

Decoupled Region Hybrid Method for Solving Propagation Parameters of Coupled Fibre Waveguide

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Abstract - This paper deals with an efficient method for calculation of normalized cutoff frequency and propagation parameters of a class of waveguides. The developed decoupled region hybrid method (DRHM) is based on suitable separation of sub regions and of subsequent application of one or more conformal mapping and using the finite element method. In this case single or multiple mapping in each sub region enables the use of rectangular mesh in the final stage. The organization of data files and computer programmes are modular and easily applicable to various fiber configurations. Illustrative calculations are made for step-index coupled core fibers.

Indexing terms - Decoupled regions, conformal mapping, rectangular regions, finite element, coupled core fibers.

I. INTRODUCTION

Finite element technique has been used with a great success in calculation of guided wave propagation constant and field of waveguides [1]. For weakly guiding fibers, the field in the core satisfy scalar Helmholtz equation and in the cladding Laplace equation. The cladding region is taken to be homogenous dielectric extended to infinity. The core can be made of variable permittivity dielectric. Conformal mapping used was rather difficult to apply because only one coordinate system was employed resulting in complicated boundaries between the sub regions. Addition difficulties are encountered in fitting the nodes of the meshes in the sub regions [2].

Single or multiple mapping of the physical domain can lead into a rectangular 'image' domain but with a complicated boundary between the sub regions. To avoid complex boundary node fitting, we propose to separate the physical sub domains and apply different mapping in each of them. The final aim of mapping/mappings is to obtain three simple rectangular sub domains with a straight-line boundary.

In this letter we apply the proposed method in solving a coupled core step index fiber system. Some other interesting cases in which the same approach can be used are found in reference [3].

II. THEORETICAL BACKGROUND

As an example of decoupled region hybrid method we consider fiber with the core made of dielectric of relative permittivity $\epsilon = \epsilon(u, v)$, where u and v are coordinates in any

suitable selected coordinate system, and the cladding made of homogeneous dielectric of relative permittivity $\epsilon = \epsilon_{cl}$. If the spatial variation of permittivity is small, the scalar wave equation can be used

$$\nabla^2 \phi - W^2 \phi + k^2 [\epsilon - \epsilon_{cl}] \phi = 0 \quad (1)$$

$$W = [\beta^2 - \epsilon_{cl} k^2]^{1/2}$$

Where ∇^2 is the transversal Laplace operator, $k = \omega(\mu_0 \epsilon_0)^{1/2}$ is the free-space wave number, β is the propagation constant and ϕ is the scalar field. At the cutoff frequency $\beta^2 = \epsilon_{cl} k_c^2$, where k_c is the cutoff wave number. Eq. (1) at the cutoff is obtained for $W=0$.

Geometry of considered fiber is shown in Fig. 1. The two core and cladding regions are denoted by R_1 , R_3 and R_2 , respectively. The boundary between the cores and cladding are C_0^1 and C_0^2 .

According to the basic variational principle, the solution of Eq. (1), subject to the boundary conditions, is equivalent to minimizing the functional:

$$F(\phi) = \int_{R+R_2} [|\nabla \phi|^2 + W^2 \phi^2 + k^2 (\epsilon - \epsilon_{cl}) \phi^2] dA \quad (2)$$

where dA is elementary area of the cross-section and the integration is taken over the whole cross-section, covering the areas R_1 , R_2 and R_3 in Fig.1.

Original sub regions have then to be mapped. In Fig.1 the core regions are directly mapped to a rectangular sub region R_1' and R_3' , while triple stage mapping is applied to infinite cladding sub region R_2 , before it becomes the rectangular sub region R_2' . Another approach would be to map full region and then separate obtained sub regions.

Multiple mapping does not require successive calculations, as it is possible to obtain the joined mapping parameters - metric coefficients and Jacobian.

The functional $F(\phi)$ given by Eq.(2), in the (u, v) coordinate system is expressed in the form, in i -th mapped R_1^i and R_3^i sub regions:

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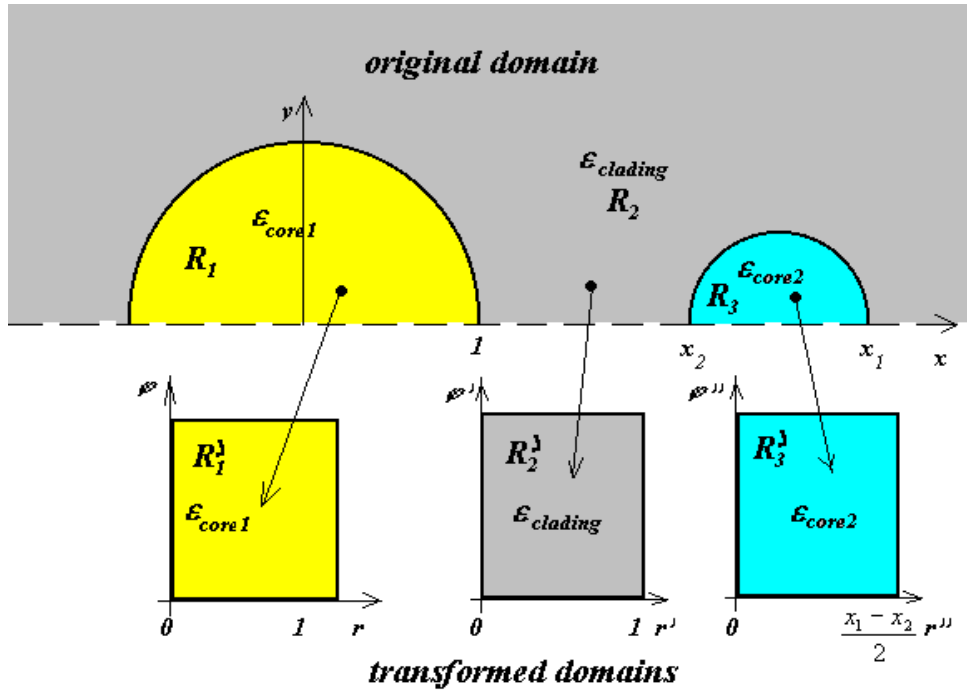


Figure 1. Illustration of the generation of transformed sub-domains in the system of coupled fibers

$$F_1(\phi) = \int_{R_1^1} \left[\left(g_{u_i u_i} \right)^{-1/2} \bar{e}_{u_i} \frac{\partial \phi}{\partial u_i} + \left(g_{v_i v_i} \right)^{-1/2} \bar{e}_{v_i} \frac{\partial \phi}{\partial v_i} \right]^2 + w^2 \phi^2 + k^2 (\varepsilon(u_i, v_i) - \varepsilon_{cl}) \phi^2 \Big] J_i du_i dv_i \quad (3a)$$

and in j -th mapped R_2^1 subregion:

$$F_2(\phi) = \int_{R_2^1} \left[\left(g_{u_j u_j} \right)^{-1/2} \bar{e}_{u_j} \frac{\partial \phi}{\partial u_j} + \left(g_{v_j v_j} \right)^{-1/2} \bar{e}_{v_j} \frac{\partial \phi}{\partial v_j} \right]^2 + w^2 \phi^2 \Big] J_j du_j dv_j \quad (3b)$$

where: u_i, v_i are i -th mapped coordinates, $\bar{e}_{u_i}, \bar{e}_{v_i}$ are unit vectors along the u_i and v_i axes, respectively, $g_{u_i u_i}, g_{v_i v_i}$ are the metric coefficients of i -th coordinate system and J_i is Jacobian of i -th coordinate system. In Table 1 all used basic mapping functions and parameters are given.

By applying the static condensation the system of inhomogenous equation is reduced to the eigenvalue problem of homogeneous equation. The functional in (3) is minimized with respect to ϕ taking into account contributions from all elements, by the application of a standard finite element procedure.

III. COUPLED FIBER SYSTEM

The choice of transformation applied in the calculation of the system of coupled fibers has no limitations with respect to geometry; it isn't necessary for the system to have a single axis system. The example illustrated here has a single plane symmetry, though it could be solved without this condition, through solving additional problems.

The illustration of the areas of coupled fiber system as well as a single-axis system is given in Figure 1.

Polar coordinate transformations have been applied in both cores.

Depending on the problem, other transformations could be applied in other cases, which might describe the core area in a simpler way and enable an easier generation for the net of final elements.

The surrounding area or blanket are described with the following transformation:

$$w = \frac{z - a}{az - 1} \quad (4)$$

In this case, however, the transformation parameters a and R are calculated as follows:

$$R = \frac{x_1 x_2 - 1 - \sqrt{(1 - x_1^2)(1 - x_2^2)}}{x_1 - x_2} \quad (5)$$

$$a = \frac{x_1 x_2 + 1 + \sqrt{(1 - x_1^2)(1 - x_2^2)}}{x_1 + x_2} \quad (6)$$

The location of the parameters x_1 and x_2 can be determined from Figure 1. The Jacobian J and the parameters g_{rr} and $g_{\phi\phi}$ are presented in the Table 1.

3.1. GENERATING THE MESH OF FINITE ELEMENTS

The generation of the mesh of finite elements is a special problem and must be separately solved for the case of coupled fiber systems.

The geometry of the ‘couplers’ can be defined with the following parameters:

primary geometrical parameters are: $x_1=2.5$ and $x_2=1.5$,
 expressed parameters are: $a=1.827934$ and $R=0.1882623$.

Table 2. gives the calculated values of cut of frequencies for the first eight modes, while Tables 3 and 4 contain values of normalized frequencies for the first two coupled modes.

The first columns of Tables 2, 3 and 4 lists the correct values for step-index fiber. In order to test the process of calculation, the calculation was done for a step-index fiber and in the Tables this case was called coupler-1. Coupler-1 has a cladding of dielectric constant 2.41, core 1 with dielectric constant 2.42 and core 2 a dielectric constant 2.41001. The difference of the indexes could not be reduced any further (to the value of 2.41 as in the core) because that would destabilize the system of equation, and the number of nodes that meets the Laplace-equation can't be increased to exceed 255. These data are contained in the Table, too (columns 2). The data for the second coupler are given in Figure 2.

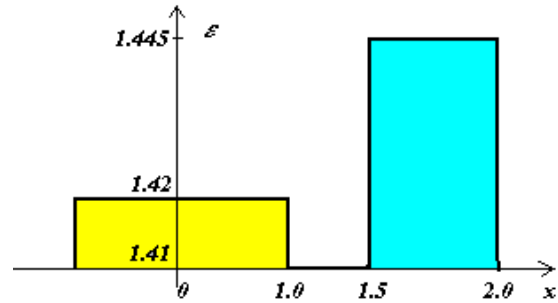


Figure 2. Illustration of dielectric constant of coupler-2

The calculation values for coupler-2 are set for dielectric constant of core 1 which appears logical. However, if the value for the maximum value of dielectric constant is to be calculated (as it is usually defined), it is necessary to multiply the calculated values with factor $f= 1.87083$, where f is calculated:

$$f = \sqrt{\frac{\epsilon_{\max} - \epsilon_{cl}}{\epsilon_1 - \epsilon_{cl}}} \quad (7)$$

TABLE 1
 JOINT MAPPING PARAMETERS

Mapped region	g_{rr}	$g_{\varphi\varphi}$	J
R_1' and R_3'	1	r^2	$J = r$
R_2'	$\frac{4}{(1 - 2r \cos \varphi + r^2)^2}$	$\frac{4r^2}{(1 - 2r \cos \varphi + r^2)^2}$	$\frac{4r}{(1 - 2r \cos \varphi + r^2)^2}$

TABLE 2
 NORMALIZING CUT OF FREQUENCY OF FIBER COUPLERS

Mode type	Correct V_c	coupler 1		coupler 2
		V_c	Rel. error	
LP_{11}	2.4048	2.40511	1.2e-4	1.26102
LP_{02}	3.8317	3.83222	1.3e-4	2.39333
LP_{21}	3.8317	3.84140	2.5e-3	2.72904
LP_{31}	5.1356	5.19171	1.1e-2	3.84023
LP_{12}	5.5201	5.52531	9.4e-4	3.92085
LP_{41}	6.3812	6.52543	2.2e-2	4.10986
LP_{03}	7.0156	7.03925	3.3e-3	4.31924
LP_{22}	7.0156	7.04560	4.3e-3	5.25965

TABLE 3
NORMALIZED FREQUENCIES OF 'LP₀₁ MODE'

w	Correct v	coupler 1		coupler 2
		v	Rel. error	
0.20236	1.0000	1.00228	2.3e-3	0.78860
0.71820	1.5000	1.50068	4.6e-4	1.19482
1.29022	2.0000	2.00029	1.5e-4	1.51952
2.42142	3.0000	3.00285	9.5e-4	2.06240
3.51620	4.0000	4.00700	1.8e-3	2.57577
5.63712	6.0000	6.01143	1.9e-3	3.59182
7.71000	8.0000	8.01185	1.5e-3	4.61411
9.75850	10.000	10.0107	1.1e-3	5.64500

TABLE 4
NORMALIZED FREQUENCIES OF 'LP₁₁ MODE'

w	Correct v	coupler 1		coupler 2
		v	Rel. error	
035828	2.5000	2.50071	2.8e-4	0.93073
1.26753	3.0000	3.00122	4.1e-4	1.99953
1.99210	3.5000	3.50455	1.3e-3	2.61709
2.65348	4.0000	4.01027	2.6e-3	3.19586
3.88075	5.0000	5.02208	4.4e-3	3.70316
5.03754	6.0000	6.02933	4.9e-3	4.16721
7.24424	8.0000	8.03220	4.0e-3	5.11796
9.37610	10.000	10.0290	2.9e-3	6.09345

Apart from the fact that the coordinate transformation describing the cladding is very unfavorable during the calculation of step-index fibers due to the asymmetric distribution of elements around core 1, the relative error during calculation is around 10^{-3} or less, so it is logical to conclude that the solutions for bonded fibers for which a coordinate system has been adapted (particularly the mesh in the cladding) will have an ever greater accuracy than in the case of step-index fibers.

IV. CONCLUSION

Conformal mapping in connection with the finite element method has been successfully used in solution of some waveguide problems [3]. In this paper the decoupled regions hybrid method (DRHM), based on suitable separation of sub regions and of subsequent application of one or more conformal mapping and using the finite element method is

presented. As illustration of the efficiency of this method we presented an approach to this problem based on the separation of core and cladding sub regions and subsequent application of conformal mapping so that, in the end, we arrive to rectangular sub regions in which mesh generation is simple and straightforward. Accuracy of calculation is below 1%.

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