# Application of Technique for De-embedding of S-parameters Based on Reflection Coefficient Calculations in WIPL-D Environment

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#### I. INTRODUCTION

Various numerical methods (MoM, FEM, TLM, FDTD) have been successfully used for the analysis of reciprocal microwave circuits [1]. As a result, one obtains the current distribution or the near field distribution and post processing of these data should be performed in order to de-embed and evaluate the circuit parameters. Let us concentrate on the circuits whose ports are in the form of a single propagating mode transmission lines.

Most often, the ports of numerically analyzed circuits are not directly excited by corresponding single propagating modes, but indirectly, by using 2-port feeding networks. In the general case, the full network used in a simulation consists of n+1 circuits, i.e., a basic *n*-port circuit and *n* 2-port feeding networks, as shown in Fig. 1.



Fig. 1. N-port circuit with n 2-port feeding networks.

For such circuits two general classes of de-embedding techniques are usually applied. The first one (applicable only for TEM lines) is based on calculation of the currents and voltages along the line conductors, and the second technique is based on calculation of the electric and magnetic fields along the dielectric material of the lines. In both cases, data are usually taken at three cuts along the line. Theoretically speaking one control point per cut can be enough for determination of *s*-parameters.

In order to minimize the de-embedding error due to local numerical errors involved in the near field calculations, a mesh of control points is used instead of one point in each cut. However there are still problems of choosing optimal number of control points and optimal distribution of the points in the

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mesh. Note that these problems should be solved again for each new transmission line used in the analysis. These problems can be avoided by a de-embedding technique, applied here, based on reflection coefficient calculations. This technique is applied first to waveguide circuits, i.e., the circuit made in one technology [2]. Here it is shown that the same method can be efficiently applied to coaxial line-to-waveguide transition, i.e., to discontinuities that combine two technologies.

## II. THEORETICAL BASIS FOR DE-EMBEDDING TECHNIQUE

The method is performed in three steps:

- 1. The *s*-parameters of the full network (including feeding structures) are obtained by 3D electromagnetic simulation.
- 2. The *s*-parameters of the 2-port feeding circuits are obtained by combining 3D electromagnetic simulation and the classical method of Deschamp for reflection coefficient measurements.
- 3. Once the *s*-parameters of the feeding circuits and the full network are determined, the *s*-parameters of the basic *n*-port network are obtained by postprocessing.
- 3D simulations are performed in software package WIPL-D [3].

#### A. S-parameters of the 2-port Feeding Network

Let us consider a 2-port feeding network excited at port no. 1 and terminated by a sliding short at port no. 2. (Sliding short is obtained in such a manner that transmission line at **port** no. 2 is extended for length l, and short-circuited) It's well known that reflection coefficient at port no. 1 can be expressed in the form

$$\rho_{\phi} = s_{11} - \frac{s_{12}^2 e^{-j\phi}}{1 + s_{22} e^{-j\phi}} \qquad \phi = 2\beta l \tag{1}$$

where  $s_{11}$ ,  $s_{12} = s_{21}$  and  $s_{22}$  are the *s*-parameters of the 2-port network,  $\beta$  is the phase coefficient of the mode propagating along the sliding short, and *l* is the length of the sliding short. The above expression can be written in the form

$$\rho_{\phi} = \frac{s_{11}e^{j\phi} + \det(\mathbf{S})}{e^{j\phi} + s_{22}} \qquad \det(\mathbf{S}) = s_{11}s_{22} - s_{12}^2 \tag{2}$$

Four positions of the sliding short are simulated in the following way. Two positions,  $2\beta l_1 = 0$  and  $2\beta l_2 = \pi$ , are

simulated by placing the transmission line end into PEC (perfect electric conductor) plane and PMC (perfect magnetic conductor) plane, respectively. In the same way another two positions are simulated,  $2\beta l_3 = \phi$  and  $2\beta l_4 = \phi + \pi$ , but with the transmission line extended for  $\Delta l = \phi / (2\beta)$ . Starting from these four reflection coefficients,  $\rho_0$ ,  $\rho_{\pi}$ ,  $\rho_{\phi}$  and  $\rho_{\phi+\pi}$ , and equation (2), we obtain the following equations

$$\rho_{\phi} e^{j\phi} + \rho_{\phi} s_{22} = s_{11}e^{j\phi} + \det(\mathbf{S})$$
 (3)

$$\rho_0 + \rho_0 \quad s_{22} = s_{11} + \det(\mathbf{S}) \tag{4}$$

$$-\rho_{\phi+\pi}e^{j\phi} + \rho_{\phi+\pi}s_{22} = -s_{11}e^{j\phi} + \det(\mathbf{S})$$
(5)

$$-\rho_{\pi} + \rho_{\pi} \quad s_{22} = -s_{11} + \det(\mathbf{S}) \tag{6}$$

Starting from these equations, and after some relatively simple transformations, feeding network parameters,  $s_{11}$ ,  $s_{12}$  and  $s_{22}$ , can be determined as

$$s_{11} = \frac{b + as_{22}}{2} \tag{7}$$

$$s_{12}^2 = \frac{a}{2}(s_{22}^2 - 1) \tag{8}$$

$$s_{22} = \frac{a - ce^{j\phi}}{d - b} \tag{9}$$

where the constants a, b, c and d are given by

$$a = \rho_0 - \rho_\pi \tag{10}$$

$$b = \rho_0 + \rho_\pi \tag{11}$$

$$c = \rho_{\phi} - \rho_{\phi + \pi} \tag{12}$$

$$d = \rho_{\phi} + \rho_{\phi + \pi} \tag{13}$$

#### B. S-parameters of a Basic n-port Circuit

Incident and reflected waves at ports of the entire network are related to the *s*-parameters by

$$(b_1) = \left(s^{(f)}\right)\left(a_1\right) \tag{14}$$

where

$$(b_{1}) = \begin{pmatrix} b_{1}^{(1)} \\ \vdots \\ b_{1}^{(i)} \\ \vdots \\ b_{1}^{(n)} \end{pmatrix} \quad (s^{(f)}) = \begin{pmatrix} s_{11}^{(f)} & \cdots & s_{1n}^{(f)} \\ \vdots & & \vdots \\ s_{n1}^{(f)} & \cdots & s_{nn}^{(f)} \end{pmatrix} \quad (a_{1}) = \begin{pmatrix} a_{1}^{(1)} \\ \vdots \\ a_{1}^{(i)} \\ \vdots \\ a_{1}^{(n)} \end{pmatrix} \quad (15)$$

In a similar way the incident and reflected waves at the ports of the basic circuit are related to the *s*-parameters by

$$\left(b^{(b)}\right) = \left(s^{(b)}\right) \left(a^{(b)}\right) \tag{16}$$

where

$$\begin{pmatrix} b^{(b)} \end{pmatrix} = \begin{pmatrix} b_{1}^{(b)} \\ \vdots \\ b_{i}^{(b)} \\ \vdots \\ b_{n}^{(b)} \end{pmatrix} \quad \begin{pmatrix} s^{(b)} \end{pmatrix} = \begin{pmatrix} s_{11}^{(b)} & \cdots & s_{1n}^{(b)} \\ \vdots & & \vdots \\ s_{n1}^{(b)} & \cdots & s_{nn}^{(b)} \end{pmatrix} \quad \begin{pmatrix} a^{(b)} \end{pmatrix} = \begin{pmatrix} a_{1}^{(b)} \\ \vdots \\ a_{i}^{(b)} \\ \vdots \\ a_{n}^{(b)} \end{pmatrix} \quad (17)$$

Finally, the incident and reflected waves at the *i*-th feeding network are related to the *s*-parameters by

$$(b^{(i)}) = (s^{(i)})(a^{(i)})$$
 (18)

where

$$\begin{pmatrix} b^{(i)} \\ b^{(i)} \\ b^{(i)} \\ 2 \end{pmatrix} \quad \begin{pmatrix} s^{(i)} \\ s^{(i)} \end{pmatrix} \quad \begin{pmatrix} a^{(i)} \\ a^{(i)} \\ a^{(i)} \\ a^{(i)} \\ s^{(i)} \\$$

Let us write equations (18) and (19) in the form

$$b_1^{(i)} = s_{11}^{(i)} a_1^{(i)} + s_{12}^{(i)} a_2^{(i)}$$
(20)

$$b_2^{(i)} = s_{21}^{(i)} a_1^{(i)} + s_{22}^{(i)} a_2^{(i)}$$
(21)

Finally, note that the incident and reflected waves at the port no. 2 of the *i*-th feeding circuit are connected to the incident and reflected waves at the port no. i of the basic circuit through

$$b_2^{(i)} = a_i^{(b)} \tag{22}$$

$$a_2^{(i)} = b_i^{(b)} \tag{23}$$

After replacing (22) and (23) into (21), incident wave at port no. 1 of the *i*-th feeding circuit can be expressed in terms of waves at port no. *i* of the basic circuit through

$$a_1^{(i)} = \frac{1}{s_{21}^{(i)}} a_i^{(b)} - \frac{s_{22}^{(i)}}{s_{21}^{(i)}} b_i^{(b)}$$
(24)

After substituting (23) and (24) into (20) the reflected wave at port no. 1 of the *i*-th feeding circuit can be expressed in terms of the waves at port no. *i* of the basic circuit through

$$b_{1}^{(i)} = \frac{s_{11}^{(i)}}{s_{21}^{(i)}} a_{i}^{(b)} + \left(s_{12}^{(i)} - \frac{s_{11}^{(i)} s_{22}^{(i)}}{s_{21}^{(i)}}\right) b_{i}^{(b)}$$
(25)

It means that the columns  $(a_1)$  and  $(b_1)$  given by (15) can be expressed in terms of columns  $(a^{(b)})$  and  $(b^{(b)})$  as

$$(a_1) = (f_1)(a^{(b)}) + (f_2)(b^{(b)})$$
(26)

$$(b_1) = (g_1)(a^{(b)}) + (g_2)(b^{(b)})$$
(27)

where  $(f_1)$ ,  $(f_2)$ ,  $(g_1)$  and  $(g_2)$  are diagonal matrices of size  $n \times n$ , i.e.

$$f_{1ij} = \begin{cases} 1/s_{21}^{(i)} & i = j \\ 0 & i \neq j \end{cases}$$
(28)

$$f_{2ij} = \begin{cases} -s_{22}^{(i)} / s_{21}^{(i)} & i = j \\ 0 & i \neq j \end{cases}$$
(29)



Fig. 2. Geometrical model of coaxial line-to-waveguide transition together with feeding circuits

$$g_{1ij} = \begin{cases} s_{11}^{(i)} / s_{21}^{(i)} & i = j \\ 0 & i \neq j \end{cases}$$
(30)

$$g_{2ij} = \begin{cases} s_{12}^{(i)} - s_{11}^{(i)} s_{22}^{(i)} / s_{21}^{(i)} & i = j \\ 0 & i \neq j \end{cases}$$
(31)

After substituting (16) into (26) and (27), and (26) and (27) into (14), and after simple manipulations, the *s*-parameters of the basic circuit are obtained from

$$(s^{(b)}) = [(s^{(f)})(f_2) - (g_2)]^{-1}[(g_1) - (s^{(f)})(f_1)]$$
(32)

In some cases, the feeding network of port no. *i* can be a part of the basic circuit. In that case de-embedding should be performed without taking into account the feeding network at port no. *i*. This can be done by using the same theory, as above, except that the *i*-th diagonal elements in the matrices  $(f_1)$ ,  $(f_2)$ ,  $(g_1)$  and  $(g_2)$  should be defined as

$$f_1^{(ii)} = 1$$
  $f_2^{(ii)} = 0$   $g_1^{(ii)} = 0$   $g_2^{(ii)} = 1$  (33)

## III. APPLICATION OF DE-EMBEDDING TO COAXIAL LINE-TO-WAVEGUIDE TRANSITION IN WIPL-D ENVIRONMENT

Let us consider WIPL-D model of coaxial-to-waveguide transition with the feeding circuits, as shown in Fig. 2. (Since the symmetry of the structure is used to decrease the number of unknowns, only half of the structure is specified.) The generators are represented by the letters "g1" and "g2". The transition is a 2-port circuit.

At the  $1^{st}$  (coaxial line) port we see that: 1) the inner conductor of the coaxial line starts with a short and thin wire, 2) there is a conical transition between the thin wire and the main part of the inner conductor, 3) the outer conductor is

capped by a metallic disk which is also connected to the wire, and 4) the generator is positioned at the junction of a metallic cap and a thin wire. It is obvious that the metallic cap, conical transition, and the thin wire represent 2-port feeding network for the coaxial line port of the transition. Such feeding network actually occurs in the case when a coaxial line of larger cross section is excited by a coaxial line of smaller cross section.

At the  $2^{nd}$  (waveguide) port we see that: 1) the waveguide is closed, 2) the thin wire is connected to the lower waveguide wall, and 3) the generator is positioned at the junction of the wire and the lower waveguide wall. It is obvious that a part of the waveguide that is closed, together with the wire probe, represents a 2-port feeding network. Such a feeding network actually occurs in the case when the wave-guide is excited by the coaxial line. After run is performed the *s*-parameters of full 3D circuit are obtained as shown in Fig. 3

In order to determine the s-parameters of 2-port feeding networks, separate WIPL-D models for each feeding network should be created, as shown in Fig. 4. The models are made obeying the following two rules: 1) the total length of the feeding network is always defined by the first symbol in the symbol list, and 2) the 2nd port of feeding network is positioned into one of the basic coordinate (symmetry) planes (i.e. X-plane, or Y-plane, or Z-plane) for any total length of the feeding network. Feeder projects are run using WIPL-D option Run\Feed, in which one should specify the length of the sliding short (see formula (1)) and in which plane (X, Y or Z) the port 2 is positioned. In such run the basic project is copied into four subprojects. In subprojects 1 and 3 the plane of port 2 is set to PEC (perfect electric conductor), while in subprojects 2 and 4 the plane of port 2 is set to PMC. Inparticular in projects 3 and 4 the length of feeder is increasing by the sliding short length. Combining reflection coefficients from these four subproject the s-parameters of



Fig. 3. S-parameters of full 3D circuit.

feeding circuits are obtained, as shown in Fig. 5.

It is easily seen from Section II.A that sliding short lengths that represent multiple of  $\lambda/4$  results in b = d, and  $s_{22}$ according (9) can not be calculated. Hence, it is recommended that that sliding short lengths should be lower than  $\lambda/4$ (quarter of wavelength) at the stop frequency. Numerical results show that optimal length at one frequency is  $\lambda/8$ . Almost identical results are obtained for any length from  $\lambda/32$ to  $7/32 \lambda$ .

Finally, combining results for full 3D circuit given in Fig. 3 and results for feeding circuits given in Fig. 5, we obtain the *s*-parameters of coaxial line-to-waveguide transition, as shown in Fig. 6. It is seen that the transition is well adjusted at 60 GHz.



Fig. 4. Geometrical models: a) coaxial feeder, and b) waveguide feeder.



Fig. 5. S-parameters: a) coaxial feeder, and b) waveguide feeder.



## IV. CONCLUSION

The paper presents application of new technique for deembedding of *s*-parameters, which is based on reflection coefficient calculations, in WIPL-D environment. The flexibility of the new technique is illustrated on the example of coaxial line-to-waveguide transition.

### REFERENCES

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