations get extremely tedious, for such filters the approaches using optimization and other techniques may advisable [9], [14-17].

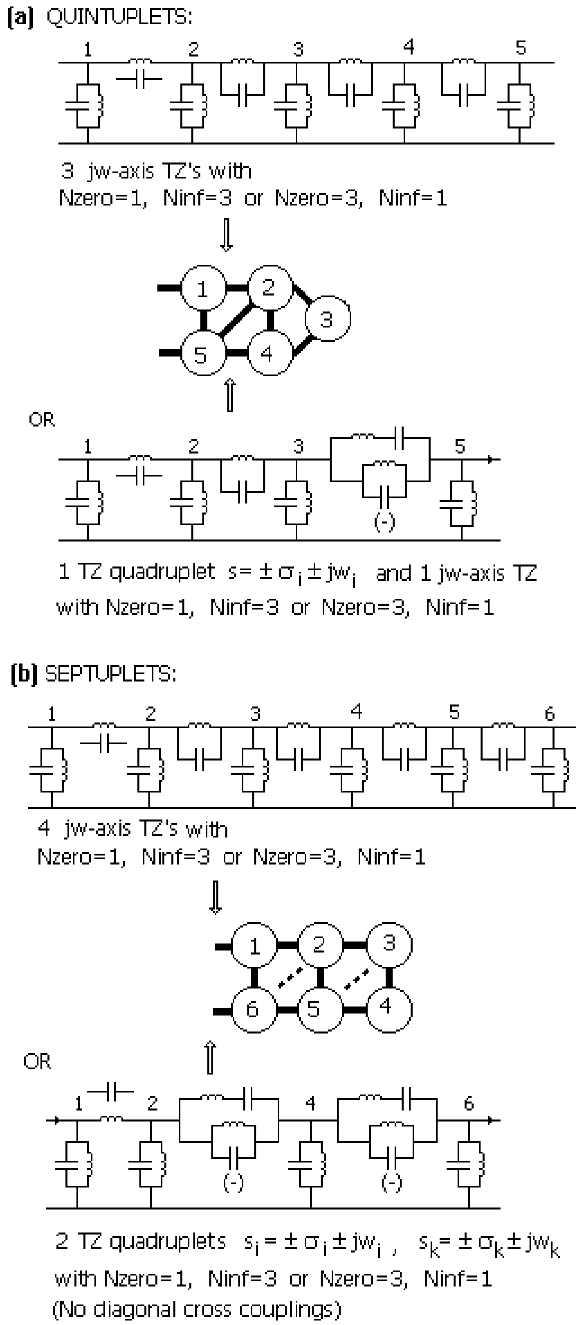

Fig. 6. Five and six resonator cross-coupled blocks

One can form Cascaded Triplet, Cascaded Quadruplet or mixed CT-CQ-quintuplet-septuplet filters by extracting the relevant circuit blocks in any order and then converting them into N-tuplet forms.

Filters made up of NR resonators will have a total degree of 2NR. Since there are Nzero+Ninf TZs at s=0 and s=∞, the total number of finite TZs will be 2NR-(Nzero+Ninf). Clearly, the single N-tuplet realization (FCC filter) will have the maximum possible number of finite TZs which is 2NR-4 because Nzero+Ninf=4 for such filters. All the other filters with the same number of resonators formed by cascading several N-tuplets will have less number of finite TZs because each N-tuplet section needs Nzero+Ninf=4, and in total the filter will have Nzero+Ninf>4. This in turn leaves 2NR-(Nzero+Ninf) degrees for realization as finite TZs which is less than that of the single N-tuplet case. The difference between Nzero+Ninf of a single N-tuplet and a multiple N-tuplet filter of the same degree reflect itself as limitations on amplitude or phase response as follows:

– When high skirt selectivity is required the single N-tuplet version is advantageous because it can collect all its $2N_R$-4 degrees near the band edges as $(2N_R-4)/2$ jω-axis finite TZs at the expense of lower minimum stopband insertion loss. Cascaded N-tuplet filter can locate less number of finite TZs there, but the level of minimum stopband loss will be higher.

– In linear phase applications flat delay bandwidth of a single N-tuplet filter can be wider than that of cascaded N-tuplet filter with the same number of resonators at the expense of less selectivity. This is because in single N-tuplet filters $2N_R$-4 degrees can be realised as $(2N_R-4)/4$ complex TZs to flatten delay. Since cascaded N-tuplet filter has $N_{zero}+N_{inf}>4$, it will have higher selectivity at the expense of narrower flat delay bandwidth than single N-tuplet filter. There will be only $[2N_R-(N_{zero}+N_{inf})]/4$ complex TZs. High selectivity forms a limitation for the flat delay bandwidth.

In short, the approach adopted in this paper enables the designers to make use the wealth of knowledge on cascade synthesis of filters for designing, modifying, evaluating and classifying cross coupled filter solutions in a systematic way. This approach will also enable one to develop alternative equivalent structures like FCC or CT-CQ or other cascaded N-tuplet versions in a controlled manner because it is clear which circuit section contributes to which TZ. Considering the number of possible alternative CT, CQ, etc. solutions and realisations that would be obtained by extracting the TZs in different orders, it is clear that the number of cross coupled solutions can assume large values. Another advantage of the approach developed in this paper is that the designer has the control for developing all the possible alternative solutions, judge the effects of various specifications of the filter on element values and hence select or tailor the best solution.

III. Formulation of Transfer Function and Element Extraction

In this section a novel method will be described for the extraction of complex TZs. The other TZs can be found as special cases of this general approach.extraction. Consider a passive, lossless, reciprocal two port circuit with resistive terminations. Using Belevitch notation [2], S-parameters S21(s) and S11(s) of a lossless, reciprocal two port can be expressed in the forms

$$S_{21}(s)=p(s)/e(s) \qquad S_{11}(s)=f(s)/e(s) \tag{1}$$

where p(s) and f(s) are even or odd polynomials with real coefficients and e(s) is a strictly Hurwitz polynomial related to f(s) and p(s) through Feldtkeller equation;

$$e(s)e(-s)= f(s)f(-s)+p(s)p(-s) \tag{2}$$

Transducer power gain of the two port generalized to s=σ+jω plane can be expressed in the form

$$S_{21}(s)S_{21}(-s)=\frac{1}{1+K(s)K(-s)} \tag{3}$$

where K(s) is termed as the characteristic function of the filter defined as

$$K(s)K(-s)=\varepsilon^2\frac{f(s)f(-s)}{p(s)p(-s)}=\pm\varepsilon^2\frac{f^2(s)}{p^2(s)} \tag{4}$$

ε is termed as passband ripple factor. f(s) and p(s) are monic polynomials (coefficients of highest order terms are unity). Response of the desired filter can be shaped by forming proper polynomials p(s) and f(s) which also set K(s)K(-s), or inversely, one can form a proper K(s) from which the polynomials f(s) and p(s) can be extracted. For typical equiripple or maximally flat bandpass filters p(s) and f(s) are of the form

$$p(s)=s^{n0}\prod\left(s^2+\omega_i^2\right)\prod\left[(s+\sigma_k)^2+\omega_k^2\right]\left[(s-\sigma_k)^2+\omega_k^2\right]$$
$$f(s)=\prod\left(s^2+\omega_r^2\right) \tag{5}$$

and hence,

$$K(s)K(-s)=\pm\varepsilon^2\frac{f^2(s)}{p^2(s)}$$
$$=\frac{\prod\left(s^2+\omega_r^2\right)^2}{s^{2n0}\prod\left(s^2+\omega_i^2\right)^2\prod\left[(s+\sigma_k)^2+\omega_k^2\right]^2\left[(s-\sigma_k)^2+\omega_k^2\right]^2} \tag{6}$$

where n_0 is the number of TZs at s=0, ω_i's and ω_r's are jω–axis TZs and reflection zeros respectively. The third factor in p(s) is formed by the complex conjugate quadruplets of TZs, $s_k=\pm\sigma_k\pm j\omega_k$. The number of TZs at s=∞ is equal to difference between degree of e(s) and degree of p(s). σ–axis TZs are excluded because they are used only in LP or HP filters. Amplitude response can be shaped by specifying the desired passband edge frequencies, passband ripple and by placing proper number jω-axis TZs with some being at s=0 and s=∞. Phase response can be shaped by placing complex TZs and tuning them while observing the response. The reflection zeros are automatically set after these specifications. That is f(s), can be recognised as the numerator of K(s)K(-s). Knowing p(s) and f(s), the polynomial e(s)e(-s) can be found from Feldtkeller equation. Then, e(s) is found using the left half s-plane roots of e(s)e(-s)=0. After e(s), f(s) and p(s) are obtained, one can form any one of the two port parameters of the circuit, like S, Z, Y, or ABCD, for element extraction. However due to severe ill conditioning, finding the roots of f(s)f(-s)=0 and e(s)e(-s)=0 are problematic because all reflection zeros are squeezed inside the passband while the roots of e(s)e(-s) are clustered near the passband edges. Such numerical accuracy problems are reduced to a lot by if the whole synthesis is carried out in the transformed frequency domain as summarized below

F. Formulation of Impedance Functions in Transformed Domain

The following frequency transformation maps the passband of bandpass filters on s=jω axis onto the whole imaginary axis of the transformed domain [3], [4]:

$$z^2 = \frac{s^2+\omega_{p2}{}^2}{s_n^2+\omega_{p1}{}^2} \qquad \text{Re}(z)>0 \quad (\text{For} \quad \text{BPF}) \tag{7}$$

where ω_{p1} and ω_{p2} are the lower and upper passband edge frequencies. This transformation separates the zeros clustered in and near the passband, easing numerical accuracy problems. The lower stopband $0 \le \omega \le \omega_{p1}$ is mapped into the range $\omega_{p2}/\omega_{p1} \le x \le \infty$ on z=x axis. The upper stopband, $\omega_{p1} \le \omega \le \infty$ is mapped into the range $0 \le x \le 1$ on z=x axis. The whole real s=σ axis is compressed into the range $1 \le x \le \omega_{p2}/\omega_{p1}$ on z=x axis. Under this transformation a TZ at s=0 maps to $Z_i=1/a$ while a TZ at s=∞ maps to $Z_i=1$. A jω-axis TZ pair, $s_i = \pm j\omega_i$ is mapped to the same point $Z_i=X_i$ on real z-axis:

$$Z_i = \sqrt{\frac{1-\omega_i^2}{a^2-\omega_i^2}} \tag{8}$$

A complex TZ quadruplet $s_i = \pm\sigma_i \pm j\omega_i$ is mapped onto z=x+jy plane as $Z_i=X_i \pm jY_i$ where

$$X_i = \sqrt{\left(X_o^2+Y_o^2\right)^{1/2}}\cos\theta/2\,,\; Y_i = \sqrt{\left(X_o^2+Y_o^2\right)^{1/2}}\sin\theta/2$$
$$X_0 = \frac{\left(1+\sigma_i^2-\omega_i^2\right)\left(a^2+\sigma_i^2-\omega_i^2\right)+4\omega_i^2\sigma_i^2}{\left(a^2+\sigma_i^2-\omega_i^2\right)^2+4\omega_i^2\sigma_i^2},$$
$$Y_0 = \frac{2\omega_i\sigma_i\left(a^2-1\right)}{\left(a^2+\sigma_i^2-\omega_i^2\right)^2+4\omega_i^2\sigma_i^2}\,,\; \theta=\tan^{-1}(Y_o/X_o \tag{9}$$

The desired characteristic function for an equiripple passband general stopband bandpass filter can be formed by using these TZs as follows:

Let's denote the transformed versions of the polynomials f(s) and p(s) by $F(z^2)$ and $P(z^2)$ respectively. Then the transformed version of K(s)K(-s) can be written as

$$K(z^2)\overline{K}(z^2) = \frac{F(z^2)\overline{F}(z^2)}{P(z^2)\overline{P}(z^2)} \tag{10}$$

Consider the polynomial V(z) defined in terms of the transformed versions of the TZs, Z_i as

$$V(z) = \prod_{i=1}^{n1}(Z_i+z)\prod_{i=1}^{n2/2}(z^2+2X_iz+X_i^2+Y_i^2) \tag{11}$$

where the first term is due to n_1 jω-axis TZs including those at s=0 and s=∞ and the second term is due to the complex TZs with n_2 being the total degree. Since complex TZs are placed as quadruples, n_2 and $n_2/2$ are even always. Consider the function formed by V(z) and V(-z):

$$K(z^2)\overline{K}(z^2) = \varepsilon^2\frac{1}{4}\left(1+\frac{V(z)}{V(-z)}\right)\left(1+\frac{V(-z)}{V(z)}\right) \tag{12 }$$

Equiripple property of this function can readily be proved by using complex algebra as follows:
Let

$$\gamma_i = \cosh^{-1}\left(\frac{Z_i}{\sqrt{Z_i^2-z^2}}\right),$$

$$\chi_i = \cosh^{-1}\left(\frac{\left|z^2+|Z_i|\right|}{\sqrt{\left(z^2+|Z_i|\right)^2-4X_i^2z^2}}\right)$$

$$\frac{(Z_i+z)}{(Z_i-z)} = e^{2\gamma_i}, \frac{z^2+2X_iz+X_i^2+Y_i^2}{z^2-2X_iz+X_i^2+Y_i^2} = e^{2\chi_i} \tag{13}$$

Using these polar forms, V(z)/V(-z) can be written as

$$\begin{aligned}\frac{V(z)}{V(-z)} &= \prod_{i=1}^{n1}\frac{(Z_i+z)}{(Z_i-z)}\prod_{i=1}^{n2}\frac{z^2+2X_iz+X_i^2+Y_i^2}{z^2-2X_iz+X_i^2+Y_i^2} \\ &= \prod_{i=1}^{n1}e^{2\gamma_i}\prod_{i=1}^{n2}e^{2\chi_i} = e^{\sum 2\gamma_i}e^{\sum 2\chi_i} = e^{2\gamma}e^{2\chi}\end{aligned} \tag{14}$$

where γ and χ are defined as

$$\gamma = \sum\gamma_i \quad \text{and} \quad \chi = \sum\chi_i \tag{15}$$

Using (14) in (12) we get

$$\begin{aligned}K(z^2)\overline{K}(z^2) &= \varepsilon^2\frac{1}{4}\left(1+e^{2\gamma}e^{2\chi}\right)\left(1+e^{-2\gamma}e^{-2\chi}\right) \\ &= \varepsilon^2\cosh^2\left[\sum_{i=1}^{n1}\cosh^{-1}\left(\frac{Z_i}{\sqrt{Z_i^2-z^2}}\right) + \sum_{i=1}^{n2}\cosh^{-1}\left(\frac{\left|z^2+|Z_i|\right|}{\sqrt{\left(z^2+|Z_i|\right)^2-4X_i^2z^2}}\right)\right]\end{aligned} \tag{16}$$

Inside the passband substitution of z=jy yields

$$K(z^2)\overline{K}(z^2) = \varepsilon^2\cos^2(\gamma+\chi) \tag{17}$$

So the condition $0 \le K(z^2)\overline{K}(z^2) \le \varepsilon^2$ is satisfied inside the passband in an equiripple manner.

After forming $K(z^2)\overline{K}(z^2)$, the z-domain counterparts of f(s), p(s) and e(s) which are denoted by $F(z^2)$, $P(z^2)$ and $E(z^2)$ respectively can be determined. The numerator and denominator of $K(z^2)\overline{K}(z^2)$ give directly $F(z^2)$ and $P(z^2)$ respectively:

$$\begin{aligned}F(z^2) &= \varepsilon V_e(z^2) \\ P(z^2) &= V_e(z^2)+zV_o(z^2)\end{aligned} \tag{18}$$

where the subscripts e and o refer to even and odd parts of the polynomial V_e. The expression for $E(z^2)$ will be extracted from the product $E(z^2)\bar{E}(z^2)$ which is formed as

$$E(z^2)\bar{E}(z^2) = \left(1+\varepsilon^2\right)V_e^2(z^2) - z^2 V_o^2(z^2) \qquad (19)$$

using Feldtkeller equation. $E(z^2)$ should contain only the left half of s-plane roots. However information on the locations of roots are lost after transformation into z-domain. Therefore $E(z^2)$ need be found through an indirect way as described in [4].

G. Element Extraction Procedure

Element extractions can be carried out from z_{11} and y_{11} parameters which can be found in terms of even and odd parts of the polynomials E(z), F(z) and P(z) as follows:

$$\frac{z_{11}}{R_1} = \frac{\mathrm{Odd}(E(z^2)) \pm \mathrm{Odd}(F(z^2))}{\mathrm{Even}(E(z^2)) + \mathrm{Even}(F(z^2))} \qquad (20)$$

$$y_{11}R_1 = \frac{\mathrm{Odd}(E(z^2)) + \mathrm{Odd}(F(z^2))}{\mathrm{Even}(E(z^2)) \mp \mathrm{Even}(F(z^2))} \qquad (21)$$

where upper and lower signs refer to symmetric and antimetric circuits respectively. The input impedances corresponding to these functions, (renamed as Z_{in}) can take one of the following four types:

Type-1: z_{in} has poles at both s=0 and s=∞:

$$z_{in}(z) = \frac{N(z^2)}{D(z^2)\sqrt{z^2-1}\sqrt{1-a^2z^2}} \qquad (22)$$

Type-2: z_{in} has zero at s=0 and a pole at s=∞:

$$z_{in}(z) = \frac{N(z^2)\sqrt{1-a^2z^2}}{D(z^2)\sqrt{z^2-1}} \qquad (23)$$

Type-3: z_{in} has pole at s=0 and a zero at s=∞:

$$z_{in}(z) = \frac{N(z^2)\sqrt{z^2-1}}{D(z^2)\sqrt{1-a^2z^2}} \qquad (24)$$

Type-4: z_{in} has zeros at both s=0 and s=∞:

$$z_{in}(z) = \frac{N(z^2)\sqrt{z^2-1}\sqrt{1-a^2z^2}}{D(z^2)} \qquad (25)$$

where N and D are real polynomials. Symmetric bandpass filters are always of Type-1 or Type-4.

In cascade synthesis each extracted element or circuit section realises a certain TZ. A jω-axis finite TZ is realised as a Brune section by zero shifting technique (partial pole removal). The pole (zero) of the admittance (impedance) function, at either s=0 (if the TZ is in lower stopband) or s=∞ (if the TZ is in upper stopband) is shifted to the frequency of the desired TZ in order to create a zero (pole) at that frequency in the remaining admittance (impedance) function. The same technique is also applied for extraction of the σ-axis TZs, leading to Darlington-C sections. Since usually σ-axis TZs appear only in lowpass filters, they are extracted by zero shifting from s=∞. Complex TZs are traditionally extracted as Darlington-D sections which are rather complicated structures for realization [5]. One novelty of this paper is that complex TZs are extracted either as a series or shunt fourth order section as shown in Fig. 7. This is possible by shifting zeros at both s=0 and s=∞ simultaneously to create TZs at the desired locations. This technique can be summarized for realisation of series arm fourth order section of a complex TZ as follows.

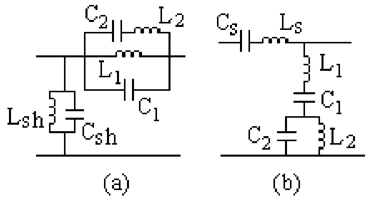

Fig. 7. Complex TZ sections.

In order to extract such a circuit section, the input admittance $1/Z_{in}(z)$ must have a zeros at both s=0 and s=∞. Hence Z_{in} is of Type-4. The zeros at s=0 and s=∞ are shifted to $s_0=\pm\sigma_0\pm j\omega_0$ which corresponds to $Z_0 = \pm X_0 \pm jY_0$ by extracting shunt inductor and capacitor, L_{sh} and C_{sh}, as shown in Fig. 7. This process leads to the relation

$$Y_1(z)\big|_{z=Z_0} = Y_{LC}(z)\big|_{z=Z_0} + Y_2(z)\big|_{z=Z_0} \qquad (26)$$

where $Y_1(z)$ is the original input admittance and $Y_2(z)$ is the admittance of remaining circuit after removal of the pair L_{sh}-C_{sh}. The values of L_{sh} and C_{sh} should be such that $Y_2(z)\big|_{z=Z_0} = 0$. The remaining impedance $Z_2(z)=1/Y_2(z)$, will have a pole at $Z_0 = \pm X_0 \pm jY_0$ which can be extracted as a series arm fourth order section whose impedance $Z_q(z)$ can be expressed in terms of four unknowns, L_1, C_1, L_2, C_2 in z-domain as

$$Z_q(z) = \frac{\left(z^2-1\right)\sqrt{z^2-1}\sqrt{1-a^2z^2}}{\left(\left(\sigma_0^2-\omega_0^2+a^2\right)^2+4\omega_0^2\sigma_0^2\right)} \times \frac{\left(\frac{1-a^2z^2}{z^2-1}\frac{1}{C_1}+\frac{1}{L_2C_2C_1}\right)}{(z-Z_{01})(z-Z_{02})(z-Z_{03})(z-Z_{04})} \qquad (27)$$

In order to have the complex TZs at the desired frequencies, the fourth order section element values should satisfy the following two equations:

$$\left(\frac{1}{L_2C_2}+\frac{1}{L_1C_1}\frac{1}{L_2C_1}\right) = -2\left(\sigma_0^2-\omega_0^2\right)$$
$$\frac{1}{L_2C_2L_1C_1} = \left(\sigma_0^2+\omega_0^2\right)^2 \qquad (28)$$

On the other hand the relation between the impedance $Z_2(z)$ and $Z_q(z)$ is

$$Z_2(z)\left[(z-Z_{01})(z-Z_{02})(z-Z_{03})(z-Z_{04})\right]\Big|_{z=Z_0} = Z_q(z)\left[(z-Z_{01})(z-Z_{02})(z-Z_{03})(z-Z_{04})\right]\Big|_{z=Z_0} \quad (29)$$

Equating the real and imaginary parts of this equation we get two more equations, thus enabling us to solve the four unkowns L_1, C_1, L_2, C_2. Extraction of the shunt arm fourth order section shown in Fig. 7 can be done in the same way, but by starting from the impedance of Type-1. In both fourth order sections of Fig. 7, the elements L_1 and C_1 come out to be negative. The negative element problems will be resolved after conversion into cross-coupled form.

The jω-axis and σ-axis TZs can be extracted through the same formulation, as special cases with either $\sigma_i=0$ or $\omega_i=0$.

IV. Design Example

In this section an example will be presented to demonstrate the new approach. It is observed that in linear phase filters the complex TZs $s_i = \pm\sigma_i \pm j\omega_i$ should be chosen as $\sigma_i \approx (\omega_{p1}-\omega_{p2})/2$ and $\omega_i \approx \omega_{p0}$ with ω_{p1} and ω_{p2} being passband edge frequencies and ω_{p0} being passband center. The passband center is customarily defined as the geometric center which is correct only for bandpass filter mapped from lowpass prototypes. However this definition is not valid for the filters with asymmetric responses. The actual passband center is affected by the number of TZs at infinity, zero and finite frequencies as well as passband edge frequencies.

Example:. Specifications of the filter are:

Passband ripple = 0.1 dB, passband edge frequencies at f_{p1}=790 MHz, f_{p2}=810 MHz, two finite jω–axis TZs are placed at 840 MHz and 860 MHz. A complex TZ is placed at ±10±j799.7 MHz. 3 TZs are placed at both f=0 and f=∞.

The TZs are extracted in the order shown in Fig. 8.a. Norton transformations are applied on series elements to convert the structure into coupled resonator form shown in Fig. 8.b. Using matrix operations the filter is then converted into CQ form as given in Fig. 8.c. Response of the filter is shown in Fig. 8.d. It is seen that phase response is linearized within 50% of the passband with only one complex TZ quadruplet.

V. Conclusion

The theory of cascade synthesis is revised to include doublets, cascaded triplets (CT), cascaded quadruplets (CQ) and other N-tuplets of coupled resonators as building blocks. The TZs are extracted in groups to form circuit sections which are readily convertible to N-tuplets of cross coupled resonators. Formulation of element extraction in transformed frequency domain is generalised to cover extraction of complex TZs, by zero shifting from both s=0 and s=∞, leading to fourth order sections that can be converted into CQ sections. Thus, CQ sections can be used for realisation of complex TZs, besides realisation pairs of jw-axis TZs. It is shown that s=σ axis TZs can be realized as CT sections in the same way as s=jω axis TZs. This approach, besides easing formation of cross coupled resonator filters, eases the usual numerical accuracy problems associated with the classical Darlington Type element extraction procedures. A design example is presented with one complex TZ and two jw-axis FTZ's in upper stopband.

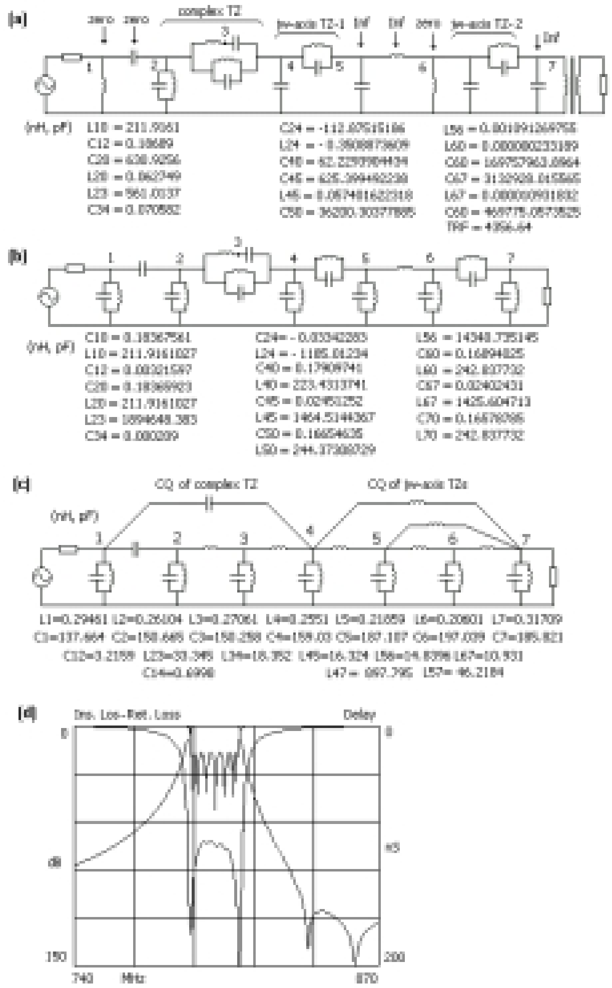

Fig. 8. Design stages of the example.

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APPENDIX: MATRIX TRANSFORMATIONS

As a demonstration of the technique for conversion of a direct coupled resonator circuit into a cross coupled one, realisation of a $j\omega$-axis FTZ as a CT (Cascaded Triplet) section will be described qualitatively, with reference to Fig. 3. Being a three resonator circuit, a CT section is a degree six circuit. Two degrees will be provided by the FTZ and the remaining four degrees will come from the transmission zeros at s=0 and s=∞. These four degrees can be shared between transmission zeros at f=0 and f=∞ as follows:

- Nzero=1, Ninf=3, leading to a circuit with a series inductor between Node-1 and Node-2, termed as inductive prototype.
- Nzero=3, Ninf=1, leading to a circuit with a series capacitor between Node-1 and Node-2, termed as capacitive prototype.

The two prototypes and conversion of the synthesized circuits into direct coupled resonator form are shown in Fig. 3.b. The direct coupled resonator circuits are then transformed into CT form using matrix operations. The Nodal Matrix of the direct coupled circuit is in the following form:

$$\begin{bmatrix} y_{11} & -y_{12} & 0 \\ -y_{12} & y_{22} & -y_{23} \\ 0 & -y_{23} & y_{33} \end{bmatrix} \quad \text{(A1)}$$

where $y_{ij} = sC_{ij}+1/sL_{ij}$ i,j = 1,2,3 and i≠j ,
$y_{ii} = sC_i + 1/sL_i + y_{i(i+1)} + y_{i(i-1)}$

In order to eliminate either L_{23} or C_{23} and to introduce coupling between nodes 1 and 3, row 2 is multiplied by m and added to row 3 and then column 2 is multiplied by m and added to column 3. Resultant matrix has the following form:

$$\begin{bmatrix} y_{11} & -y_{12} & -my_{12} \\ -y_{12} & y_{22} & -y_{23}+my_{22} \\ -m\times y_{12} & -y_{23}+my_{22} & y_{33}-2my_{23}+m^2y_{22} \end{bmatrix} \quad \text{(A2)}$$

Proper choice of m results in single element (either capacitive or inductive) coupling between Node--2 and Node-3 while introducing a bridge type coupling between Node-1 and Node-3. The cross coupling element between nodes 1 and 3 will be the same type as the element between Nodes 1 and 2. That is, an inductor L_{12} of inductive prototype will lead to inductive cross coupling element L_{13} and a capacitive coupling element C_{12} of capacitive prototype will lead to a capacitive cross coupling C_{13}. The type of coupling element between resonators 2 and 3 is set by the position of the FTZ with respect to passband. An upper stopband transmission zero results in an inductive coupling L_{23} while a lower stopband transmission zero results in a capacitive coupling. For each prototype two more solutions can also be obtained which involve approximations. This is obtained by multiplying the third row and column of the final nodal matrix given in Eq. (A2) by –1. This approach yields negative L_{13} for inductive prototype and negative C_{13} for capacitive prototype. In narrowband filters the negative inductor (capacitor) can be replaced by positive capacitor (inductor) with negligible distortion in response.