Spectral Domain Analysis of Open Planar Transmission Lines

Ján Zehentner, Jan Mrkvica, Jan Macháč

Abstract – **The paper presents a new code calculating the basic characteristics of modes propagating on 18 types of open/shielded planar transmission lines – the microstrip line and the modifications of the slotline and the coplanar waveguide. The code outputs are: the complex propagation constant, the distribution of an electric field (current) across a slot (strip) or slots, the distribution of the vector of an electric field in the transversal plane of the line, 3D animation of the mode electric field, and the characteristic impedance. These quantities can be calculated for all possible modes: the bound mode, the surface leaky mode, the space leaky mode and higher order modes of these three groups of modes. The line analysis is based on the method of moments applied in the spectral domain. To show the code versatility a selection of examples of its outputs is presented in the paper.**

Keywords – **spectral domain approach, planar transmission lines, dispersion characteristic, field distribution.**

I. Introduction

A transmission line is a technical tool for transferring power from one place to another. Low losses, negligible signal distortion, high transmitted power capability, electromagnetic compatibility, a wide operational frequency band, high electromagnetic resistance and pure dominant mode propagation are requirements set on any kind of transmission line. Closed transmission lines, such as waveguides or coaxial lines, satisfy all the above mentioned demands. In contrast, open transmission lines, such as two wire lines, striplines, microstrip lines, image guides, uniplanar lines, etc., have reduced transmitted power, higher losses, lower electromagnetic resistance, cross-talk to neighbouring circuits and occasionally strong dispersion. Short sections of transmission lines are basic circuit elements, and in this case they have the appearance of a printed circuit line. A change in their cross-sectional size, material parameters or bending from the straight direction results in many types of inhomogeneities, which form sources of undesired radiation, power leakage and propagation of higher modes. Generally available professional SW packages devoted to analysis and design of these lines mostly have not taken these effects into consideration, as has already been observed and commented [1]. They provide solutions satisfactory only for the purely bound modes at low frequencies. At higher frequencies, when leakage effects or higher order modes appear, the designer must either accept the limits of the code validity, or rely on the obtained results without being able to check them. This factor in many cases leads to discrepancies between theoretically predicted and measured characteristics of circuits or systems containing active devices or passive elements.

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The aim of this paper is to fill in the gap in the code supply accessible to researchers involved in problems related to the analysis and design of planar transmission lines in wide frequency bands. Our principal approach to this task is based on the spectral domain method applied to printed circuit lines made on dielectric substrates. The spectral domain approach (SDA) was introduced and demonstrated on a shielded microstrip line in [2]. An analogous derivation was presented for the slotline [3] and summarized in [4]. Analysis in the same manner of a coplanar waveguide was performed in [5]. The method applicable to fully open planar transmission lines uses the Fourier integral to transform functions into the spectral domain. As a consequence, a planar line longitudinally homogeneous with an infinitely wide substrate is assumed. Fourier series are used when laterally shielded lines are investigated. A line confined to finite width of the substrate by conducting planes is assumed in this case. The Fourier integral and simultaneously Fourier series are used in calculating the propagation constant and the field distribution of modes on partly shielded lines. The line is now composed of two parts, one of finite width confined by electric walls and the other with an infinitely wide substrate or opened space.

The Analysis of Planar Transmission Lines (APTL) package contains an analysis of 18 different planar transmission lines, the cross-sections of which are shown in Fig. 1. The code provides the frequency-dependent propagation constant of a selected mode, the characteristic impedance when it can be defined, the field distribution in the cross-section and the distribution of the transversal field component within a slot, or the current density component on the strip. All possible modes, including bound modes, surface leaky modes, space leaky modes, and higher order modes of all kinds, are treated by the APTL code.

II. Some Details of Computation of the Propagation Constant and Field Distribution

Analysis of lines implemented in the APTL code is based on the method of moments applied in the spectral domain. This method is well known and is described, e.g., in [2]. The main constraint which limits the usability of the SDA follows from application of the boundary conditions. Therefore zero thickness and infinite conductivity of conductive layers are assumed. On the other hand, dielectric losses are accounted for.

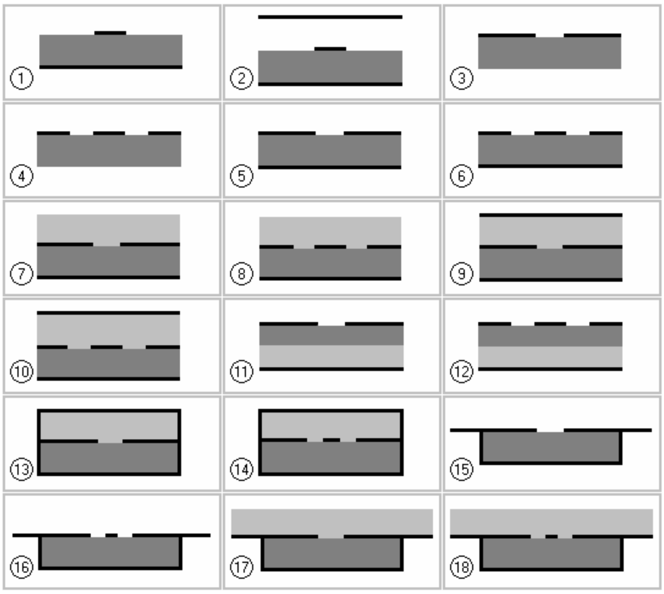
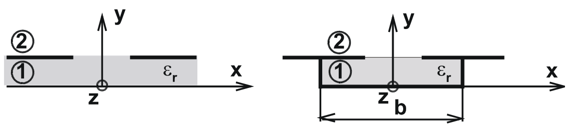


Fig. 1 Cross-sections of planar transmission lines analyzed by the APTL program.

Let us hint at several crucial points in the calculation of the propagation constant and in the reconstruction of the modal fields. We distinguish even and odd modes according to the transversal component of the electric field $E_x(x)$ within the slot parallel to the slot plane with regard to the slot axis, or according to the transversal component of the current density $J_x(x)$ on the strip with respect to the strip axis.



(a) (b)

Fig. 2 Cross-section of a slotline (a) and a conductor-backed slotline with finite width of the substrate metallized on the side walls (b).

Let Hertz's electric Φ^e and magnetic Φ^m potential represent the electric field **E** and magnetic field **H** on the transmission line. In the laterally unbounded regions, e.g., ① and ② in Fig. 2a, or ② in Fig. 2b, we choose Φ^e, Φ^m in the y-direction, while in the laterally bound regions, e.g., ① in Fig. 2b, they are taken in the z-direction. Hereafter we will drop the upper indices of the potentials. Then the Fourier transform $\tilde{\Phi}$ of the original Φ with respect to the x-axis in the laterally unbounded case is

$$\tilde{\Phi}(\xi, y, z) = \int_{-\infty}^{\infty} \Phi(x, y, z) e^{-j\xi x}\, dx \qquad (1)$$

and in the bound region

$$\tilde{\Phi}(\xi_n, y, z) = \frac{2}{b} \int_{-b/2}^{b/2} \Phi(x, y, z) e^{-j\xi_n x}\, dx \qquad (2)$$

where

$$\xi_n = \begin{cases} 2n\pi / b & \text{even modes} \\ (2n-1)\pi / b & \text{odd modes} \end{cases} \qquad (3)$$

and $n=0, \pm 1, \pm 2, \ldots \pm\infty$. The backward Fourier transform provides the original potential

$$\Phi(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\Phi}(\xi, y, z) e^{j\xi x}\, d\xi \qquad (4)$$

and

$$\Phi(x, y, z) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \tilde{\Phi}(\xi_n, y, z) e^{j\xi_n x} \quad , \qquad (5)$$

respectively.

The propagation constant of a particular mode is a solution of the dispersion equation obtained from the boundary conditions on the interface of media carrying conductive strips applied in the Fourier transformed domain. This homogeneous determinant equation contains integrals consisting of Green's functions and basis functions, both in the spectral domain.

In the space domain we use a sinusoidal function modified by an edge-condition term as the basis functions. For odd modes on a microstrip line and on modifications of a microstrip line they are

$$j_{xn}(x) = \frac{\sin[2n\pi x / w]}{\sqrt{1-(2x/w)^2}} \qquad n=1,2,3,\ldots \qquad (6)$$

$$j_{zn}(x) = \frac{\cos[2(n-1)\pi x / w]}{\sqrt{1-(2x/w)^2}} \qquad n=1,2,3,\ldots \qquad (7)$$

where w denotes the strip width. On a slotline and on modifications of a slotline for even modes

$$e_{xn}(x) = \frac{\cos[2(n-1)\pi x / w]}{\sqrt{1-(2x/w)^2}} \qquad n=1,2,3,\ldots \qquad (8)$$

$$e_{zn}(x) = \frac{\sin[2n\pi x / w]}{\sqrt{1-(2x/w)^2}} \qquad n=1,2,3,\ldots \qquad (9)$$

while for odd modes

$$e_{xn}(x) = \frac{\sin[(2n-1)\pi x / w]}{\sqrt{1-(2x/w)^2}} \qquad n=1,2,3,\ldots \qquad (10)$$

$$e_{zn}(x) = \frac{\cos[(2n-1)\pi x / w]}{\sqrt{1-(2x/w)^2}} \qquad n=1,2,3,\ldots \qquad (11)$$

where w is the slot width. For a coplanar waveguide with the center strip width s and the slot width w for even modes

$$e_{xn}(x) = \pm \frac{\cos[n\pi(x \pm p)/w]}{\sqrt{1-[2(x \pm p)/w]^2}} \qquad n=0,2,4, \qquad (12)$$

$$e_{zn}(x) = \frac{\cos[n\pi(x \pm p)/w]}{\sqrt{1-[2(x \pm p)/w]^2}} \qquad n=1,3,5, \qquad (13)$$

while for odd modes

$$e_{xn}(x) = \frac{\sin[n\pi(x \pm p)/w]}{\sqrt{1-[2(x \pm p)/w]^2}} \qquad n=1,3,5, \qquad (14)$$

$$e_{zn}(x) = \pm \frac{\sin[n\pi(x \pm p)/w]}{\sqrt{1-[2(x \pm p)/w]^2}} \qquad n=0,2,4, \qquad (15)$$

where p=s/2+w/2, the upper sign holds for x<0 and the lower sign is valid for x>0.

The path of integration, introduced by Galerkin's method, determines the propagation constant of the constituent modes. We integrate individual terms in the dispersion equation in the complex plane of the spectral variable ξ. Once the propagation constant is known, the amplitudes of the basis functions can be computed.

By the backward Fourier transform we get the field distribution, and again the choice of the integration path determines the field of the selected mode. The integrands in question have poles and branch points due to their presence in Green's functions. Therefore in the first step the code calculates these singular points and the user picks the required mode and determines the form of the integration path. Integration along the real axis leads to the dominant mode, while integration encompassing one or more poles provides a mode leaking power into the substrate. When in addition the integration path is round the branch point, we get a space leaky mode.

The choice of the basis functions and the number of basis functions influence the final results, so the user can make sure of the proper field distribution in the slot, or the current density on the strip. The total picture of the field on the plane transversal to the line axis then confirms the expected mode. For a better image, 3D visualization is available. The characteristic impedance of the bound modes is calculated by its power-current definition for lines with strips, and by the power-voltage definition for lines with slots.

Normalization of the propagation constant to the wave number of the free space and normalization of the frequency to the substrate thickness ensure validity of the results in wide frequency bands. At the moment the code contains an analysis of 18 lines, the cross-sections of which are shown in Fig. 1. Particular subroutines and their convergence have been tested with respect to the number of selected basis functions and the upper limits of integration and addition of terms in the series.

III. Illustrations of the APTL Program Outputs

Selected examples of the APTL program outputs are presented in order to acquire a better view of its functionality. Typical dispersion characteristics, i. e., normalized phase β/k_0 and leakage α/k_0 constants of the microstrip line encompassing the dominant bound mode and surface leaky mode are shown in Fig. 3. The characteristic impedance of the line is plotted in Fig. 4. The electric field of the dominant bound mode of the microstrip line on the cross-sectional plane is plotted in Fig. 5. The dispersion characteristics of a slotline with a wide slot are drawn in Fig. 6 and encompass the even dominant bound mode, the first higher order odd bound mode and the first odd space leaky mode. The transversal electric field components within the slot for the even dominant bound mode and the first odd space leaky mode are sketched in Fig. 7. The phase constant of the even bound mode and the characteristic impedance of a slotline with a narrow slot are illustrated in Fig. 8. The electric field of the even dominant bound mode on the cross-sectional plane of the slotline is shown in Fig. 9, while Fig. 10 shows the electric field of the surface leaky mode. The electric field of the first space leaky mode on the slotline determined in Fig. 6 is shown in Fig. 11. The dispersion characteristics of a coplanar waveguide, ④ in Fig. 1, for the bound mode and first surface leaky mode are shown in Fig. 12. The characteristic impedance of the dominant bound mode propagating on the coplanar waveguide is given in Fig. 13. Fig. 14 represents the electrical field distribution of the dominant bound mode on the transversal plane of that line. The dispersion characteristics of the inverted conductor-backed slotline ⑦, when the permittivity of the upper layer is higher than the permittivity of the bottom layer, is plotted in Fig. 15. The electric field transversal plane distribution of the bound mode on the line ⑦ determined in Fig. 15 is shown in Fig. 16.

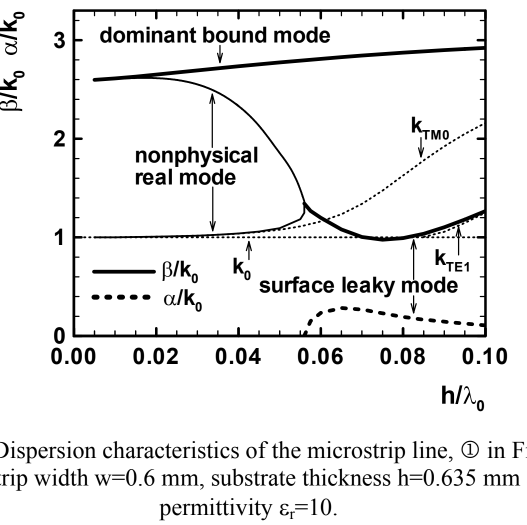


Fig. 3 Dispersion characteristics of the microstrip line, ① in Fig. 1, with strip width w=0.6 mm, substrate thickness h=0.635 mm and permittivity ε_r=10.

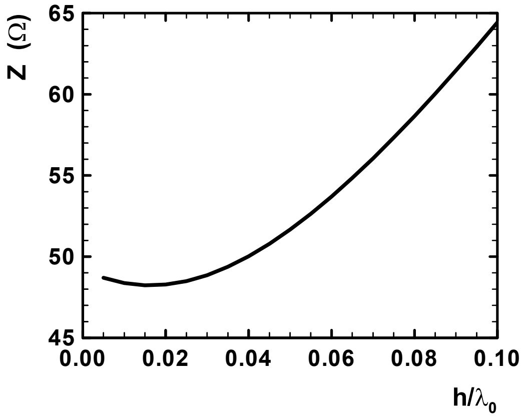


Fig. 4 The characteristic impedance of the microstrip line specified in Fig. 3.

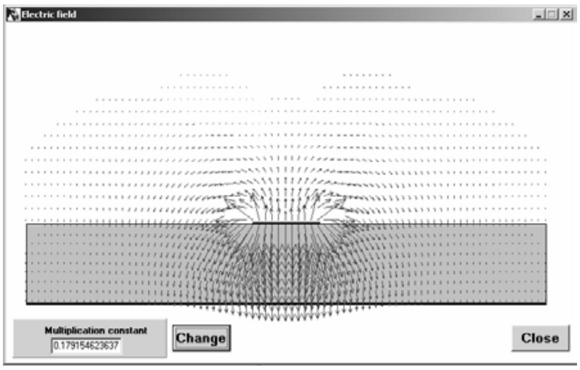


Fig. 5 Electric field of the dominant bound mode on the cross-sectional plane of the microstrip line, ① in Fig. 1, when w=1 mm, h=1 mm and ε_r=2.2.

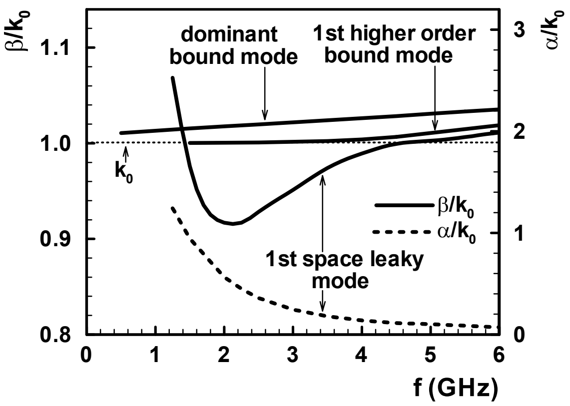


Fig. 6 Dispersion characteristics of a slotline, ③ in Fig. 1, with slot width w=60 mm, substrate thickness h=1.2 mm and permittivity ε_r=2.6.

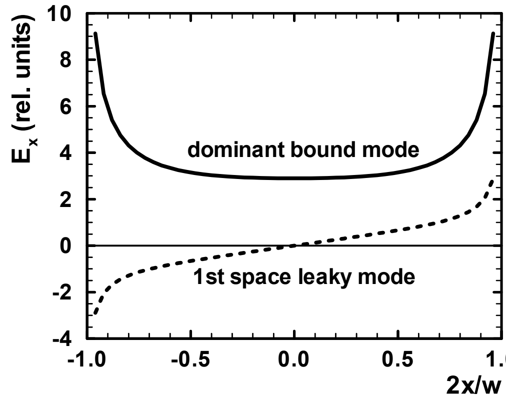


Fig. 7 Transversal electric field component within the slot of the slotline specified in Fig. 6 when h/λ_0=0.012.

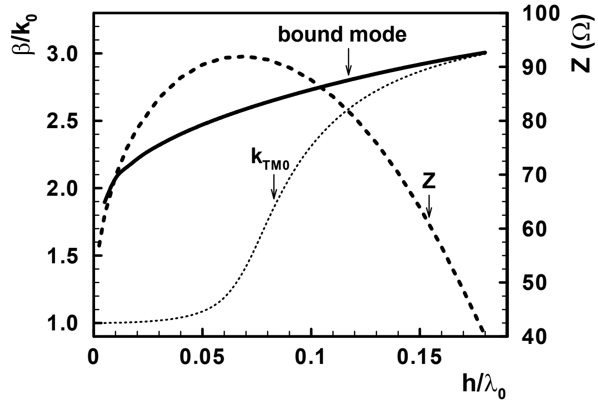


Fig. 8 Phase constant of the dominant bound mode and the characteristic impedance of a slotline, ③ in Fig. 1, with w=0.15 mm, h=0.625 mm and ε_r=10.8.

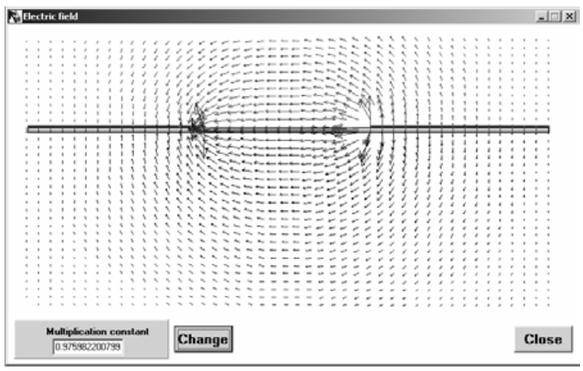


Fig. 9 Electric field of the even dominant bound mode on the slotline specified in Fig. 6 at h/λ_0=0.012.

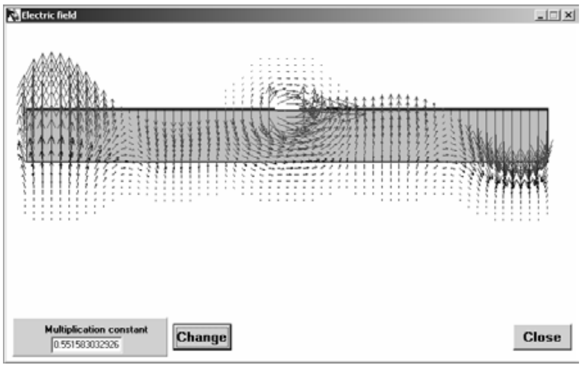


Fig. 10 Electric field of the even surface leaky mode on a slotline with w=5.65 mm, h=14.6 mm, ε_r=2.6 and h/λ_0=0.4.

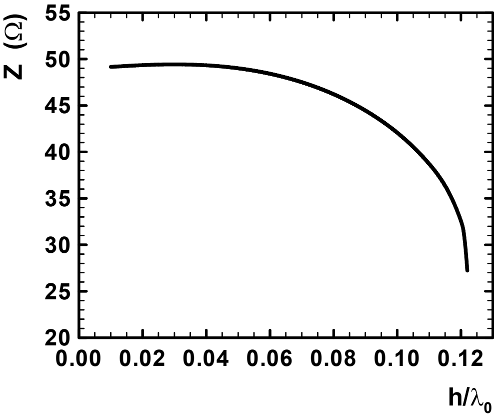


Fig. 13 Characteristic impedance of the coplanar waveguide specified in Fig. 12.

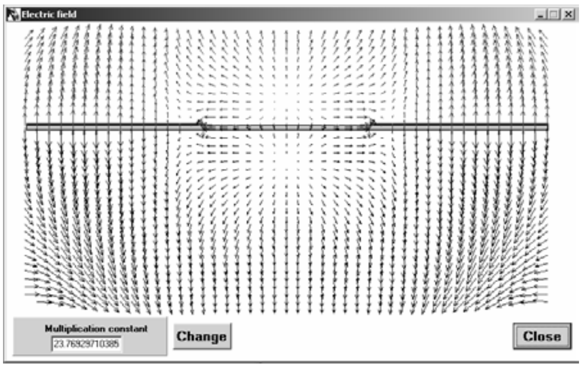


Fig. 11 Electric field of the odd 1st space leaky mode on the slotline specified in Fig. 6 at h/λ_0=0.012.

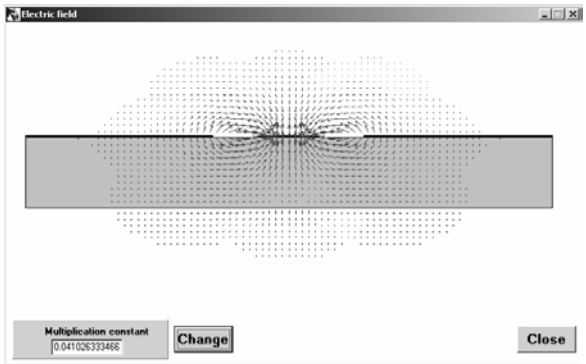


Fig. 14 Electric field of the dominant bound mode on the cross-sectional plane of the coplanar waveguide specified in Fig. 12.

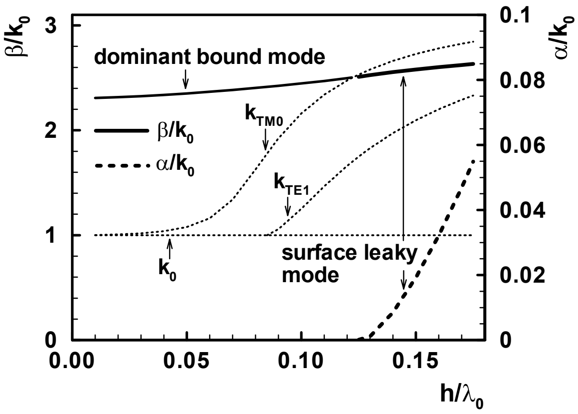


Fig. 12 Dispersion characteristics of a coplanar waveguide, ④ in Fig. 1, when central strip width s=0.6 mm, slot width w=0.25 mm, substrate thickness h=1 mm and permittivity ε_r=10.

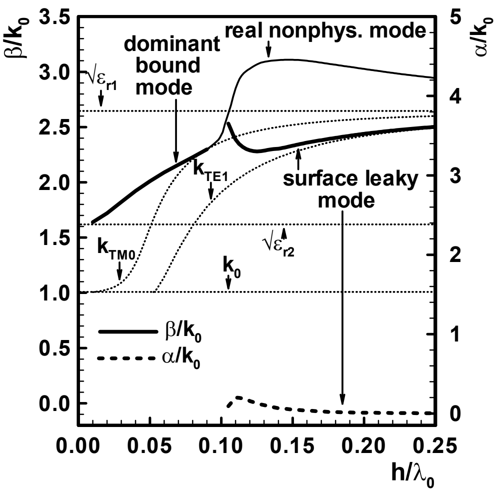


Fig. 15 Dispersion characteristics of a modified conductor-backed slotline ⑦ in Fig. 1, when slot width w=6 mm, substrate thicknesses h_1=6 mm, h_2=12 mm, and permittivities ε_{r1}=2.6, ε_{r2}=7 and ε_{r3}=1.

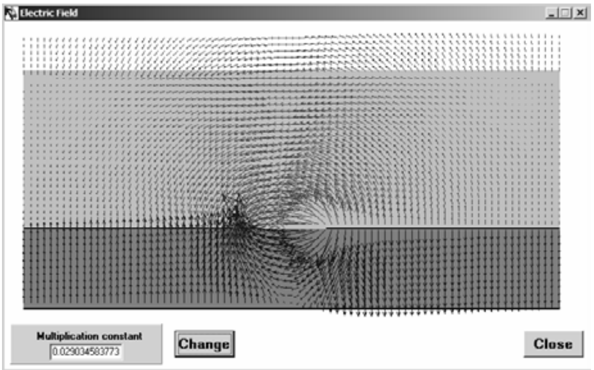


Fig. 16 Electric field of the dominant bound mode on the cross-sectional plane of the modified conductor-backed slotline specified in Fig. 12.

When the permittivity is the same in the whole cross-section of the completely shielded conductor-backed slotline, ⑬ in Fig. 1, we have a waveguide with a conductive slotted partition in the H plane. The dispersion characteristics of the even dominant slot type and waveguide type modes are drawn in Fig. 17. The characteristic impedance of that line is shown in Fig. 18. The electric field of an even slot type dominant mode on the transversal plane of a wide slotted line is shown in Fig. 19. The electric field of an even waveguide type dominant mode on the cross-sectional plane of the wide slotted line is given in Fig. 20. Fig. 21 shows the dispersion characteristic of an even bound mode on a conductor-backed slotline with finite width of the substrate short circuited on the side walls compared with the characteristic of the dominant TE_{10} mode on a slotted rectangular waveguide. The field distribution of this mode is given in Fig. 22.

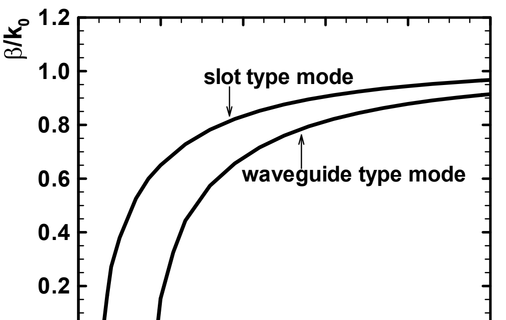


Fig.17 Normalized phase constant of the even dominant slot type mode, and waveguide type mode on a shielded conductor-backed slotline,⑬ in Fig. 1,when slot width w=20 mm, height of the shield 2h=4 mm, its width b=50 mm and permittivity ε_r=1.

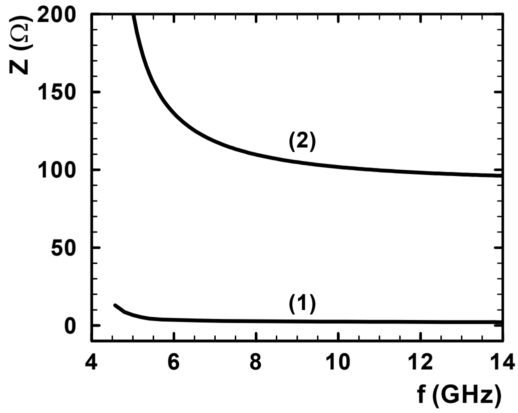


Fig. 18 The characteristic impedance (1) of the line specified in Fig.15 with the slot type mode, and (2) of the line when w=1 mm, 2h=10 mm, b=23 mm, ε_r=1, on which the slot type mode propagates.

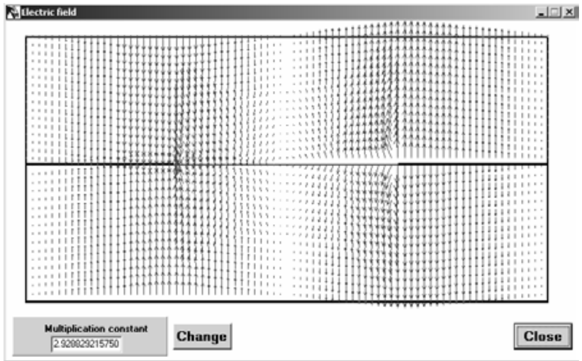


Fig. 19 Electric field of the even slot type dominant mode on a cross-sectional plane of the wide slotted line specified in Fig. 17 at f=7.5 GHz.

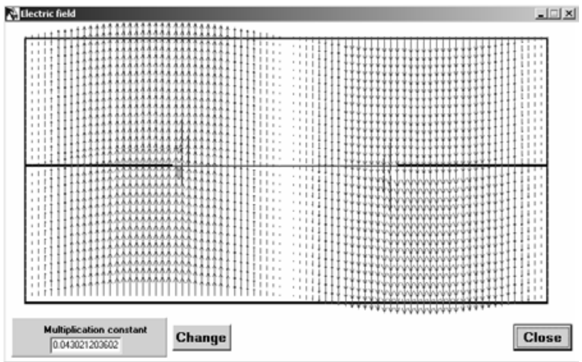


Fig. 20 Electric field of an even perturbed waveguide type mode on the cross-sectional plane of the wide slotted line specified in Fig. 17 at f=7.5 GHz.

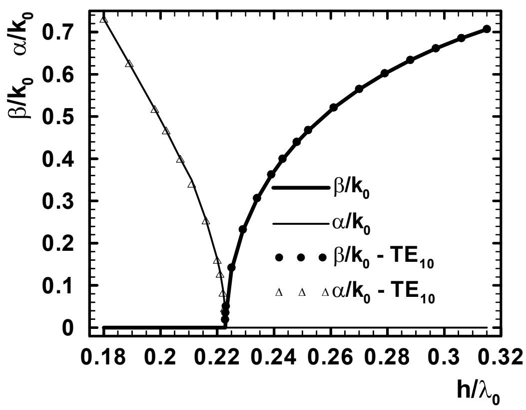


Fig. 21 The dispersion characteristic of the mode on the slotted rectangular waveguide compared with the characteristic of TE_{10} mode. The slot width is 1 mm, and the internal dimensions are 22.86 x 10.12 mm.

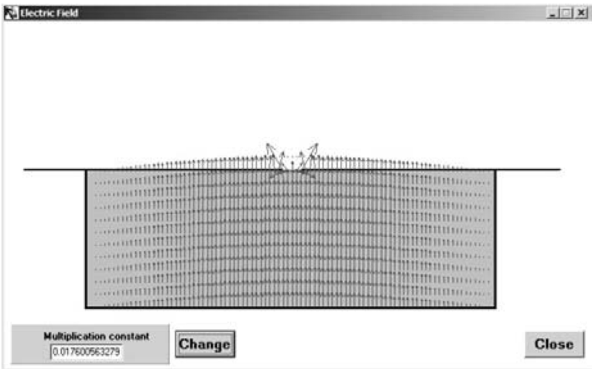


Fig. 22 Electric field of the mode on the slotted waveguide specified in Fig. 21.

The APTL program is still under development and is undergoing additional testing. It enables the user to find simply whether the line transmits a bound mode, whether the first and higher order leaky modes set up, determines the width of the pure bound mode propagation, and calculates the characteristic impedance. 2D or 3D visualization of the field distribution enables a better understanding of its behaviour on the investigated lines. The full wave approach provides more trustworthy results than do the quasi-TEM approach or analytical-experienced solutions. It turned out that lines with wide slots or wide strips are suitable for space leakage, particularly when an odd space leaky mode is excited. They can be applied as radiators or elements of arrays. A completely shielded conductor-backed slotline retains the merits of waveguides combined with planar technology.

IV. Conclusion

The authors think that the APTL program provides a much more complex view on the line behaviour from more points of view than other programs, and gives a realistic picture of their characteristics, including leakage effects occurring in the substrate or in space. The current menu of the code results from our recent research demands. However, the system is open and can be extended at any time by adding further types of lines. Particular modes are studied in the source-free region on longitudinally and mostly also transversally unbounded lossless lines. The influence of dielectric losses has been incorporated. Conductor losses have been cut out due to the negligibly small thickness of the metallization, which is a basic assumption of the spectral domain method. In addition, we verified experimentally the wave propagation on some open planar transmission lines made on enlarged models.

Acknowledgement

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