Calculation of Electromagnetic Field inside a Wire Structure in a Pulse Field of Lightning Discharge

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Abstract – **Calculation of electric field and potential inside a wire structured object in external pulse electromagnetic field of lightning discharge is presented in this paper. Conductive structure is solved as a receiving antenna using program SPAN [1,2] for an arbitrary antenna configuration, consisting of lineic cylindrical segments, in external electromagnetic field. One simple approximation [3] for external pulse field is used, which parameters are calculated for standard pulse function. FFT (Fast Fourier's Transform) is performed on this function and antenna response in time domain is obtained using IFFT (Inverse Fast Fourier's Transform) to the results of program SPAN in frequency domain. Results for electric field and for potential, as the consequences of induced currents, are presented in the paper in the form of graphics.**

Keywords – **Pulse electromagnetic field, receiving antenna, lightning discharge.**

I. INTRODUCTION

Pulse response of a wire antenna structure, consisting of lineic cylindrical segments, is obtained using programs SPAN and FAS[4]. Integral equation of Hallen's type is used for determining currents along conductive segments and for obtaining receiving antenna response for each frequency of incident field. Currents are approximated by polynomials with complex coefficients. Collocation Method i.e. Point Matching Method is used for approximate numerical solving of the equations system. In the case of pulse field of lightning discharge current, one simple approximation of pulse function representing external electric field is chosen. FFT is performed on this function, program SPAN is used in frequency domain and, afterwards, IFFT is used for obtaining results in time domain. The results for electric field in some points inside a chosen wire structure are presented in the paper.

II. CALCULATION OF ELECTROMAGNETIC FIELD OF RECEIVING ANTENNA STRUCTURE

For an arbitrary conductive configuration in external electromagnetic field of a plane wave of arbitrary frequency, on condition that all parts can be treated as thin cylindrical lineic segments with equivalent radii, following equation system is obtained, for determining complex constants and coefficients in the polynomial approximations of the cur- rents along conductive segments:

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$$\sum_{l=1}^{n}\int_{0}^{h^{(l)}}\underline{I}_{z^{(l)}}(z^{(l)\prime})\underline{K}_{m,l}(z^{(m)},z^{(l)\prime})\,\mathrm{d}z^{(l)\prime}-\frac{4\pi\underline{C}_{1m}}{\mu}\cos(kz^{(m)})-$$

$$-\frac{4\pi\underline{C}_{2m}}{\mu}\sin(kz^{(m)})=\frac{4\pi}{\mathrm{j}c\mu}\int_{0}^{z^{(m)}}\underline{E}_{z^{(m)}e}\sin\left[k(z^{(m)}-s)\right]\mathrm{d}s$$

$$\text{for } m=1,\ldots,n, \quad (1)$$

where: k –is the phase constant for frequency f,

$\underline{C}_{1m}$ and $\underline{C}_{2m}$ –are the complex constants,

$(x^{(m)},y^{(m)},z^{(m)})$ –is the matching point in Descartes' coordinate system with m–th segment as z–axis,

n –is the number of conductive cylindrical lineic segments of radii $a^{(m)}$, $m=1,\ldots,n,$

$\theta_{l,m},\psi_{l,m}$ and $\varphi_{l,m}$ –are mutual Euler's angles for corresponding Descartes' coordinate systems for l–th and m–th segments as z–axes and

$\underline{E}_{z^{(m)}e}$ –is the tangential component of external electric field in the matching point on m–th segment. Matching points are chosen equidistantly along segments, including both ends of segments.

For $l \neq m$, kernel of the integral in (1) is:

$$\underline{K}_{m,l}(z^{(m)},z^{(l)\prime})=\cos(\theta_{l,m})\left.\frac{\mathrm{e}^{-\mathrm{j}kr_{l,m}}}{r_{l,m}}\right|_{\substack{x^{(m)}=a^{(m)}\\ y^{(m)}=0}}-$$

$$-\int_{0}^{z^{(m)}}\cos\left[k(z^{(m)}-s)\right]\left[\frac{\partial}{\partial z^{(l)\prime}}+\cos(\theta_{l,m})\frac{\partial}{\partial z^{(m)}}\right]\left.\frac{\mathrm{e}^{-\mathrm{j}kr_{l,m}}}{r_{l,m}}\right|_{\substack{x^{(m)}=a^{(m)}\\ y^{(m)}=0,\ z^{(m)}=s}}\mathrm{d}s, \quad (2)$$

and for $l=m$:

$$\underline{K}_{l,l}(z^{(l)},z^{(l)\prime})=\left.\frac{\mathrm{e}^{-\mathrm{j}kr_{l,l}}}{r_{l,l}}\right|_{\substack{x^{(l)}=a^{(l)}\\ y^{(l)}=0}}, \quad (3)$$

where $r_{l,m}$ is the distance between the matching point on m–th segment and the point on l–th segment with current $\underline{I}_{z^{(l)}}(z^{(l)\prime})$ on the axis.

Polynomial approximations for the segments' currents are:

$$\underline{I}_{z^{(m)}}(z^{(m)}{}') = \sum_{t=0}^{p_m} \underline{B}_{mt} \left(\frac{z^{(m)}{}'}{h^{(m)}} \right)^t , \quad \text{for } m = 1,\ldots,n, \quad (4)$$

where: $h^{(m)}$ –is the length of m–th conductive segment,

$z^{(m)}{}'$ –is the point coordinate along current source,

$\underline{B}_{mt}$ –are the complex coefficients and

p_m –is the polynomial degree for the current approximation along m–th segment of the structure.

Electric field in the points in near zone of the structure is the result of external field, $\underline{\boldsymbol{E}}_e$, and of field $\underline{\boldsymbol{E}}$, that is the consequence of induced currents, which components can be determined from the results for magnetic vector potential, according to the equation:

$$\underline{\boldsymbol{E}} = -\operatorname{grad}\underline{\varphi} - \mathrm{j}\omega\underline{\boldsymbol{A}} = -\mathrm{j}\frac{c}{\beta}\operatorname{grad}\operatorname{div}\underline{\boldsymbol{A}} - \mathrm{j}\omega\underline{\boldsymbol{A}}. \quad (5)$$

III. FFT OF PULSE EXCITATION

One simple analytical approximation of pulse function from Ref. [3] is used,

$$\frac{y(t)}{y_{\max}} = \begin{cases} [\tau \exp(1-\tau)]^a, & 0 \le \tau \le 1 \\ [\tau \exp(1-\tau)]^b, & 1 \le \tau < \infty \end{cases}$$

$$\text{for } \tau = t/t_m, \quad (6)$$

which, for the standard pulse 1.2/50, that lasts about $50\,\mu\mathrm{s}$ with rising time $1.2\,\mu\mathrm{s}$, has the parameters values of $a = 4$ and $b = 0.0312596735$, while $t_m = 1.906398381\,\mu\mathrm{s}$. Time t_m is the time in which pulse function achieves maximum value and after that time the intensity decreases to null (Fig. 1).

In order to analyze conductive structure's response to pulse electromagnetic field excitation of a lightning discharge current, FFT is performed in 8192 points, because of specific pulse excitation function with these characteristics: short upward and long lasting downward part of the function representing this field.

For sampling of the function $y(t)/y_{\max}$ from Fig.1, the interval $\Delta T = 19.06398381\,\mathrm{ns}$ is chosen. This sampling interval and the number of points $N = 8192$ for FFT corresponds to transforming of the function $y(t)/y_{\max}$ by FFT from time domain, from interval $[0,T]$, where $T = N \cdot \Delta T = 156.17215537152\,\mu\mathrm{s}$, to the frequency domain and to interval $[-f/2, f/2]$. Program **FAS** [4] is used for FFT and for IFFT also. Sampling frequency is $f = 1/\Delta T \cong 52.455\,\mathrm{MHz}$ and corresponding interval in frequency domain is $\Delta f = (N \cdot \Delta T)^{-1} = T^{-1} \cong 6.4\,\mathrm{kHz}$. Real and imaginary parts of FFT for pulse function presented on Fig.1 are presented on Fig.2 and Fig.3. It is not necessary to take into account all of the points in order to obtain IFFT, which is very useful for decreasing computation time in frequency domain.

IV. RESULTS

The influence of pulse field of lightning current on the field nearby some objects and installations, which act as non–intending receiving antennas, can be calculated using the presented procedure. One conductive structure shaped as parallelepiped, with basis dimensions $a = 9\,\mathrm{m}$, $b = 12\,\mathrm{m}$ and height $c = 6\,\mathrm{m}$, is chosen. Radii of all conductive segments are $a^{(i)} = 0.03\,\mathrm{m}$, for $i = 1,\ldots,n$. For this object on ground surface, its plane mirror figure has to be added, while ground half–space excluded and taken into account with reflected field added to the incident field. Cage structure as in Fig.4 has to be treated in frequency domain, with minimum 12 segments.

Pulse electric field is given in the first node (Fig.4), $E_\theta = E(t)$, $E_\psi = 0$, $\theta = 90^{\mathrm{o}}$, $\psi = 0$, where θ and ψ are cylindrical coordinates. For maximum value of the incident field $E_{\max} = 1\,\mathrm{V/m}$, all of the results presented in the form of graphics. So, all of the values of the resulting field have to be multiplied by factor $\mathrm{MPF} = 6000$ for the distance about $200\,\mathrm{m}$, i.e. with 1800 for the distances about $500\,\mathrm{m}$ from the lightning current of maximum $I = 10\,\mathrm{kA}$, as presented in Ref. [5]. It is sufficient to take $p_m = 2$ as the polynomial degree for the approximation of currents. Program SPAN is modified so that it calculates the response for all of the frequencies for FFT, i.e. in all points of discretization in frequency domain. IFFT is used to determine time domain response on the basis of obtained SPAN results. Electric field inside the conductive structure is of pulse shape, but amplitudes are different in different points inside the structure, depending on coordinates.

Chosen points in which electric field is calculated are: A (–6 m, 4.5 m, 8 m), B (–4 m, 3 m, 8 m), C (–4 m, 8 m, 8 m), D (–10 m, 3 m, 8 m), E (–10 m, 8 m, 8 m), F (–6 m, 4.5 m, 10 m), G (–4 m, 3 m, 10 m), H (–4 m, 8 m, 10 m), I (–10 m, 3 m, 10 m) and J (–10 m, 8 m, 10 m). The results for electric field components are presented in Fig.5–10, for the points A (Fig.5), B (Fig.6), C (Fig.7), D (Fig.8), H (Fig.9) and I (Fig.10). First node is the origin of the Descartes' coordinate system with segment (1) as z–axis. These graphics present pulse character of electric field inside parallelepiped structure, but maximum values are different in different points. These values enable estimation of electric field values for the purpose of protection equipment against EM field influence inside objects.

$N = 8192$ points for FFT prolongs the calculation time, but much smaller number than this is also enough for obtaining response for chosen example of structure. Using results for $N/10$ points, results can be obtained using IFFT and adding nulls for the rest of the points, because the expected solution for field components in each point has to be also of pulse shape. It also possible that the number of the points of interest for obtaining solution can be even smaller and this should be investigated, because it decreases the time for calculations, but that is also related to desired accuracy.

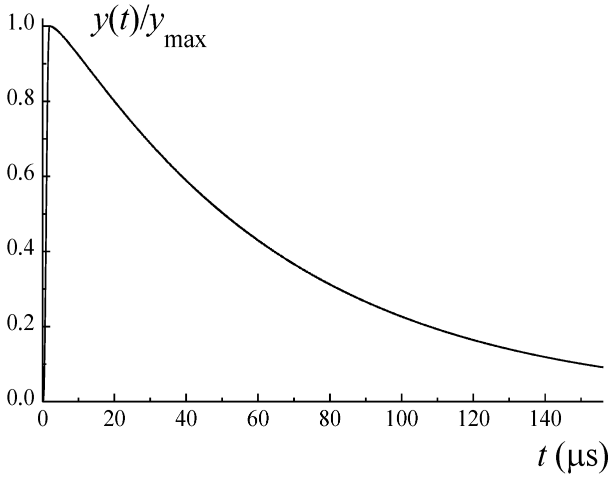

Fig.1 *Normalised value of pulse electric field*

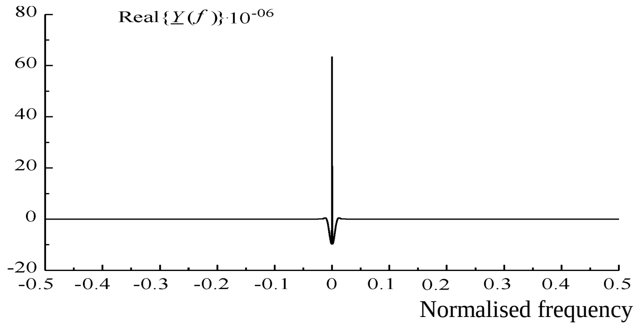

Fig.2 *Real part of FFT for pulse excitation (Fig.1)*

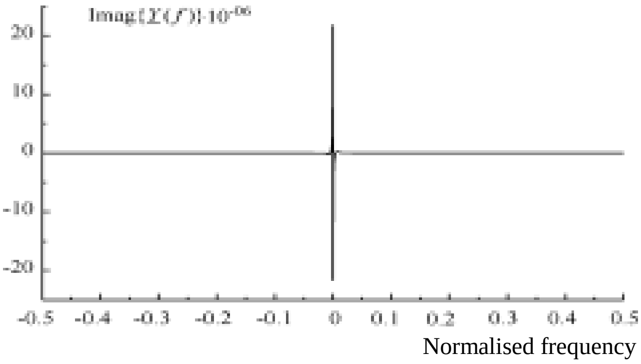

Fig.3 *Imaginary part of FFT for pulse excitation (Fig.1)*

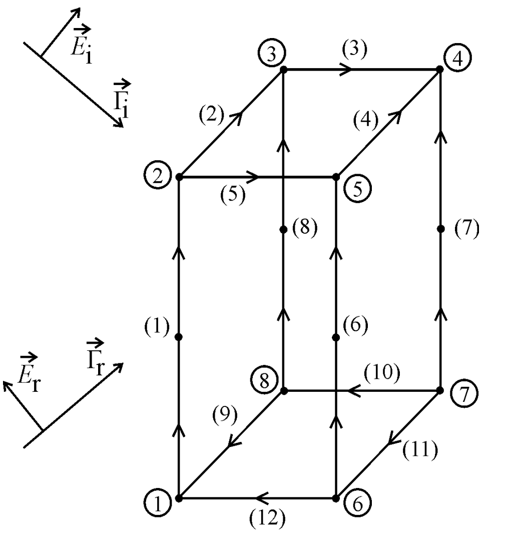

Fig.4 *Conductive structure in a pulse EM field*

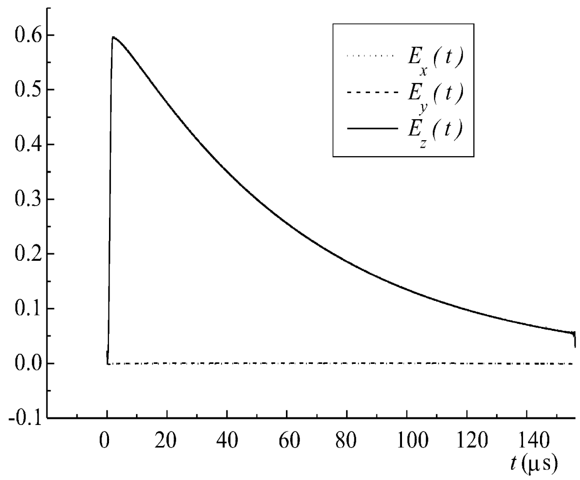

Fig.5 *Electric field in the point* A (–6 m, 4.5 m, 8 m)

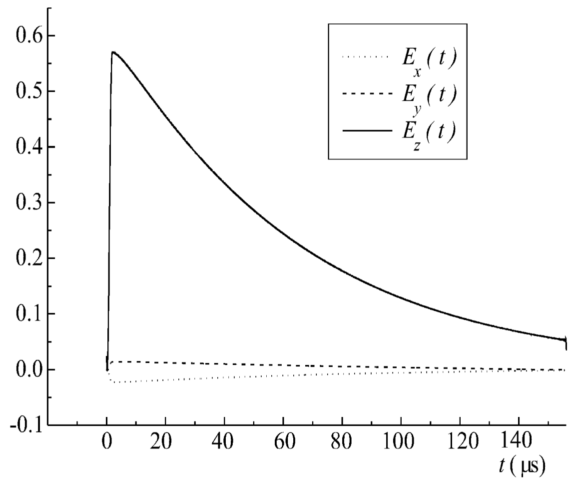

Fig.6 *Electric field in the point* B (–4 m, 3 m, 8 m)

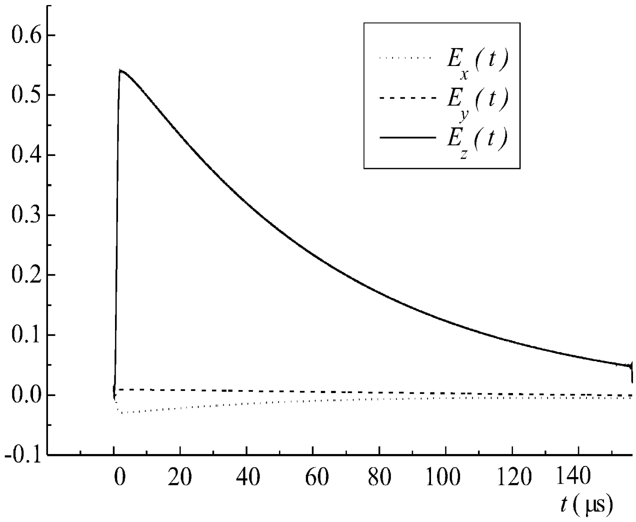

Fig.7 *Electric field in the point* C (–4 m, 8 m, 8 m)

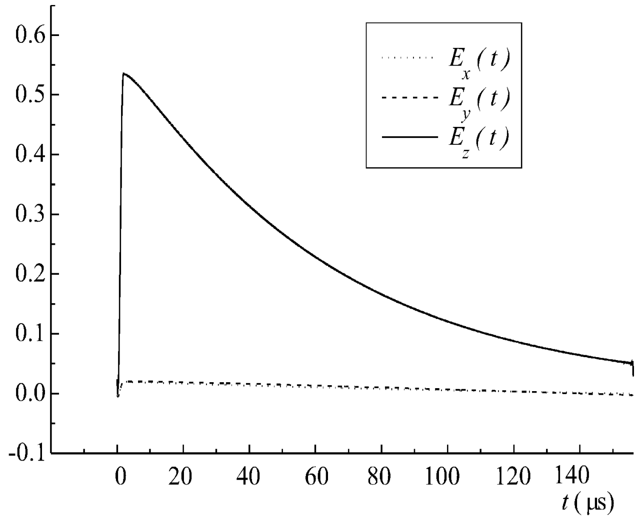

Fig.8 *Electric field in the point* D (–10 m, 3 m, 8 m)

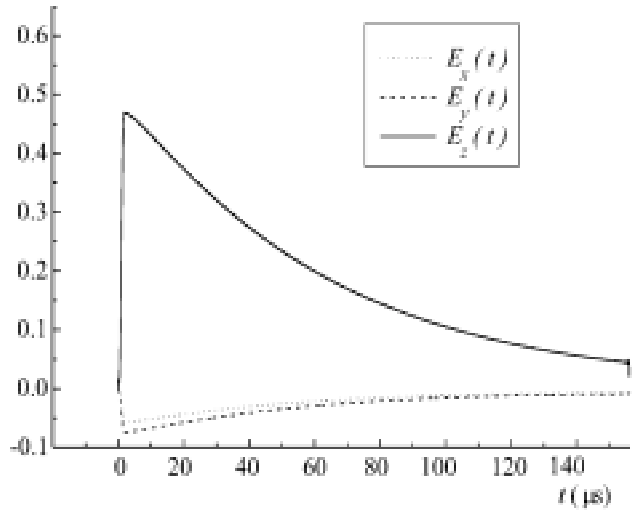

Fig.9 *Electric field in the point* H (–4 m, 8 m, 10 m)

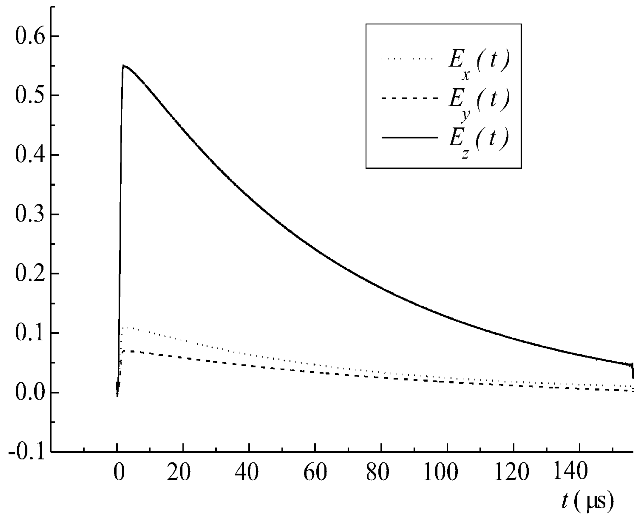

Fig.10 *Electric field in the point* I (–10 m, 3 m, 10 m)

V. CONCLUSION

Using simple approximation for the pulse excitation, program FAS for FFT and program SPAN for the frequency domain, pulse response of receiving antenna structure in external EM field is obtained. The chosen example can represent one small building configuration with conductive edges, but this procedure can be used for arbitrary conductive configuration consisting of arbitrary positioned lineic segments. Results for the field in some points inside object present pulse character of the electric field function, but also show that some points inside object are more protected from external field then other.

The procedure that is used for calculating electric field in the case of lightning discharge, can be used in the cases of other pulse excitations with respectable energy dissipation, such as EM pulse of nuclear explosions (EMINE), and other important analyses of EM field influence on certain objects.

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