Negative Refractive Index Metamaterials: Principles and Applications

Zoran Jakšić[1], Nils Dalarsson[2], Milan Maksimović[3]

Abstract – **We review structures for microwave and optical range containing 'left-handed' metamaterials – artificial composites with simultaneously negative effective permittivity and permeability which achieve negative values of refractive index. Attention has been given to the fundamentals of negative index materials, the main design strategies and proposed applications which include subwavelength resonant cavities, sub-diffraction limited near-field lenses (superlenses) and phase compensators. Some practical implementation in microwave are considered.**

Keywords – **Metamaterials, left-handed materials, negative refractive index, superlenses, subwavelength resonant cavities.**

I. Introduction

The advent of microsystem technologies and nanotechnologies enabled breakthroughs in many different areas of science and technology, offering functionalities well beyond the natural ones. It enabled structuring of materials for electromagnetic and optical applications in manners previously unimaginable. Among probably the best known examples of novel electromagnetic structures are photonic crystals (e.g. [1]) and the negative refractive index metamaterials, popularly known as 'left-handed' materials [2], [3]. These enabled extension of the operation of passive and active elements for microwave and optical applications beyond the limits previously deemed possible. Another result was an extreme miniaturization of components, sometimes even three to four orders of magnitude.

Negative refractive index metamaterials (NRM) are artificial composites, characterized by subwavelength features and negative effective value of refractive index [2], [3]. These materials were theoretically predicted in 1967 by Veselago [4] (in English translation of his text it is erroneously stated that the first results on this topic were published in 1964). However, most of the ideas connected with negative refraction appeared even earlier. L.I. Mandelshtam described negative refraction and backward propagation of waves in his textbook published in 1944 [5]. Backward-wave transmission lines were described by Malyuzhinets in 1951 [6]. The early history of the field is described in some detail in [7]. The main problem with all of the early works was that due to the technological limitation these ideas remained only a scientific curiosity and thus did not attract much attention.

With the arrival of micro- and nanofabrication, new possibilities opened for practical implementation of different metamaterials and the field became intensely studied by a number of research teams. Extremely influential were seminal texts by Pendry [8],[9],[10]. A further boost to the field came when the existence of NRM was experimentally confirmed by Smith, Shelby et al [11],[12],[13],[14]. The applicability of NRM for lensing which avoids the diffraction limit by utilizing both periodic and evanescent electromagnetic waves, as proposed by Pendry in 2000 [15] even further increased the interest for NRM. The field continued to expand owing to the fact that the Maxwell equations are scalable, thus practically the same strategies can be used for the microwave and the optical range, including the transmission line approach.

Today the number of the teams studying NRM and the number of published treatises on this topic are both increasing exponentially (see e.g. [2],[3] and references cited therein). The *Science* journal included these materials among the ten most significant breakthroughs of the recent years [16].

The aim of this paper is to present a comprehensive review of the state of the art in the rapidly expanding field of NRM. We examine electromagnetics and physics of these materials, focusing our attention to some issues which may appear counter-intuitive. We systematize the most important approaches to the design and application of the NRM.

The text is organized as follows. In the first part fundamentals of materials with negative refractive index are reviewed from the phenomenological point of view. The second part handles electromagnetic and optical design strategies to reach negative effective refractive index, i.e. describes the basic building blocks of NRM while putting an accent to the structures for the optical range and their nanofabrication. Finally, the third part shortly describes the most interesting proposals for practical applications.

II. Fundamentals of Negative Refraction

A. Some Definitions

Since one often encounters slightly dissimilar, but sometimes even outright contradictory descriptions of different structures and phenomena connected with negative refraction, we start this Subsection by giving some definitions to be used throughout this paper.

First we define metamaterial as an artificially structured material furnishing properties not encountered in nature.

Electromagnetic metamaterial is a metamaterial furnishing tailored electromagnetic response which surpasses that of natural structures. It may be an ordered structure (periodic, quasiperiodic, aperiodic, fractal) or disordered (random)

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structure. Artificial dielectrics [17]were historically the first electromagnetic metamaterials. Examples of periodic electromagnetic metamaterials include electromagnetic and photonic crystals and left-handed metamaterials.

Electromagnetic metamaterials intended for ultraviolet, visible or infrared range are termed optical metamaterials.

Materials that can achieve negative effective value of their refractive index were historically first termed left handed metamaterials (LHM). These can be defined as artificial composite subwavelength structures with effective electromagnetic response functions (permittivity and permeability) artificially tuned to achieve negative values of their real part. They are also denoted as negative refractive index materials (NRM), to make a distinction between these and chiral materials, which are also sometimes called 'left-handed'. Another name often met in literature is double negative materials (DNG), to stress the difference between these and the so-called single negative materials (SNG), i.e. structures characterized either by negative effective permeability or permittivity, but not both at the same time. Also NPV (negative phase velocity) media is used. Other names encountered in literature include Veselago media and backward media. Further details on terminology can be found in [18],[19]. In this text we will mostly stick to the term negative refractive index materials (NRM).

B. Negative effective refractive index

The complex refractive index of a given medium is defined as the ratio between the speed of an electromagnetic wave through that medium and that in vacuum and can thus be written as $n^2 = \mu\varepsilon$, where μ is complex relative magnetic permeability and ε complex relative dielectric permittivity. If both ε and μ are negative in a given wavelength range, this means that we may write $\mu = |\mu| \exp(i\pi)$ and in an equivalent fashion $\varepsilon = |\varepsilon| \exp(i\pi)$. It follows that

$$\begin{aligned} n &= \sqrt{|\mu||\varepsilon|\exp(2i\pi)} \\ &= \sqrt{|\mu||\varepsilon|}\sqrt{\exp(2i\pi)} \\ &= -\sqrt{|\mu||\varepsilon|} \end{aligned} \tag{1}$$

i.e. the refractive index of a medium with simultaneously negative μ and ε must be negative. A more detailed consideration based on causality may be found e.g. in [20].

Since no known material inherently possesses negative permeability and permittivity, NRM metamaterial is a composite of two materials which individually show $\varepsilon<0$ and $\mu<0$. This raises a question when it is permissible to describe such a composite as a medium with negative effective index.

Starting from the Maxwell equations in their integral form (Gauss and Ampere law)

$$\int_C \vec{H}d\vec{l} = \frac{\partial}{\partial t}\int_S \vec{D}d\vec{S} \tag{2}$$

$$\int_C \vec{E}d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}d\vec{S} \tag{3}$$

one can define average fields in quasistatic approximation as

$$\begin{aligned} \langle H\rangle_{x_i} &= \frac{1}{a_{x_i}}\int_0^{x_i}\vec{H}d\vec{r} \\ \langle B\rangle_{x_i} &= \frac{1}{a_{x_i}}\int_0^{x_i}\vec{B}d\vec{S} \end{aligned}, \quad x_i = x, y, z \tag{4}$$

so that the effective μ across the considered volume is

$$\left(\mu_{eff}\right)_{x_i} = \frac{\langle B\rangle_{x_i}}{\mu_0\langle H\rangle_{x_i}} \tag{5}$$

Analogously, the effective permittivity can be written

$$\left(\varepsilon_{eff}\right)_{x_i} = \frac{\langle D\rangle_{x_i}}{\varepsilon_0\langle E\rangle_{x_i}} \tag{6}$$

A practice often met in literature when investigating NRM is to analyze the cases with frequency-independent ε and μ [21]-[24]. However, any real NRM must be dispersive and lossy in order to preserve causality principle [25].

An often utilized form for effective ε and μ is the lossy Drude model [2] according to which polarization is

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\Gamma_e)} \tag{7}$$

and magnetization

$$\mu_{eff}(\omega) = 1 - \frac{\omega_{pm}^2}{\omega(\omega + i\Gamma_m)} \tag{8}$$

where plasma frequency ω_p and dumping constant Γ are usually assumed to be equal for both polarization and magnetization, $\omega_{pe} = \omega_{pm} = \omega_p$, $\Gamma_{pe} = \Gamma_{pm} = \Gamma_p$. For low-loss model ($\omega_p >> \Gamma$) the refractive index becomes

$$n_{eff}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)} \approx 1 - \frac{\omega_p^2}{\omega^2} + i\Gamma\frac{\omega_p^2}{\omega^3} \tag{9}$$

An often met assumption is that dumping is negligible, which is an acceptable approximation even from the experimental point of view [11]. Finally, if lossy metamaterial is explicitly considered, a common assumption is that the dumping factor is given as a fraction of plasma frequency [26].

Another model often encountered in literature is the Lorentz model [27], applicable for some of the experimental implementations of negative permeability structures.

A more detailed consideration of constitutive relations in NRM can be found in [28].

C. Basic Properties of NRM

Many interesting phenomena not appearing in natural media are observed in double negative materials. These are shortly reviewed below.

The most important and probably the most thoroughly considered effect in NRM is the modification of the Snell's law, i.e. negative refraction [2], [4]. Fig. 1 shows the refraction of electromagnetic plane wave incident from vacuum (or air) onto an electromagnetically thicker medium with its real part of effective refractive index positive (Fig. 1 top) or negative (bottom). The range of refractive index values which are positive, but < 1 is also shown in top figure.

Wavevector and Poynting vector are parallel in the positive index case and antiparallel in the NRM case. Thus a beam arriving at the interface between the positive and the negative index material will be refracted to the same side it came from. At first sight this may appear counter-intuitive. A consequence is that a plane parallel slab of a NRM, rather to scatter beams arriving from a distance d_1, will focus them at a distance d_2 if $d_1 + d_2 = d$, where d is the thickness of the slab. In other words, a convex NRM lens will diverge a plane wave, while a concave lens will converge it.

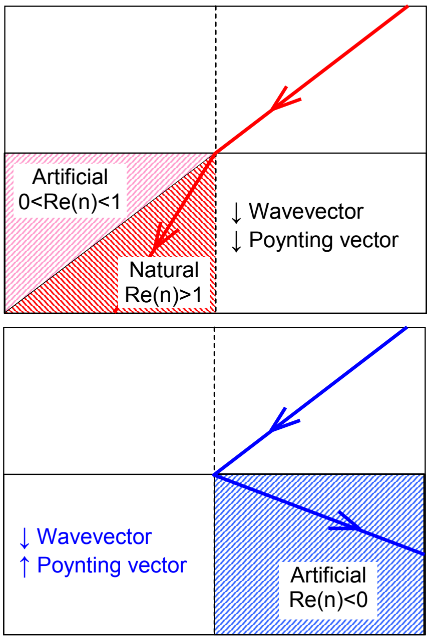

Fig. 1. Refraction in conventional material (top figure, wavevector and Poynting vector parallel) versus that in NRM material (bottom figure, wavevector and Poynting vector antiparallel).

It should be mentioned that the angle of the reflected beam remains unaffected by the NRM.

Another effect in NRM is the reversal of Čerenkov radiation [29]. It is known that a charge moving through a medium at a speed larger than the speed of light in that medium emits Čerenkov radiation in a certain angle cone. In NRM this radiation is emitted in backward direction, i.e. at an obtuse angle.

The Doppler shift in NRM will also be reversed in a NRM medium [2]. This means that the detected shift in an approaching NRM object will be red, and that in a receding blue, counter to the behavior in a conventional medium.

If a beam is incident to optically rarer medium, there is a critical angle beyond which its propagation is evanescent and a beam is then reflected with a certain shift of the beam in the transverse direction – the so-called Goos-Hänchen shift. It has been shown that the Goos-Hänchen shift is negative for reflection from NRM [30].

Since phase and group velocity have opposite signs, a NRM medium is described by the backward propagation. One should note, however, that under no circumstances the causality is violated and that in that respect the propagation through NRM corresponds to that in positive index media.

D. Mesoscopic Versus Subwavelength Negative Refraction

A misconception is that negative refraction of electromagnetic waves is equivalent to the existence of negative refractive index. The former may occur in electromagnetic and photonic crystals (the superprism effect) [31], but also in conventional diffractive gratings and generally in holographic optical elements. In that case it is a mesoscopic phenomenon entirely due to diffractive effects, i.e. Bragg scattering, and the phase velocity and wavevector have the same direction as the Poynting vector, the same as in conventional dielectric case.

However, in metamaterials with negative refractive index the wavevector and the Poynting vector have the opposite directions, the process is due to the subwavelength nature of the structures and a plethora of different effects occur which do not take place in diffractive structures [32].

Another important question is that of homogenization. Although there have been attempts to describe photonic crystals by effective refractive index, especially near the photonic band edges, a NRM-like homogenization is difficult to apply [33], thus negative refractive index cannot be even formally introduced. Still, PBG can be utilized for all-angle negative refraction and one may expect devices based on it.

E. Reversal of Fermat Principle

An interesting consideration of refraction in NRM from the point of view of the Fermat's principle [34] can be done by analyzing a propagating wave using ray tracing. The optical path length is $OPL = \int_A^B n(\vec{r})ds$, where $n(\vec{r})$ is an arbitrary dependence of refractive index on the position vector $\vec{r} = \vec{r}(x, y, z)$ and ds is a differential element of the path length. The time taken by light to travel from a point A to B is proportional to the optical path. The least action principle [35] states that the most probable path is determined by the path of stationary phase, which corresponds to an extremum in the spatial derivative of the total travel time through all possible paths. An extremum means that the variation in optical path is zero, $\delta\int_A^B n(\vec{r})ds = 0$. Generally, this extremum could be a minimum, a maximum, or a point of inflection [35]. The Fermat principle states that it must be a minimum [35], .

Consider the path for a ray incident from medium with an index of refraction n_1 to an medium n_2 (a derivation can be found in [36],[37]). For $n_2 > 0$, the curvature is positive, indicating that Fermat's result indeed gives the minimum time and distance. For $n_2 < 0$, the curvature is negative, pointing to an apparently counter-intuitive conclusion that a maximum travel time is necessary for a wave to cross a path in NRM material. In terms of the least action principle, such a solution for the path is adequate after recognizing that negative index materials are causal in an energy sense but not in a temporal sense [36], [2]. Thus for the case of NRM the Fermat principle has to be reformulated.

III. Design Strategies for NRM

A. Basic 'Particles' of NRM

It is known that highly conductive metals with permittivity dominated by plasma-like behavior (as described by Drude model) show negative dielectric permittivity in a narrow range at UV/visible frequencies. However, typical materials with negative permeability or negative permittivity are composites consisting of a large number of the basic building blocks described as unit cells (to retain correspondence with single crystals of natural materials). These building blocks are also referred to as electric ($\varepsilon<0$) or magnetic ($\mu<0$) 'particles' [2] and are often composed of dielectric with metal inclusions.

The characteristic dimensions of NRM particles have to be much smaller than the operating wavelength (in order for the effective medium approach to be applicable), but still macroscopic, i.e. much larger than the atomic or ionic dimension of their constituent materials.

All NRM structures fabricated until now were highly dispersive and dissipative. The main cause of dissipation is large absorption due to conduction losses in the metal parts of the NRM.

The main structures utilized to obtain NRM include thin metallic wires, metal cylinders, 'Swiss roll' structures, split ring and complementary split ring resonators (SRR), omega structures, broadside-coupled or capacitivelly loaded SRRs, capacitivelly loaded strips, space-filling elements, etc. Of these, only the most important ones will be presented here.

A. Thin Metallic Wires

Thin metallic wires were described as one of the earliest structures with negative permittivity. The media with embedded thin metallic wires as an artificial dielectrics for microwave applications were reported in 1953 [17].

The structure with $\varepsilon < 0$ described by Pendry [9] consists of a square matrix of infinitely long parallel thin metal wires embedded in dielectric medium (Fig. 2)

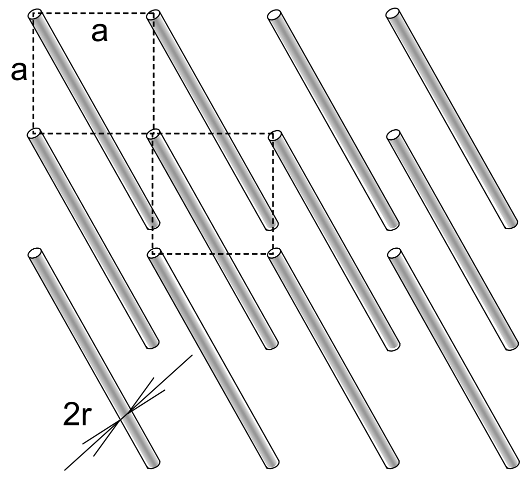

Fig. 2. Metallic wire mesh with negative dielectric permittivity

In the situation shown in Fig. 2 the medium is vacuum or air, the unit cell length is a and the radius of a single wire is $r<<a$. If plasma frequency for the longitudinal plasma mode is

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \ln(a/r)} \tag{10}$$

the effective dielectric permittivity can be written as

$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega\left[\omega - i\left(\omega_p^2 a^2 \varepsilon_0\right)/\sigma\pi r^2\right]} \approx 1 - \frac{\omega_p^2}{\omega^2} \tag{11}$$

i.e. it becomes negative for $\omega<\omega_p$. The approximate value at the right-hand side of the above expression is valid if conductance $\sigma\rightarrow\infty$.

B. 'Swiss Roll' Structures

The induced currents in a particle (both real and displacement ones) contribute to its effective magnetization through their magnetic moments. This contribution is non-negligible if at the same time their electric polarizability is small. For instance, if the effective permeability of the structure of metal cylinders is considered, similar to that shown in Fig. 3, one obtains that its effective permeability cannot reach negative values. However, the introduction of capacitive elements into the structure furnishes $\mu<0$.

This can be practically done by rolling up a metal sheet into spiral coils which assume the form of a cylinder [10] (Fig. 3). This is the popularly know Swiss roll structure.

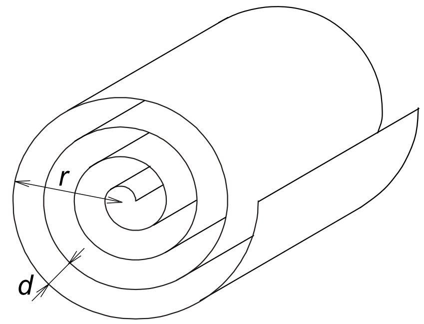

Fig. 3. Swiss roll structure with negative permeability

The sheets in the Swiss roll coils are separated by an insulator with a thickness d. If the number of coils is N and their per unit length resistance is ρ, the effective μ becomes

$$\mu_{eff} = 1 - \frac{\pi r^2 / a^2}{1 - \left[\frac{da^2}{2\pi^2 r^3}(N-1)\omega^2 + i\frac{2\rho}{\mu_0 \omega r (N-1)}\right]} \tag{12}$$

Swiss roll structures are especially convenient for low frequency operation.

C. Split Ring Resonators

For decades, and starting in the early 1950s, different ring or ring-like structures with negative permeability were of interest as building blocks for artificial chiral materials in microwave. A split ring was described in this context in the textbook by Schelkunoff and Friis [38].

A double split ring resonator (SRR) (Fig. 4) is a highly conductive structure in which the capacitance between the two rings balances its inductance. A time-varying magnetic field applied perpendicular to the rings surface induces currents which, in dependence on the resonant properties of the structure, produce a magnetic field that may either oppose or enhance the incident field, thus resulting in positive or negative effective μ. In other words, the operation of a SRR represents an 'over-screened, under-damped' [2] response of material to electromagnetic stimulation.

For a circular double split ring resonator (Fig. 4a) in vacuum and with a negligible thickness the following approximate expression is valid [2]

$$\mu_{eff} = 1 - \frac{\pi r^2 / a}{1 + \dfrac{2\sigma i}{\omega r \mu_0} - \dfrac{3d}{\pi^2 \mu_0 \omega^2 \varepsilon_0 \varepsilon r^3}} \tag{13}$$

where a is the unit cell length, and σ is electrical conductance.

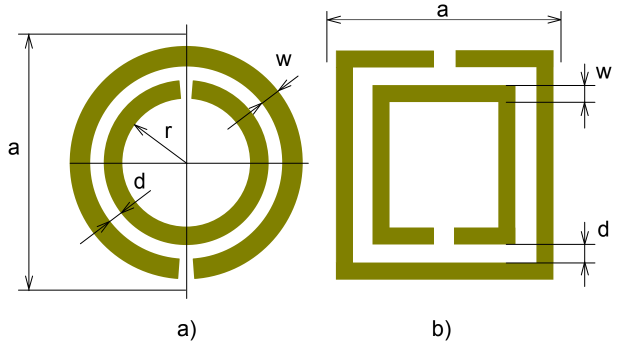

Fig. 4. Planar geometries of negative permeability material unit cells based on the split ring resonator; a) circular structure; b) square structure. Dark: thin metal film.

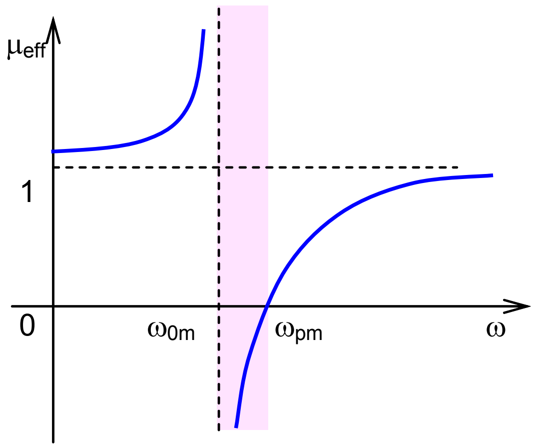

Fig. 5. Frequency dependence of effective permittivity for a split ring resonator. Shaded area denotes negative μ range

An detailed calculation of the effective magnetic permeability of split ring resonators can be found e.g. in [13].

The shape of the frequency dependence of (6) is shown in Fig. 5. It can be seen that there is a narrow frequency range where the effective permeability is below zero.

The resonant frequency (for which $\mu_{eff} \rightarrow \pm\infty$)

$$\omega_{0m} = \sqrt{\frac{3dc_0^2}{\pi^2 r^3}}, \tag{14}$$

while the magnetic plasma frequency (for which $\mu_{eff} \rightarrow 0$)

$$\omega_{pm} = \sqrt{\frac{3dc_0^2}{\pi^2 r^3 \left(1 - \pi r^2 / a^2\right)}}. \tag{15}$$

For a dielectric with ε and a ring width w

$$\omega_{0m} = \sqrt{\frac{3dc_0^2}{\pi \varepsilon r^3 \ln(2w/d)}}. \tag{16}$$

Split ring resonator is probably the most often used and analyzed negative permeability building block for the NRM.

D. Complementary Split Rings

Structures complementary to double split rings were designed and produced by applying the Babinet principle to the split rings [39]. In this way structures with apertures in metal surface are obtained, as shown in Fig. 6. These complementary split rings (CSRR) create negative ε instead of μ in a narrow range near the resonance frequency.

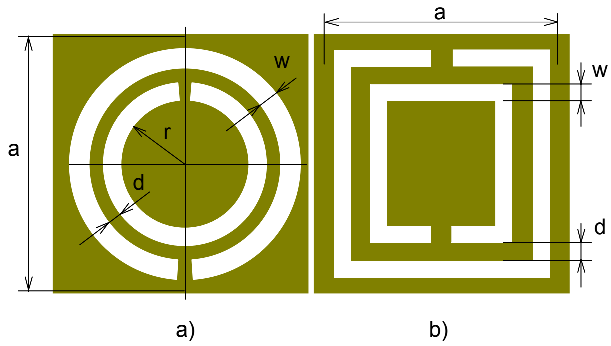

Fig. 6. Planar geometries of negative permittivity material unit cells based on the complementary split ring resonator; a) circular structure; b) square structure. Dark: thin metal film.

E. Transmission Line NRM

An alternative approach to using NRM 'particles' was simultaneously proposed in [40], [40] and [42] by three different teams and subsequently elaborated in [43], [44]. It utilizes the well-known duality between filters and distributed networks to produce the so-called transmission line metamaterials (TLM). Contrary to the split ring resonator-based metamaterials, the TLM are of non-resonant type.

There is a direct analogy between the voltage/current in a transmission line and the components of the electric and magnetic fields. While the impedance and the admittance of a homogeneous and isotropic medium material are defined as

$$\begin{aligned} Z &= i\omega\mu \\ Y &= i\omega\varepsilon \end{aligned} \tag{17}$$

these variables for a LH TLM are defined in an analogous manner as

$$\begin{aligned} Z' &= 1/j\omega C \\ Y' &= 1/j\omega L \end{aligned} \tag{18}$$

In an ideal case, conventional material multilayer filters are equivalent to distributed L-C networks with series inductance and parallel capacitance. A transmission-line based

NRM is equivalent to a dual distributed network with series capacitance and shunt inductance (Fig. 7).

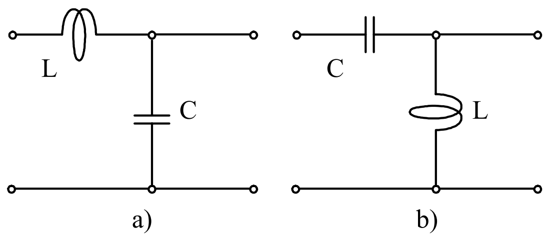

Fig. 7. Duality of (a) conventional (right-handed) and (b) left-handed transmission lines

This is actually a high-pass filter supporting backward wave propagation. [43].

A unit cell of a realistic lossless CRLH structure includes parasitic series inductance and shunt capacitance (Fig. 8).

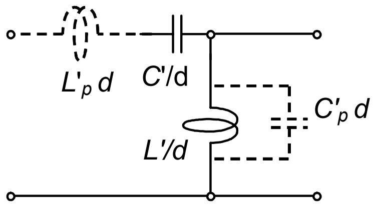

Fig. 8. Equivalent circuit for a lossless CRLH unit cell with a length *d*. Index "p" denotes parasitic inductance and capacitance (dashed lines). Per unit length parameters are shown (primed)

A name coined for such realistic TLM composites containing both a left-handed and a right-handed part is CRLH (composite Right/Left Handed) materials [45], [46].

The per-unit length impedance Z' and admittance Y' for the 1D unit cell in Fig. 1 are

$$Z'(\omega) = j\left[\omega L'_p - \frac{1}{\omega C'}\right],\ \ Z'(\omega) = j\left[\omega C'_p - \frac{1}{\omega L'}\right] \qquad (19)$$

The propagation along the CRLH is described by the well-known telegrapher's equation (e.g. [43]). The complex propagation constant γ is

$$\gamma = \alpha + j\beta = (Z'Y')^{1/2} \qquad (20)$$

where β is the phase constant. For $L'_p C' = L'C'_p$, we have the balanced case. The dispersion of β for the balanced case is

$$\beta(\omega) = \omega\sqrt{L'_p C'_p} - \frac{1}{\omega\sqrt{L'C'}} \qquad (21)$$

The dispersion relation of left-handed transmission lines is drawn in Fig. 9.

It can be seen that a CRLH TLM behaves as a purely "left-handed" for low frequencies (below the resonant frequency ω_0), and as purely "right-handed" for high frequencies (above the resonant frequency).

The low frequency range where no propagation occurs (the bottom part of Fig. 9) is the "left-handed" bandgap, since this part of the dispersion curve is defined by the left-handed characteristics of material.

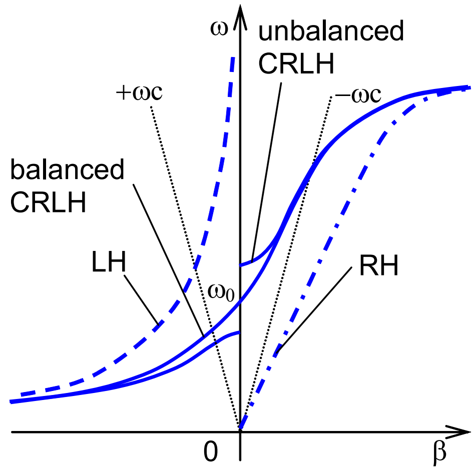

Fig. 9. Dispersion diagram for left- and right-handed transmission lines. Dashed-dotted: conventional (right-handed) transmission line; dashed: idealized left-handed TLM; solid: composite right/left-handed TLM for balanced case (continuous line) and unbalanced (discontinuity at β=0)

For the unbalanced case, there is an additional bandgap between the left and the right arm of the unbalanced dependence. Finally, there is a third gap above the dispersion curves at the right side – this is the "right-handed" bandgap.

Since the propagation constant of a material is defined as $\beta = \omega(\mu\varepsilon)^{1/2}$, the refractive index of a TLM is given by

$$n = c / v_p = c\beta / \omega \qquad (22)$$

Besides offering larger bandwidths and lower losses, the unit cells of TLM can be equipped with lumped circuit elements, allowing an additional degree of freedom in design, and are suitable for e.g. filtering applications [47].

F. Scaling to Optical Frequencies

An important issue to be considered is the high frequency scalability of the electric and magnetic particles for NRM. The thickness of the metal layers for the SRR/CSRR must not be smaller than the skin depth [26] (approx. 20 nm for silver at 100 THz). Moreover, at wavelengths approaching the optical range there is an additional inductance determining the plasma frequency, termed inertial inductance, which is a consequence of the electron mass and the currents through the SRR being almost purely ballistic [2]. For the scaled-down dimensions the inertial inductance becomes prevailing and the negative permeability/permittivity effect completely disappears. Several methods were proposed to overcome this problem [26]. The simplest one is to add more capacitive gaps to the original SRR design. An implementation of this principle to the square geometry SRR and CSRR is shown in Fig. 10.

Experimental NRM structures with near-infrared response have been demonstrated in various implementations (e.g. [26], [48], [49]). Among the most recent results are those on NRM metamaterials for 1.5 nm range with double-periodic array of pairs of parallel gold nanorods, achieving a value of negative refractive index of about –0.3, published by Shalaev et al in December, 2005 [50].

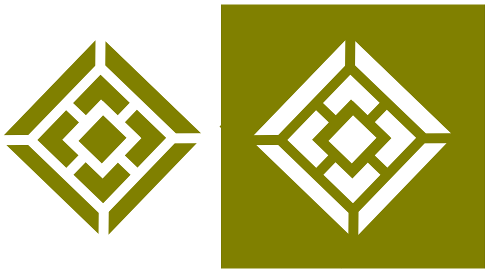

Fig. 10. Magnetic particles modified for high frequency design by adding capacitive gaps. Left: double SRR; right: double CSRR

G. Anisotropy and Homogenization Issues

The negative ε and the negative μ parts of a NRM are assumed to be electromagnetically independent. However, in reality they may interact and thus affect the performance of the composite, or even completely remove its NRM properties. A solution to this problem is to model the composites of negative μ and ε simultaneously when performing electromagnetic modeling to determine when the approximation of independence of its parts hold. In this way one can maximize NRM effects and remove undesired interference.

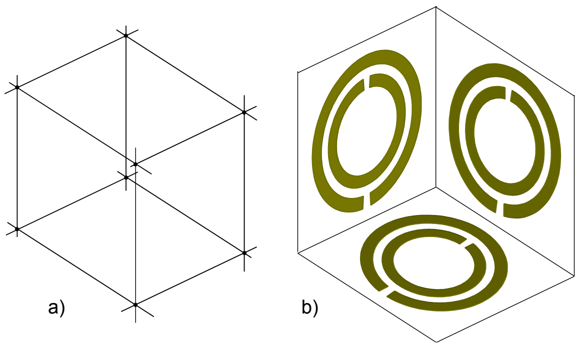

Fig. 11. Unit cells of three-dimensional isotropic structures. a) thin wire spatial mesh structure for negative permittivity; b) double split ring 3D structure for negative permeability

Another issue of interest for the existence of NRM is that of homogenization. The magnetic and electric elements of NRM are typically strongly anisotropic and with a very narrow bandwidth. The simplest and the most straightforward method to obtain a spatially isotropic effective medium is to repeat the same planar element in all three orthogonal orientations in space. Thus the obtained structure is approximately isotropic (although it would be best described by a band structure). Basic structures utilized to homogenize negative permittivity and negative permeability [10] are depicted in Fig. 11.

IV. Perfect Lens and Superlens

A. Principles of Perfect Lensing

It was shown in Subsection II C that the very fact that Snell's law is reversed in NRM must result in focusing properties of a plane-parallel metamaterial slab. However, due to the interaction of plasmons at the NRM surfaces with evanescent (near-field) components of objects located near the surface and the ensuing resonant enhancement, the NRM slab also focuses these evanescent waves. Thus a NRM slab acts as a lens focusing all components of electromagnetic radiation originating from the object and therefore surpasses the diffraction limit. Such a lens enables subwavelength resolution and is referred to as the perfect lens.

The concept of perfect lens was introduced in the classic paper by Pendry [15]. A perfect lens may be described as an optical or electromagnetic element which enables imaging in both near and far field, i.e. it forms image using both propagating and evanescent modes, resulting in an almost perfect image of the original.

The dispersion relation in free space can be written as

$$k_x^2 + k_y^2 + k_z^2 = \varepsilon_0\mu_0 \frac{\omega^2}{c^2} = k_0^2 \quad (23)$$

since $k_i = 2\pi/\Delta_i$ (where Δ_i is spatial distance over the coordinate i), if Δ_x, Δ_y are small, this means that the corresponding k_x, k_y must be large. If k_x, $k_y > k_0$, then k_z must be imaginary and thus the wave in z direction must be evanescent, i.e. decay exponentially with z. If the lens is at a distance larger than the operating wavelength, it will be unable to “see” this k_z. Therefore k_0 (and thus λ) remains the basic limitation in the far-field approximation – this is the well-known diffraction limit.

Let us further consider the transmission and reflection of a slab with $\varepsilon = -1$, $\mu = -1$ (which exactly compensates the positive permittivity and permeability of the surrounding free space). One can apply the well-known relations for transmission and reflection in case of multiple reflections

$$T = \frac{t_{21}t_{32}\exp(ik_{z2}d)}{1 - r_{12}r_{21}\exp(2ik_{z2}d)} \quad (24)$$

$$R = \frac{r_{21} + r_{32}\exp(2ik_{z2}d)}{1 - r_{12}r_{21}\exp(2ik_{z2}d)} \quad (25)$$

where t, r are the complex Fresnel transmittance and reflectance coefficients. The incident medium is denoted by 1, the slab material by 2, and the exiting medium is 3.

The wavevector k_{z2} can be written as

$$k_{z2} = \pm\sqrt{\frac{\varepsilon\mu\omega^2}{c^2} - k_x^2 - k_y^2} \quad (26)$$

where + sign is used for propagating, – sign for evanescent waves. In case of propagating waves one obtains $R = 0$ and $T = \exp(-ik_{z1}d)$, identical to the positive material case. However, if one replaces the expression for evanescent waves, the result is that $R = 0$ and $T = \exp(+ik_{z1}d)$. This means that NRM increases exponentially the amplitude of the evanescent wave and thus completely restores it, acting as a perfect complementary medium.

B. Losses and Active Compensation

It was mentioned before that to satisfy the causality principle a NRM must be lossy (α>0). A perfect lens with losses is referred to as superlens. The existence of absorption and dissipation significantly impairs the resolution of the superlenses and may completely remove the desirable effects. Ramakrishna [51] proposed a method to remove absorption losses and dissipation by optical amplification, e.g. by introducing lasing medium instead of dielectric into NRM-positive material composite and to utilize alternating NRM-amplifying layers instead of bulk NRM. Although there are problems with this approach (for example, regarding the influence of surface plasmons and extreme localization of fields to optical amplification), it appears that it could furnish improved results.

C. "Poor Man's" Superlens

It is well known that metals have negative dielectric permittivity in a range of frequencies in UV or visible spectrum. If the distances are much smaller than the wavelength, then the quasi-static (or extreme near-field) limit is observed and single-negative materials can be used as superlens (negative ε material will be applicable for p polarization, and negative μ for s polarization).

It was proposed to use very thin sheets of silver (a fraction of the wavelength) to obtain near-field lensing to p-polarized radiation [2]. In this way one avoids the diffraction limit in ultraviolet/visible part of the spectrum, where this feature is potentially the most applicable.

D. Transmission Line-Based Superlens

Planar left-handed media isotropic for perpendicular polarization of electric field were first described by Eleftheriades [52]. The first proposed solution was single-interfaced and consisted of transmission lines loaded with series capacitors and shunt inductors, where inductors were connected to the ground. Double interface transmission line lens (dual lens, shown in Fig. 12) was theoretically described in [53] and its design and testing results were presented in [54]. Resolution enhancement of that lens is proportional to the quality factor of the series load capacitors divided by the electrical thickness of the lens.

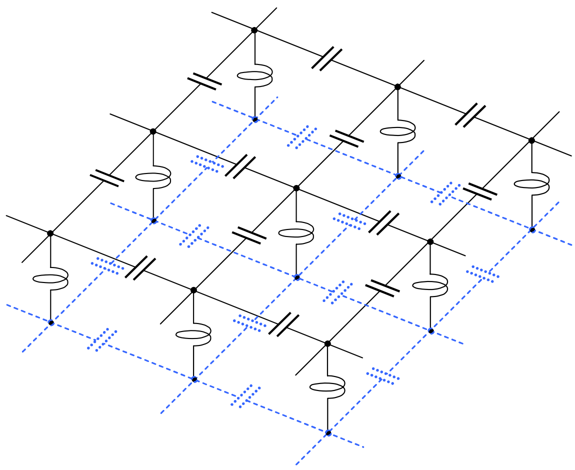

Fig. 12. 2D double-interface superlens based on transmission line loaded with series capacitors and shunt inductors. Dashed: bottom interface metamaterial-dielectric

E. Experiments on Superlens Nanolithography

Experimental use of superlenses in lithography is almost exclusively dedicated to planar silver superlenses. Zhang's group [55], [56] used a chrome mask on quartz support and a 35 nm Ag layer as superlens under UV illumination at 365 nm. Focused ion beam lithography was used to pattern chrome on quartz. These experiments demonstrated super-resolutions with 60 nm half-pitch, about one-sixth of the illumination wavelength. They produced parallel lines, but also wrote the word 'nano' in 40 nm wide straight-line segments, each letter 1.5-2 μm wide.

Another series of experiments was done by Blaikie's group [57]. They reported the application of a modified form of conformal-mask photolithography. They used a tungsten-on-glass mask. Dielectric spacers and 50 nm thick planar silver superlens were coated onto the mask. The operating wavelength was 365 nm. They produced parallel lines with periods down to 145 nm, a resolution enhancement of more than two.

V. Phase Compensators and Subwavelength Resonators

Phase compensation may be defined as partial or complete removal of phase shift of electromagnetic wave propagating through a structure containing both positive and negative refractive index material.

The simplest structure to be considered as phase compensator consists of two slabs, one of them positive index material with a thickness d_1 and refractive index n_1, the other NRM, described by d_2 and n_2. If $\vec{k}_0$ is the wave vector in free space, then wave vector in the material is $\vec{k}_i = \vec{k}_0 n_i$ (i=1,2) if impedance-matched to free space. After passing through both slabs, the phase is $\phi = \vec{k}_1 d_1 + \vec{k}_2 d_2 = \vec{k}_o \left(n_1 d_1 + n_2 d_2 \right)$ and thus the phase difference introduced by the structure is $\phi = k_o \left(n_1 d_1 - |n_2| d_2 \right)$. Since n_1>0 and n_2<0, the total phase difference becomes zero if

$$\frac{n_1}{|n_2|} = \frac{d_2}{d_1} \tag{27}$$

In the case of materials with complex refractive index $N_i = n_i - i k_i$ (the extinction coefficient $k_i < 0$ for lossy structures and $k_i > 0$ for amplification), the condition for phase compensation is

$$\phi = k_o \left(n_1 d_1 - |n_2| d_2 \right) - i \left(k_1 d_1 + k_2 d_2 \right) = 0 . \tag{28}$$

If the first material is conventional material with gain and second material is metamaterial with losses, the phase compensation conditions are [57], [59]

$$\frac{n_1}{|n_2|} = \frac{d_2}{d_1} , \quad \frac{k_1}{|k_2|} = \frac{d_2}{d_1} . \tag{29}$$

An important conclusion is that the phase compensation in NRM-positive material composite is not dependent on the absolute value of slab thickness d_1, d_2, but only on their ratio.

Such a structure is denoted as a phase compensator. It is also called beam translator, because it does not introduce any phase changes in the beam, and only "translates" it in space.

If a phase compensator/beam translator is placed between two perfect reflectors, the structure behaves as a resonator. However, due to the fact that the phase compensator behavior depends on NRM-positive slab thickness ratio only, the structure will act as a subwavelength resonator [60], i.e. it will possess nonzero modes even for the case d_1, $d_2 \ll \lambda/2$. Obviously, this property is extremely useful for long-wavelength applications, where their dimensions may be extremely reduced in comparison to the conventional ones.

The extension of the concept of subwavelength resonators was done from 1D to different other geometries, including spherical and cylindrical one [3].

VI. Guided Wave Applications

The described effects of phase compensation and the existence of subwavelength resonators have been the basic underlying principle for numerous proposed applications in the microwave range.

Various kinds of NRM-containing waveguides, phase shifters, power splitters, mixers, couples, resonators, etc. were described in the open literature. In this text we shortly review a few of these.

A. Subwavelength Waveguides

As described by Alu and Engheta [68] and Grbic and Eleftheriades [69], the concept of sub-wavelength cavity resonators and transmission line-based metamaterials can be applied to parallel-plate waveguides and generally guided-wave structures to obtain functional structures with lateral dimensions smaller than the diffraction limit. Such waveguides incorporating NRM support a larger range of propagation constants, even very high ones. They offer a tighter confinement of waves compared to the conventional ones. The concept has also been described for the case of cylindrical guides [70].

Caloz and Itoh described and fabricated a microstrip-based TLM CRLH where the unit cell (as shown in Fig. 8) is implemented as a series interdigital capacitor and a shorted (via) stub inductor, Fig. 13.

Similar to metamaterial superlenses, in practical NRM-based subwavelength guides the main limitation to a decrease of their dimensions are losses.

B. Dual-Band Branch Coupler

In [71] it has been proposed to replace conventional transmission lines in branch couplers with CRLH ones, thus enabling arbitrary tailoring of the phase response of the coupler and ultimately the fabrication of a dual-frequency branch-line coupler.

In the conventional branch couplers the phase response is a straight line, meaning that the very choice of one design frequency at -90^0 automatically means the determination of the next frequency at -270^0 as three times the design frequency. However, in the dual band coupler the effect of phase compensation is straightforwardly used to change the slope of the phase response and thus to set the second operating frequency at an arbitrary value. By setting the dc offset of the CRLH transmission line in the coupler, one can tailor both the design frequency and the dual frequency.

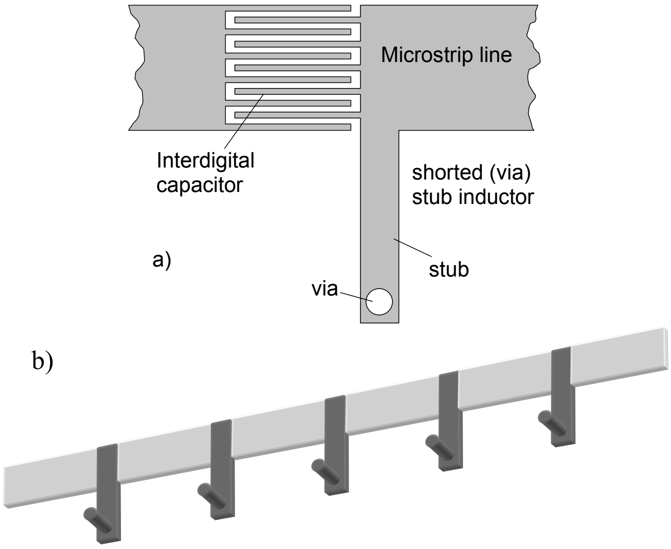

Fig. 13. A microstrip implementation of CRLH transmission line metamaterial. a) Unit cell of a microstrip subwavelength waveguide with interdigital capacitor and stub inductor; b) a segment of a transmission line utilizing the presented TLM unit cell

A CRLH TLM branch coupler enabling a tailorable dual-band operation is shown in Fig. 14.

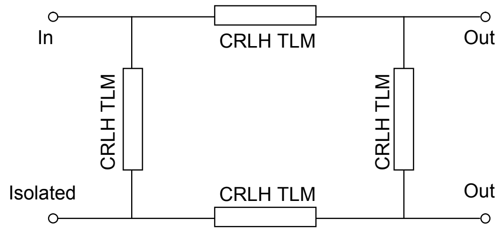

Fig. 14. Physical implementation of left-handed transmission line-based dual band branch coupler

The CRLH-based dual band branch couplers were proposed for the use in dual band wireless communications.

C. Directional Couplers

The NRM-based subwavelength guide structures offer a possibility for the "backward" coupling between such guides and the conventional ones. Such directional couplers were described in [72] and [73]. A high efficiency directional coupler is described in [75]. If one of the coupled waveguides carries power in one direction, the second waveguide, placed in its vicinity, will send some of this power in the opposite direction. The coupling efficiency of the redirected power increases exponentially as the distance between the two guides decreases. The two power flows are isolated each in its channel and flow separately.

An illustration of a metamaterial-based directional coupler is shown in Fig. 15.

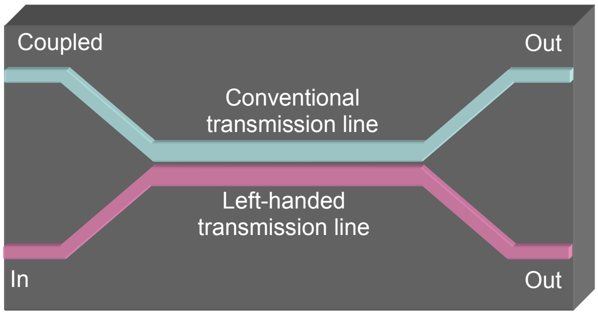

Fig. 15. Presentation of a metamaterial-based directional coupler in microstrip implementation

The metamaterial-based directional couplers, besides showing improved coupling, also show a wider range than in conventional implementations and a smaller coupling length.

D. Zero Order Resonator

A practical implementation of a subwavelength resonator as described in Section V was presented in [45], [74]. The fact is used that a phase constant β equal to zero can be achieved at frequencies different from zero. At $\beta=0$ no phase shift exists across the resonator, since it is determined by the product of propagation constant and the resonator thickness. Strongly subwavelength dimensions of the resonator can be obtained, while retaining a quality factor equal to that of a half-wavelength resonator. The authors of [74] designed a microstrip structure by placing a stub inductor between two interdigital capacitors (as the structure shown in Fig. 13a). They denoted their structure as zeroth order resonator (ZOR). The quality factor of the ZOR resonator is given as

$$Q = \frac{1}{G}\sqrt{\frac{C}{L}} \tag{30}$$

where G is the shunt conductance of a lossy CRLH TL. The above value does not depend on the number of unit cells used in the resonator.

E. Phase-Shifting Lines

The phenomenon of phase compensation in structures containing both conventional and "left-handed" parts has been utilized in transmission lines to design and fabricate transmission lines in which the transmission phase is tailored to a desired positive or negative value or is completely removed. Such a phase-compensating structure is fabricated by loading a conventional transmission line with lumped series capacitors and shunt inductors, as shown in Fig. 16.

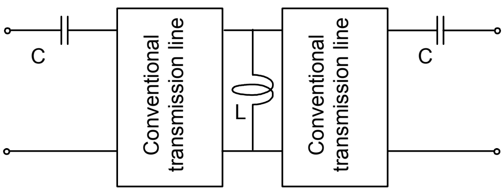

Fig. 16. Unit cell of a phase-shifting line containing metamaterial compensator

Since the phase compensation effect is not connected with the physical length of the structure, the phase-shifting lines can be very compact. The short electrical length implies a broadband response.

If the host transmission line is described by its inductance L and capacitance C, while the "left-handed" part is characterized by L_0 and C_0, then the condition for a balanced structure is [76]

$$Z_0 = \sqrt{\frac{L_0}{C_0}} \tag{31}$$

and if this condition is valid, the total phase shift introduced by a single unit cell is

$$\beta_{eff} \approx \omega\sqrt{LC} - \frac{1}{\omega\sqrt{L_0 C_0}} \tag{32}$$

F. Antenna Feeding Lines and Baluns

The phase shifting lines have been put to use in series-fed antenna arrays as their feeding networks, as proposed by Eleftheriades [76] (Fig. 17). In conventional arrays antenna elements can be fed in phase, although physically separated by a distance of λ/2, by using long meandered feed lines which introduce a phase of –2π rad. However, such arrays are strongly cross-polarized, and any changes of the operating frequency causes beam squinting. On the other hand, since metamaterial-based phase shifting lines can have any desired phase, they can be adjusted to zero-phase shift and thus be used as very compact and non-squinting alternative to the conventional feed lines. If operated in the backward-wave mode, they do not emit any radiation, and at the same time their propagation constant is larger than that of the free space.

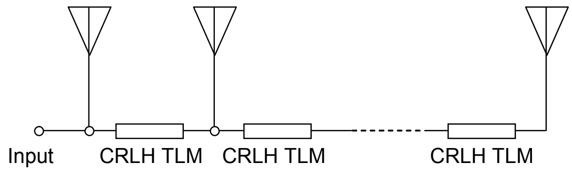

Fig. 17. Antenna array fed by CRLH TL metamaterial feeding lines

Another example of the situation where metamaterial phase-shifting lines are useful are baluns for feeding two-wire antennas, where currents in each branch are balanced through metamaterial lines and thus symmetrical radiation patterns with a given polarization are obtained. A broadband Wilkinson balun implemented by CRLH TL metamaterial phase-shifting lines was proposed in 2005 by Antoniades and Eleftheriades [77]. The conversion of a single-ended input into a differential output over a large bandwidth was demonstrated. The top branch of the Wilkinson divider was equipped with a metamaterial phase-shifting line introducing a $+90^0$ phase shift, and the bottom branch by -90^0 shift.

G. Dispersion Compensators

The idea for dispersion compensation in transmission lines using NRM was described in [78]. Elimination of dispersion can be done by adjusting the NRM part to match the frequency dependence of the effective permittivity of any particular type of transmission line. A partial dispersion compensation has been done experimentally for microstrip lines [3], and full dispersion compensation appears theoretically possible.

VII. ELECTRICALLY SMALL ANTENNAS USING NEGATIVE INDEX METAMATERIALS

Analyses of the applicability of NRM-containing resonant structures to improve the performance of antennas were published in [61]. The idea was to enclose a small emitting dipole into a shell of NRM-containing material which would match it to the surrounding free space (i.e. act as a matching network), thus obtaining an electrically small antenna with large radiated power (the “super-gain” effect) [61]. Here the compensating behavior of NRM was utilized, reducing to zero the potential barrier and thus the reactive power near the antenna. The effect was that the antenna appeared to have a larger aperture than its physical size.

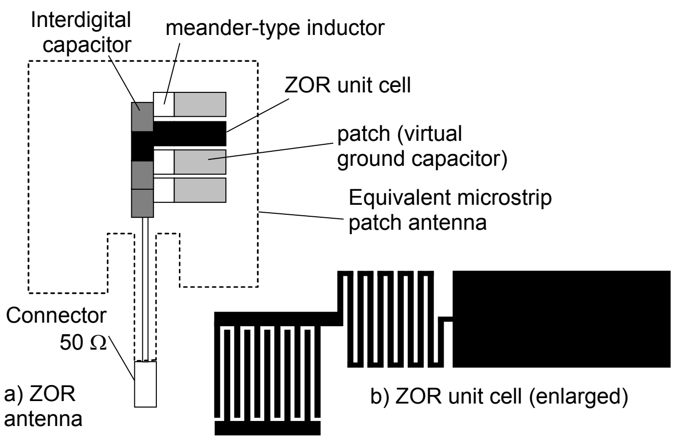

Fig. 18. a) Subwavelength-dimensioned ZOR-based antenna with four unit cells. Solid fill denotes a single unit cell. To compare dimensions, dashed lines represent a microstrip patch antenna with similar characteristics. b) single ZOR unit cell (enlarged)

A. Zero-Order Resonator Antenna

Probably the simplest NRM-containing antenna in microstrip implementation is obtained by utilizing a zero-order resonator. Its unit cell consists of an interdigital series capacitor and a shunt meander-type inductor connected to a rectangular patch which acts as a virtual ground plane. The ZOR-type antenna was presented in [45]. An implementation of such antenna is shown in Fig. 18.

The dimensions of the ZOR antenna do not depend on the operating wavelength, but only on the reactive loading of its building block, the ZOR resonator unit. Thus the antenna dimensions can be smaller that $\lambda/2$ while retaining full performance of a larger antenna. A dimensional comparison to a conventional patch antenna is shown in Fig. 18. The authors of [45] fabricated a ZOR antenna for an operating wavelength of 4.88 GHz with a size of 10 mm (each unit cell 2.5 mm wide).

B. Leaky-Wave Antennas

An important implementation of NRM metamaterial transmission lines are steerable leaky wave antennas (LWA) [45], [62].

One of the proposed applications utilizes fully balanced CRLH transmission line for frequency scanned leaky wave operation [63]. Such antenna can function in the fundamental mode because in this case it encompasses the fast wave region. This is in contrast to the conventional LWAs which must operate at higher order mode. By varying the operating frequency around the transition between the left-handed and the right-handed radiation mode (ω_0 in Fig. 9), the antenna scanning angle can take any value from backfire (-90^0) to endfire ($+90^0$). At exactly the transition frequency the antenna radiates broadside. Owing to the operation in the fundamental mode, feeding of CRLH-based LWA antennas is more efficient and simpler.

To obtain efficient channelizing, however, a LWA should operate at fixed frequency. To this purpose another concept was proposed, a transmission line with an array of CRLH unit cells, each one of them with its separate voltage control [64]. The antenna operation mode will depend on the control voltage distribution across the unit cell array. For a uniform control voltage across the cells maximum directivity is obtained, and if the voltage changes simultaneously along the whole TLM, effective scanning is obtained. On the other hand, the introduction of nonuniform distribution (and its maintenance while simultaneously varying control voltage on all unit cells) enables beamwidth control.

NRM structures in general appear very promising for antennas, thus many different applications were proposed in literature. We may also mention antennas with single negative materials as described in [65], and multi-horn antennas presented in [66]. Highly directive antennas using the superlens principle have also been described [67].

VIII. FILTERING AND BEAM SHAPING

A. Emittance and Absorptance Tailoring

Negative index materials have been proposed for various filtering applications, typically in the form of 1D photonic crystals consisting of alternating negative and positive index strata. The use of such NRM-containing structures for emittance and absorptance modification of Planck radiation sources has been described in [80].

A trait typical for different kinds of NRM-based spectral filters is that their spectral response is significantly flatter than that of positive-index structures, while at the same time the angular dependence is usually much less pronounced.

NRM structures with a gradient of refractive index have been described in the context of improved filtering applications [82], [83]. Similar approach is used to that in conventional filters where graded structures enable a wider range of high-reflection or antireflection properties, but in this case enhanced by omnidirectionality and spectral flattening typical for NRM metamaterials.

The use of NRM-containing 1D photonic crystals for passive filters with fractal or quasiperiodic periodicity was described in [84], [85].

B. Active NRM filters

Amplifying media in NRM-containing structures have been described as a way to overcome the absorptive and dissipative losses. The use of such structures as active filters was described in [79]. Three possible configurations are shown in Fig. 19.

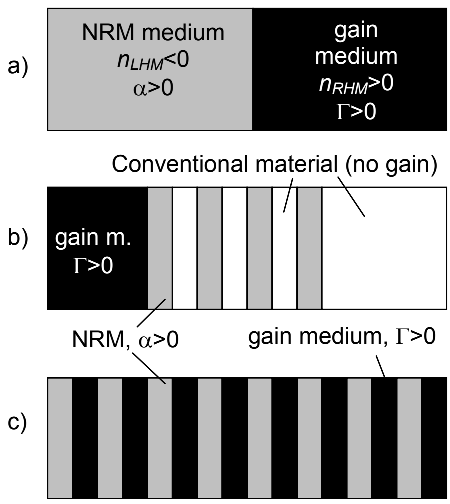

Fig. 19. Different implementations of active filters incorporating metamaterial stacks. a) single layer pair; b) sandwich structure; c) DFB active filter

The problem of tailoring the spectral response of the active medium was proposed to be dealt with by using compound semiconductors with continuously tunable bandgap.

C. Antireflection and High Reflection coatings

Antireflection (AR) and high reflection (HR) coatings based on NRM were proposed in [21]. One of their greatest advantages is their relative insensitivity to the incident angle and a much wider bandwidth than in the conventional AR and HR layers and structures. Their applicability both in microwave and in optical range has been described.

D. Beam Shaping and Directing Structures

The use of superlenses and generally complementary media has been proposed for near-field applications where a beam should be focused or simply shifted, but also for efficient channeling of beams with different shapes onto a predefined surface [15]. It can be either with curved surfaces (if a collimated/flat beam is used), or, in many situations, even plane parallel, which is an advantage in certain geometries. Further, NRM-based beam shapers can bend incident waves to a greater extent although only small absolute values of the refractive index are used compared to the conventional refractive and diffractive shapers. Finally, NRM shapers are by definition impedance-matched to the incident medium and offer subwavelength resolutions.

IX. Conclusion

In this paper we shortly review the basic properties and applications of the popular 'left-handed' metamaterials. Over a very short period the field has rapidly developed into a state where the knowledge of its fundamental processes reached its maturity. This enabled a host of different applications to be proposed, while NRM physics slowly transits to technology. Currently the focus is shifting from microwave applications to terahertz and optical frequencies. The endeavor to create optical NRM metamaterials and applications will require a number of innovative techniques, including intensive use of micro- and nanofabrication. The speed of the development of the NRM and the number of teams participating its research anticipates fruitful results and a bright future for the field.

Acknowledgement

This work has been partially supported by the Serbian Ministry of Science and Environmental Protection within the framework of the project TR-6151.B.

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