Metamaterials: a New Concept in the Microwave Technique

Béla Szentpáli

Abstract - **The history of the metamaterial concept is outlined. The story started with the intuition of Veselago and continued almost thirty years later with the works of Pendry and co-workers. Their constructions resulting negative relative dielectric constant and negative magnetic permeability are also reviewed. The difference between the metamaterial structures and the photonic crystals is described. The experimental verifications are also presented.**

Keywords - **Metamaterial, negative index of refraction, left-handed medium, microwave technique**

I. Introduction

Metamaterials exhibit qualitatively new electromagnetic response functions, which can not found in the nature. To be more exact, the non-natural electromagnetic response means that the index of refraction in these materials is negative.

These substances are artificially fabricated periodic metallic structures. The length of the period is smaller than the wavelength at which the strange properties occur. These structures can be fabricated easily for the microwave domain, where the characteristic dimensions range from a few mm to a few cm. In the optical domain the structures should have dimensions in the micrometer range, or less. Therefore the manufacture of optical metamaterial is not yet possible; it is the task of the future. In this paper we will review the development of this concept roughly in the chronological sequence. Before doing it some general properties of the plane waves are recalling to mind.

II. Plane Waves

In a plane wave the electromagnetic field vectors $\vec{E}$ and $\vec{H}$ can be described as:

$$\vec{E}, \vec{H} \approx \exp(i\vec{k}\vec{r} - \omega t) \tag{1}$$

where $\vec{k}$ is the wave number.

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The Maxwell equations tell:

$$\mathbf{rot}\,\vec{E} = \frac{-1}{c}\frac{\partial \vec{B}}{\partial t} = \frac{-\mu}{c}\frac{\partial \vec{H}}{\partial t} \tag{2}$$

and

$$\mathbf{rot}\,\vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{\varepsilon}{c}\frac{\partial \vec{E}}{\partial t} \tag{3}$$

where $\varepsilon=\varepsilon_r\varepsilon_o$ and $\mu=\mu_r\mu_o$. Substituting (1) into (2) and (3) gives:

$$\vec{k} \times \vec{E} = \frac{\omega}{c}\mu\vec{H} \tag{4}$$

and

$$\vec{k} \times \vec{H} = -\omega\varepsilon\vec{E} \tag{5}$$

Namely in the case of plane waves the forming of the rotation of $\vec{E}$ and $\vec{H}$ is the same as the vector product of $\vec{k}$ and $\vec{E}$ or $\vec{H}$. This is because the partial derivation of the exponential function produces the multiplication of $\vec{E}$ by the components of $\vec{k}$ and the indexing of the rotation formation is the same as the indexing of the vector product. Equations (4) and (5) means that the set of vectors $\vec{k}$, $\vec{E}$ and $\vec{H}$ is a right handed set. In the usual case, when ε and μ are positive, $\vec{k}$ is parallel to the direction of the power flow, described by the Poynting vector:

$$\vec{S} = \vec{E} \times \vec{H} \tag{6}$$

The refraction index is:

$$n = \sqrt{\varepsilon_r \cdot \mu_r} \tag{7}$$

III. Veselago's Intuition

In 1967 V. G. Veselago, a physicist from the Lebedjev Physical Institute in Moscow published a paper [1], in which he speculated about the electrodynamics of materials having simultaneously negative ε_r and μ_r, these are the so called double negative material. As a consequence of Esq. (4) and (5) the direction of the $\vec{k}$ vector turns round and the set of vectors $\vec{k}$, $\vec{E}$ and $\vec{H}$ will be left handed. It means that the direction of the phase velocity is opposite to the direction of the power flow, i.e. opposite to the group velocity.

All phenomena connected to the phase of the wave will change to the opposite. An example of this is the Doppler effect, in a double negative medium the red shift will occur under approaching conditions and the blue shift when the source and receiver move away. Similarly in the case of the Cherenkov radiation the angle of emission will change to concave. In the metamaterial the emitted radiation propagates not forward, but backward.

The index of refraction is an important parameter when the wave propagation is described in the frame of the geometrical optics. This is also phase sensitive and in the case of left handed substance the square root in Eq. (7) should taken with negative sign [1]. The general sign rules are given in Table 1.

In normal material, when both ε_r and μ_r are positive the sign of n is also positive, in the case of double negative, left handed metamaterial when both ε_r and μ_r are negative n is also negative, in the cases when one of the ε_r or μ_r is positive, the other negative (off- diagonal terms in the table) the value of n is imaginary, so there is no wave propagation.

TABLE 1
Sign rules of the refraction index

	$\varepsilon_r>0$	$\varepsilon_r<0$
$\mu_r>0$	+	i
$\mu_r<0$	i	-

Substituting the negative n into the Snell's law it will result in a refracted beam at negative angle. The consequence of this fact is that a slab of left handed material will focus the radiation twice: in the material and the outgoing rays will focus again. Fig. 1. shows the refractions of the rays.

Within the medium the $\vec{k}$ vector points opposite to the Poynting vector. This means that $\vec{k}$ will change its phase at the interface. This phase reversal enables the medium to refocus the radiation.

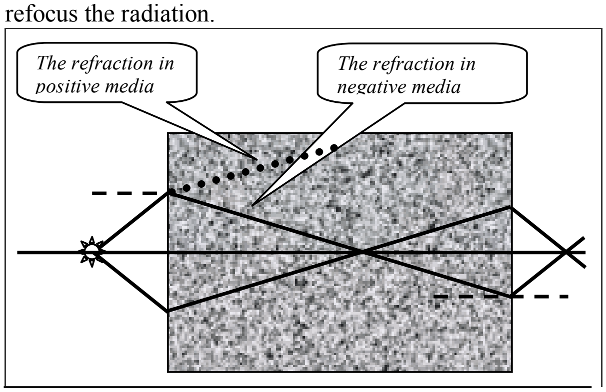

Fig. 1. The refraction in a slab of left-handed material. The negative refractive index occurs a bending of the rays to negative angles with the surface normal. The formerly diverging beams of a point source will converge back to a point in the material, if it is thick enough. Released from the metamaterial the rays reach a second focus. Of course when the source is placed into the left handed substance, then the two focal points form outside.

This lens effect can be improved very much if the values of εr and μ_r are equal [2]. In this case the wave impedance:

$$Z = \sqrt{\varepsilon / \mu} \tag{8}$$

will not change, it has the same positive value (120π Ω) within the left handed region as in the vacuum. Therefore the medium is in perfect match to the surrounding and no reflection will occur at the interface. The total energy is transmitted into the medium, only the $\vec{k}$ vector will change the polarity at the interface. As it has been pointed out [2] in this case the focusing effect is more perfect than in any traditional optics. The focal point will be a realistic projection of the source, i.e. the radiation can be focused to a smaller volume than the wavelength if the source was similar. Also the depth of focus is not limited. The only remaining limitation is the extension (diameter) of the material, as the diffraction on the periphery decreases the resolution.

This property of the media with negative refraction index offer interesting applications. In next chapters we will see that the negative index of refraction can be reached now only in the microwave domain. It seems that a slab of such substance can replace antennas on a very straightforward way. Also it is possible to fabricate near field electrostatic, or magnetostatic lenses operating at GHz frequencies.

IV. THE PENDRY'S PROPOSALS

The Veselago's intuition remained without any echo for 29 years. In 1996 J. B. Pendry from the Imperial College London and his co-authors from the GEC-Marconi published a paper [3] about an artificial metallic construction, which exhibit negative ε_r. The understanding of this concept starts from the nature of the dielectric constant of metals.

A . The ε_r in metals

The so-called Drude-model of metals tells that the conduction electrons are free, however, there is a small friction to the lattice, which is represented by a relaxation time [4], and then the differential equation of the electron motion in electric field is:

$$\dot{\vec{v}} + \frac{\vec{v}}{\tau} = \frac{e}{m^*}\vec{E} \tag{9}$$

where $\vec{v}$ is the velocity of the conduction electron, having e charge, m* effective mass and τ is the mean time between two collisions when the electron losses the drift velocity. The current density is:

$$\vec{j} = Ne\vec{v} \tag{10}$$

where N is the density of conduction electrons. From Eqs. (9) and (10) follows:

$$\dot{\vec{j}} + \frac{\vec{j}}{\tau} = \frac{Ne^2}{m^*}\vec{E} = \varepsilon_o \omega_p^2 \vec{E} \tag{11}$$

where ω_p is the so-called plasma frequency:

$$\omega_p^2 = \frac{Ne^2}{\varepsilon_o m^*} \qquad (11a)$$

ω_p is the frequency of the collective vibration of the electron cloud in respect to the lattice. E.g. an electrical, or even a mechanical shock will remove the centre of electron charges from the centre of positive ions (lattice), and a damping vibration of the mobile electron cloud will occur. This vibration is called as plasmon, having a frequency ω_p. Because in a pure metal the electric field and the current are in phase, we can write:

$$\vec{E} = \vec{E}_o e^{-i\omega t} \qquad (12a)$$

$$\vec{j} = \vec{j}_o e^{-i\omega t} \qquad (12b)$$

$$\dot{\vec{j}} = -i\omega\vec{j} \qquad (12c)$$

and

$$\vec{j} = \sigma(\omega)\vec{E} \qquad (12d)$$

where $\sigma(\omega)$ is the frequency dependent conductivity. Substituting into Eq. (11), we obtain:

$$\sigma(\omega) = \frac{\varepsilon_o \omega_p^2 \tau}{1 - i\omega\tau} \qquad (13)$$

We see here that at low frequencies the conductivity is proportional with τ; i.e. if the electrons collide rarer then their average velocity is larger. The dielectric constant of this lightly bonded electron cloud can define through the Maxwell equations in the following way:

Eq. (3) will be completed by the current term:

$$\mathbf{rot}\,\vec{H} = \dot{\vec{D}} + \vec{j} \qquad (3a)$$

Substituting Eq. (12d) and taking into account Eq. (12a) it gives:

$$\mathbf{rot}\,\vec{H} = \varepsilon_o \left(1 + \frac{\sigma(\omega)}{\omega\varepsilon_o}\right)\dot{\vec{E}} \qquad (14)$$

This expression offers the definition that the term in the bracket is the relative permeability of the metal described by the Drude model. Substituting Eq. (13) we obtain:

$$\varepsilon(\omega) = \varepsilon_o \left(1 - \frac{\omega_p^2}{\omega^2 + \frac{i\omega}{\tau}}\right) \qquad (15)$$

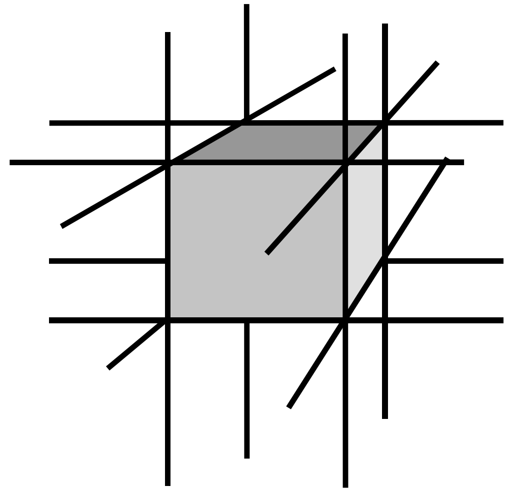

Figure 3. The periodic wire frame structure for obtaining negative ε. The cube is shadowed only for the better visualization, he structure is empty.

It is worthwhile to note here that the plasma frequency of the ionosphere can be derived in a very similar way [5]. There the electron density is between $10^{10}\ldots10^{12}$ m^{-3}, which is much less than in this wire construction. In accordance the plasma frequency is in the 6…60 MHz range. This fact, which is known for a long time and also the more recent work of Pendry and co-workers indicate the general feature of the electromagnetic response: a dilute, low-density conductor has low plasma frequency.

C. The negative μ

This proposal gives a useful tool to design and fabricate structures, which exhibit in a given frequency range negative ε. However, what about the negative μ? Three years later in 1999 Pendry and co-authors published a similar solution for negative μ [6]. The idea is he same, a periodic structure was proposed with a length of periodicity less than the wavelength of the exiting electromagnetic signal. Then the average value of $\vec{B}$ and $\vec{H}$ was calculated and μ was determined as their ratio. The arrangement is also cubic, however, the elementary structure occupy the sides of the cube. It consist of two concentric circles of metal, each of them is split. So the induced current can not close within one ring, but only in the capacitive coupled other one. The draft of the structure is shown in Fig. 4.

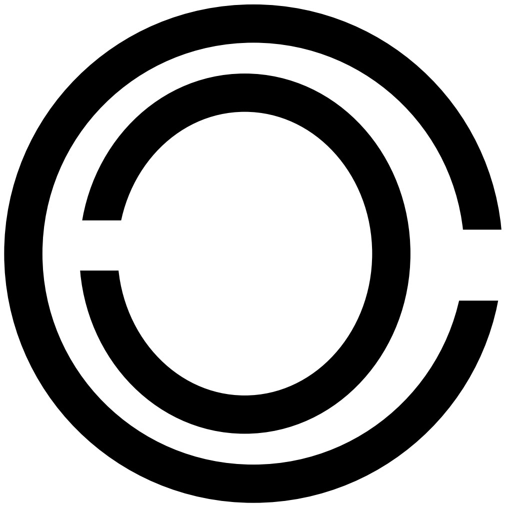

Figure 4. The mould of flat metallization for negative μ. The proposed fabrication techniques are the screen printing or etching of printed circuit board. The metal is on the dark area. The structure is periodically repeated in three dimensions.

The structure is resonant, and at frequencies slightly over the resonant frequency the mentioned average μ has a negative real part. The imaginary part of μ has a maximum at the resonant frequency, the width of this peak depends on the conductivity of the metal; and it is less if the conductivity is better. The width of the transition region of the real part of μ behaves opposite; it is broader for more conductive metal. Using highly conductive metal (cooper of the printed circuit board) there will be a narrow region of frequencies in which the real part of μ is negative and the imaginary part is not too large. Figure 5. shows the characteristic frequency dependence of the real and imaginary parts of the μ.

In the original paper the ring structure was investigated with the following geometrical parameters: inner radius 2mm, width of each ring 1mm, spacing between rings 0.1 mm, lattice constant 10 mm. For this arrangement the resonant frequency was 13.5 GHz.

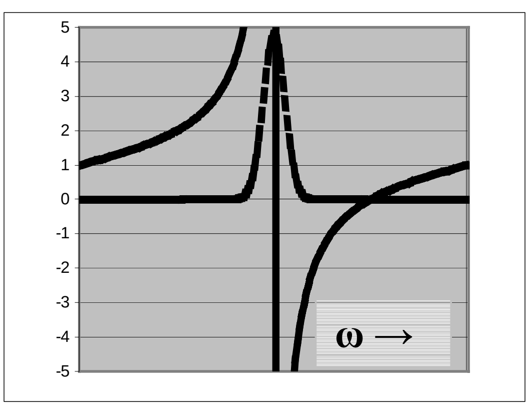

Figure 5. The plots of real (continuous) and imaginary (doted) parts of μ in the function of frequency. The width of the peak and the transition around the resonance depends on the conductivity of the metal.

D. Why metamaterial?

The question is while we term these cages as material? A material or substance used to be continuous. The fact is, that for wavelengths around 3 cm and larger the average of the structure appears in the electromagnetic response. In other words the lattice spacing of 0.5 cm is small enough compared to the wavelengths in question. As we have seen in Fig. 2. the interesting region of frequencies range down from the plasma frequency in a tight decade. (At more lower frequencies the dissipation asserts itself.) For these wavelengths the structure can not be resolved, it appears as an effectively homogenous dielectric medium whose internal structure appears only in the value of ε. At more shorter wavelength, where the resolution is better the structure will not be continuous any more, e.g. in the extreme case of visible radiation there will be no special effects; the cage is seen. The other extreme case is the DC excitation, when the response will be the Joule heat arising from the finite resistance of the structure. Naturally the structure will not behave as a continuous, homogenous substance in any other physical or chemical event. So the nomination of metamaterial regards only to the electromagnetic response in a not very wide frequency range.

It should be clarified here the difference between the metamaterial and photonic crystal. They are somehow similar; both consist of a periodic array of simple structures. The metamaterials were characterized above, they exhibit a negative index of refraction in a limited frequency band in which the wavelength of the electromagnetic radiation is larger than the lattice constant of the structure. However their ratio needs not to be rational. The physics of the phenomenon is in the average of ε and μ, which are calculated as the ratio of the average fields. In the case of photonic crystal the effect is in the Bragg reflection of the electromagnetic radiation in the periodic structure. Here the ratio of the wavelength and lattice constant is rational. The photonic crystals results in a sharp interference on the electromagnetic radiation. At determined frequencies the radiation is allowed, or even forbidden only in special directions. They are similar to the classical microwave filters. While the metamaterials behave like an isotropic and homogenous refracting media. However, the two phenomena may have a common range [7] too, but these conceptions are beyond the frame of the present review.

V. Experimental Verifications

The first experimental verification was made by Shelby, Smith and Schultz at the University of California in 2001 [8]. The left-handed material was prepared by the standard shadow mask & etching circuit fabrication technique on 0.25 mm thick fiber glass reinforced circuit board material. The structure consists of square split ring resonators and flat wire strips, both from copper. The rings and wires were on the opposite sides of the board. After processing the boards were cut and assembled an interlocking unit, which is shown in Fig. 6.

Figure 6.The metamaterial used in the experimental verifications at the Univ. of Calif. The construction is fabricated from PC board.

The lattice constant of the structure was 5 mm. The negative ε and μ occurred simultaneously between 10.2 GHz and 10.8 GHz. At the low-frequency side of the left-handed band the index of refraction was expected to be very negative, passing through zero at the high-frequency side and reaching 1 at more high frequencies.

The verification of the negative index of refraction was made in the set-up shown in Fig.7.

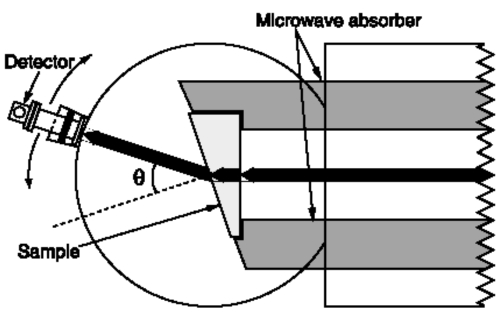

Figure 7. The measurement set-up viewing from above.

The microwave signal is guided between two aluminum plates spaced from each other at 1.2 cm. The lateral confinement of the beam was ensured by two absorber strips, places at 9.3 cm apart. A prism was cut from the left-handed cage and fitted to the end of the waveguide. The aluminum plates ended in two circular plates, made also from aluminum and the prism was placed in the center of these discs having 30 cm diameter. The out coming signal was caught by an X-band waveguide-coaxial adapter and led to the detecting instrument. As the adapter moved at the periphery of the circle the angle of the propagation was determined. As a control a prism with equivalent geometry was made from Teflon and measured in the same set up. Fig.8. shows the obtained angular dependence. The negative refraction of the beam is evident from the figure. Taking into account the refraction index of Teflon (n=1.4) the absolute value of the negative refraction index was calculated. It was derived to be -2.7.

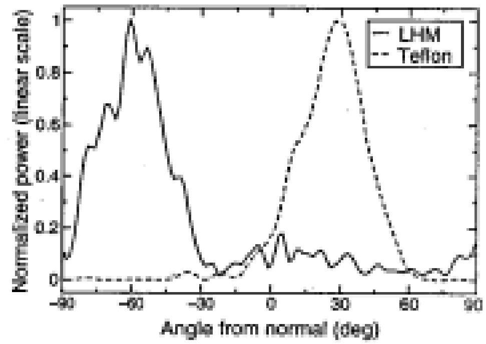

Figure 8. The angular dependence of the transmitted power in the case of the metamaterial and also for Teflon. The notches on the curve of the metamaterial can be attributed to the diffraction effects due to the rather rough unit cell structure.

The experiment seems very convincing, but not everybody agreed. In 2002 Valanju and his group from the University of Texas criticized the concept of left-handed media and negative refraction index on theoretical basis. Their stated that the experiment was an artifact due to the too short distances between the waveguide and metamaterial prism. In this year a second experimental verification appears [9], which refutes the Valanju's objections. Parazzoli and his co-workers at the Boeing Phantom Works in Seattle used a metamaterial structure a slightly different to the Shelby's one. Their left handed band was between 12.6 GHz and 13.2 GHz. The negative index of refraction was demonstrated also by the refraction on a prism, however the microwave beam was focused to the prism by a horn antenna/lens combination from a distance of about 30 cm, and also the detector was placed at distant 33-66 cm far away from the prism. This experiment verified also convincing the existence of the negative index, and consequently the concept of Veselago and Pendry.

VI. Conclusions

The idea of left-handed material and the negative refraction arose more than thirty years ago, when Victor Veselago, the Russian physicist speculated about the electromagnetic response of materials having simultaneously negative ε and μ. In the recent years the purposeful theoretical work of the Pendry's group has given the possibility of constructing such materials. The structure is similar to the photonic crystals: periodic lattice of wire/metal foil cage, however the functioning is different.

Surprisingly few periods – only 3 - 6 - of the elementary structure results in the bulk properties. The left-handed properties fulfils only in a narrow frequency band. May be that in the future other elementary structures will be invented, which will have broader frequency bands.

At present the most interesting application would be the super lens. It can substitute very effectively the antennas or can find applications in microwave image analysis. Also directive emission can be obtained from an isotropic source embedded in a slab of metamaterial [9]; The outgoing power will be concentrated in a narrow cone.

The available electronic technologies allow the fabrication of left-handed mediums in the microwave frequency domain. Here the preparation is rather easy: mask etching of printed circuit boards and assembling them in a simple 3D lattice, having a lattice constant several times smaller than the wavelength. At lower frequencies the self sustaining cage structure can be applied too. In the optical domain the analogue structure would have dimensions which can not easily fabricated by the present technologies. Therefore it seems that the applications will start first for microwaves.

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