Power Amplifier Behavioral Modeling Strategies Using Neural Network and Memory Polynomial Models

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Abstract — This paper discusses the performance comparison of an artificial neural network (ANN) model and a memory polynomial (MP) model for modeling the dynamic nonlinear input-output characteristics of power amplifier (PA) with memory. The ANN model was based on time delay neural network (TDNN) and the memory polynomial model was developed using analytical polynomial function. Both models were developed to fit the dynamic AM-AM and AM-PM conversions of the PA obtained from QPSK digital modulated signal. Furthermore, the conventional TDNN model topology was extended by introducing an additional input to take into account the frequency tone spacing (for two-tone excitation condition) of the stimulus signal, to incorporate memory-effect behavior. The comparison results show that the two variants of PA models are applicable to model the PA, however, the TDNN model, compared to the memory polynomial model, can give better modeling results.

Keywords — TDNN, memory polynomial, PA behavioral modeling, memory effects.

I. INTRODUCTION

In recent years, the development of cellular market has world-wide initiated researchers to investigate the implications of power amplifiers (PAs) in the telecommunication system. With the increasing importance of spectral efficiency, an RF PA, used in third generation (3G) mobile communication, needs to be highly linear to meet stringent spectrum emission requirements of 3GPP standards. In this regard, PA modeling is the most important step in the design of communication systems wherein non-constant envelope digital modulation techniques are adopted. System-level amplifier behavioural modeling allows the design and simulation of more complicated structures like transmitter and/or linearizer with different complex modulated signal. The first step in designing or developing any linearization techniques is to precisely characterize and model the nonlinearity of the amplifier [1], [3-11]. Behavioral modelling is often used for modelling PA nonlinearity because it provides an efficient means to compute input-output nonlinear relation without the need of physical analysis of the device or system.

Amplifying an input signal to levels required for reliable transmission using currently available PA introduces amplitude and phase distortion. In narrowband applications, such as global system for mobile communication (GSM), the nonlinearity of the PAs is usually expressed by amplitude modulation to amplitude modulation (AM-AM conversion) and amplitude modulation to phase modulation (AM-PM conversion) characteristics as shown in Fig. 1. In this case, AM-AM and AM-PM conversions are only function of the input signal level and independent of its envelope frequency (see Fig. 1). A PA having such characteristics is called quasi-memoryless PA where it is assumed that the output of the PA is only function of the instantaneous input signal. The characteristics of such a PA will cause symmetrical IMD in the output spectrum when the input signal of the PA is multi-tone [4]. Similarly, we get a symmetrical spectral regrowth in the output spectrum when input to the PA is a digitally modulated signal. Conventional PA models, such as Saleh model and polynomial [5], can be used in modeling such behavior.

![AM-AM and AM-PM characteristics of the PA used in simulation for single tone input at 850 MHz.](image)

However, for a nonlinear system with memory, the output of the PA is not only a function of its instantaneous input but also its past inputs. Consequently, the corresponding single-tone AM-AM and AM-PM characteristic response does not give sufficient information about the system nonlinearity. Thus an accurate characterization and modelling techniques are essential to precisely describe the system nonlinearity.

The use of Volterra series representation is one recommended method to model such dynamic systems with memory [6]. But the computation of Volterra kernels is often difficult when the system has complex nonlinearity. Thus it is necessary to implement a simpler model for most complex PA modelling applications.

Analytical equations, such as memory polynomial, which will be discussed in section II, was analysed in a detailed manner in [7] to model PAs with memory, however, it is difficult to compute and optimize the fitting parameters of the
analytical functions for dynamic (envelope frequency dependent) input-output measured data. Additionally, the complexity of the model increases with increasing memory and nonlinearity order. In this case, a large number of coefficients should be extracted for better approximation.

Furthermore, some authors [8] have published papers on Wiener model, a two-box model approach used for PA modeling with memory, which is a cascade connection of linear time invariant (LTI) system and memoryless nonlinear system. However, this approach is applicable for modeling AM-AM and AM-PM nonlinearities for a particular modulation frequency of the excitation signal.

In modern wideband applications involving wideband code division multiple access (WCDMA) stimulus signals, which are highly sensitive to nonlinearities, PA memory effects cannot be neglected. In simple terms, memory effects are defined as the variation in the amplitude and phase of distortion components caused by changes in modulation frequency of the input signal. These effects distort the symmetry in the output spectrum of the amplifier and hence make the PA linearization methods less effective [1], [4].

In general, memory effects depend on signal envelope frequency ranging from 10 kHz to several MHz and can be defined as dynamic behaviour of the output signal as function of the envelope frequency. As discussed in [3], the main causes of memory effect in high power amplifiers (HPAs) are:

- Electro-thermal memory effects: caused at low modulation frequencies due to self-heating and surface trap induced dispersion effects [9] of transconductance and channel conductance of the active device.
- Electrical memory effects: caused by varying envelope, fundamental or second harmonic impedances, arising from matching network and/or bias circuits, at different modulation frequencies.

Fig. 2 shows how the 3rd-order intermodulation distortion (IMD3) is a function of power and frequency spacing between two tones of the input signal where the frequency swept from 0.01-20 MHz tone spacing. Furthermore, this figure presents the asymmetry between lower and upper IMD. This non-constant distortion behavior is related to the memory effect in PA as discussed in [1], [8] and [10]. The difficulties involved in modeling nonlinear amplifier with memory effects originate not only from the nonlinearity as function of the input power level such as AM-AM and AM-PM, but also from the nonlinearity as function of the envelope frequency. In this case, Wiener model, for instance, cannot model envelope frequency dependent dynamic characteristics. As a result, a more comprehensive modeling approach based on parallel Wiener model (see Fig. 3) should be developed which is discussed in detail by H. Ku et al [8].

Another reliable technique in order to overcome these limitations would be to develop a new PA modeling approach using artificial neural network (ANN) [7], [10-15]. ANN model, which has the ability to learn by example, makes them very flexible, powerful and reliable for nonlinearity and...
memory effects modeling of the PA. Furthermore, it has the
capability to learn a complex nonlinear behavior of a dynamic
system without the need to understand the internal
mechanisms of the system. Consequently, ANN can be an
alternative and attractive approach for PA modeling with
memory effects.

Section II and III of this paper describe memory polynomial
(MP) model and time delay neural network (TDNN) model
for modeling the PA including memory effects, section IV
summarizes the modeling results and section V gives a
conclusion.

II. DESCRIPTION OF MEMORY POLYNOMIAL
MODEL

Polynomial function is one of the most frequently used
method for PA modeling because regenerated spectral
components can be calculated analytically based on the
polynomial coefficients. Nowadays, baseband memory
polynomial model is widely used to describe nonlinear effects
in a PA, which exhibit memory effects [3], [7].

The general form of a MP behavioral model with a unity
delay tap denoted by $Z^{-1}$, which is used to fit the discrete
complex measured data of the PA, can be written as

$$V_{out}(n) = \sum_{q=0}^{Q} \sum_{k=1}^{K} \tilde{a}_{kq} \cdot V_{in}(n-q) (2n-1)$$  \hspace{1cm} (1)

where $\tilde{a}_{kq}$ are complex MP coefficients, which can be
estimated by a simple least-squares method and $q=0,1,\ldots,Q$ is
the memory interval which is equal to the sampling interval.
$V_{in}(n)$ and $V_{out}(n)$ are measured complex $n^{th}$ sample of input
and output signals. $Q$ and $K$ are the maximum memory- and
polynomial-order respectively. This equation can be
represented by a block diagram shown in Fig. 4 and can also
be rewritten in a compact form if we let

$$F_{q}(n-q) = \sum_{k=1}^{K} \tilde{a}_{kq} \cdot V_{in}(n-q) (2n-1)$$  \hspace{1cm} (2)

so that

$$V_{out}(n) = \sum_{q=0}^{Q} F_{q}(n-q)$$  \hspace{1cm} (3)

In this model the output signal is dependent on the
instantaneous and previous input signal, which allows
modeling the memory in the system.

III. DESCRIPTION OF TDNN MODEL

ANN can be viewed as a model capable of mapping inputs
to outputs by learning (by optimization) the behavior of a
system from a given environment. These NNs operate by
adjusting their weights iteratively after knowing the error
between the actual and the desired output. NNs have drawn
the attention of several RF and microwave CAD design

![Fig. 4. Memory polynomial power amplifier model.](image)

![Fig. 5. TDNN for power amplifier model.](image)

researchers due to its immense potential of modeling complex
nonlinear dynamic systems.

A. Conventional TDNN Topology

The topology of a three-layered feedforward TDNN model
with time delay taps is depicted in Fig. 5, which is commonly
used for representing nonlinear PA model with memory. In
general, these feedforward NNs implement back propagation
(BP) algorithm to approximate the nonlinear relationship
between the input and the output signal of PA [11]. The
TDNN PA model is analogous to the classical Wiener model,
which can be viewed as a cascade of FIR filter (LTI system)
and an MLP (nonlinear memoryless system) block. The
topology of the conventional TDNN model was realized in
MATLAB.

However, this conventional TDNN model can only be
implemented, for instance, for a multi-tone excitation signal
or realistic telecommunication digital modulated signal, in
general, which has a fixed envelope bandwidth frequency. In other words, it models the dynamic AM-AM and AM-PM characteristics for a particular input baseband envelope frequency. For example, let us assume that a TDNN PA model is developed for a two-tone stimulus signal having $\Delta f_1$ tone spacing. Now, if the tone spacing of the input signal envelope is varied to $\Delta f_2$, and then tested for the same PA model, the TDNN will give incorrect result, in which case, the response of the PA model will be different from the desired output. This is because the TDNN model is trained for a specific tone spacing of the input two-tone stimulus. This, in turn signifies that, the model cannot predict memory effects using conventional TDNN topology.

Therefore, the TDNN model should be suitably modified to predict memory effect phenomenon in case of dynamic stimulus signals with varying envelope frequencies. Accordingly, a new improved TDNN model is proposed as shown in Fig. 6, which incorporates an additional input $\Delta f$ for defining the tone spacing of the signal envelope.

### B. Improved TDNN Topology Exhibiting PA Memory-Effects

In case of the improved TDNN model, the additional input vector, which signifies the tone spacing of the stimulus signal, is calculated by scaling the corresponding tone spacing frequency in the range of [-1,1]. This computation procedure is discussed in detail in section IV.

In commonly used ANN models, the complex input-output data are processed separately either in the rectangular form (real and imaginary) or converted to polar form (magnitude and phase). Although the authors [12] have proposed complex-value based NNs, the training algorithm becomes very complicated because the activation functions, which are used in the NN, must also be complex. Therefore, two separate TDNNs are implemented, as illustrated in Fig. 7, to model the dynamic AM-AM and AM-PM nonlinear characteristics (Fig. 6). Levenberg-Marquardt BP algorithm was used for training these TDNN models. To develop such model, only measured complex input and output data are required without the need to understand the internal mechanisms of the PA. Consequently, such model has the capability to fit a complex nonlinear behavior in PAs.

### IV. MODELING RESULTS OF THE DEVELOPED MODELS

In this section, the modeling results of two variant PA models are discussed. Both models were realized in MATLAB. The developed memory polynomial PA model has a polynomial order of seven and a memory order of five. On the other hand, the developed TDNN model has 7-neurons at the input layer, 15-neurons in the hidden layer and one neuron at the output layer. The activation functions used for the input-output layers and hidden layers are linear transfer function and hyperbolic tangent sigmoid activation function, which is a nonlinear function, respectively.

The measurement data, used for comparing modeling results, is extracted from ADS® designed class AB amplifier. This PA circuit model, which exhibits memory effects, uses a Motorola MOSFET.

Concerning the procedure to calculate additional input vector for the improved TDNN model, let $x_{\text{min}}$ and $x_{\text{max}}$ represent the minimum and maximum input vectors of simulated data and $\bar{x}_{\text{min}}$ and $\bar{x}_{\text{max}}$ represent the minimum and maximum scaled vectors. Subsequently, the linear scaled result [13] is computed as

$$\bar{x} = \bar{x}_{\text{min}} + \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} (\bar{x}_{\text{max}} - \bar{x}_{\text{min}})$$

(4)

wherein for a given value of $x$, scaled vector $\bar{x}$ is obtained which is denoted as $\Delta f$ in Fig. 6. As an example, Table I shows the unscaled and scaled input vectors, which indicates the tone spacing of the two-tone stimulus signal, having boundary conditions $\{x_{\text{min}}, x_{\text{max}} = 0, 20 \text{ MHz}\}$ and...
For PA modeling including memory effects, more practical and application oriented stimulus signals, such as digital modulated signals with non-constant envelope, should be considered. Moreover, these signals have larger PAR compared to the two-tone signal and can extract more information about nonlinearity from broadband PA. For this reason, around 6000 samples of input and output measurement data was considered, obtained from QPSK input signal having 12 dBm input power level, 850 MHz center frequency, 6 MHz bandwidth and roll-off factor of 0.35.

### TABLE I

<table>
<thead>
<tr>
<th>Unscaled ((\Delta f)) in MHz</th>
<th>Scaled ((\Delta f'))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-0.98</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.92</td>
</tr>
<tr>
<td>1</td>
<td>-0.90</td>
</tr>
<tr>
<td>3</td>
<td>-0.70</td>
</tr>
<tr>
<td>5</td>
<td>-0.50</td>
</tr>
<tr>
<td>10</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The dynamic AM-AM and AM-PM modeling results for both models are shown in Fig. 8. For the memory polynomial PA model, the maximum error between the measured and the modeled data was found equal to \(10^{-2}\) and \(10^{-5}\) for TDNN model trained for 100 iterations. Hence negligible difference is observed between measured and modeled data in Fig. 8. Furthermore, by increasing the order \(K\) of the polynomial in memory polynomial model and the number of iterations, in case of TDNN model, better results can be achieved. Nevertheless, it is important to note that usage of very high polynomial order might lead to computational instability caused by the matrix inversion in least square solution. To overcome this drawback, conventional polynomial can be replaced by orthogonal polynomial [14].

### CONCLUSION

In this paper, two variants of PA models, memory polynomial and TDNN, for PA modeling with memory effects have been described and compared. Both models can be used to model a nonlinear PA with memory. Further, a novel TDNN topology was proposed, wherein an additional input was added to the existing conventional model, which takes into account the tone spacing of the stimulus signal. The modeling results show that the TDNN model shows excellent agreement with the measured data compared to the memory polynomial model. This is due to the difficulty in fitting a dynamic behavior using analytical function, especially when the system memory becomes prominent and possess a complicated dynamic form. Furthermore, since TDNN model is more accurate and reliable, it seems to be an effective solution for amplifier modeling including memory effects.

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### REFERENCES


