Investigation of a Left-handed Microstrip Line

Antoniya R. Georgieva

Abstract – **This paper presents the results of an experimental investigation of transmission properties of a left-handed microstrip line. The microstrip line is loaded by complementary split-ring resonators (CSRR), acting as resonant electric dipole particles which give rise to negative effective permittivity, and periodical capacitive gaps generating negative effective permeability. The transmission measurements of the structure show a pass-band arising near the resonance frequency of the CSRR where negative effective permittivity and permeability coexist. The presented results are of interest in the design of compact devices based on left-handed materials.**

Keywords – **left-handed materials, metamaterials, complementary split-ring resonators, negative effective dielectric permittivity and magnetic permeability.**

I. INTRODUCTION

In recent years, a novel kind of artificial materials named left-handed materials has become a subject of growing interest in microwave and millimetre wave engineering because of their extraordinary electrodynamic properties, not found in nature. Usually constructed by small, compared to the wavelength, elements arranged in periodical manner, the left-handed materials exhibit negative effective dielectric permittivity and magnetic permeability in a limited frequency range. The negative constitutive parameters give rise to a number of extraordinary macroscopic properties of the material, theoretically studied in 1968 by V.Veselago [1]. Due to the coexistence of negative permittivity and permeability, the wave vector forms a left-handed triplet with the vectors of the electric and magnetic field intensity, in contrast to ordinary materials where these vectors obey the right-hand rule. This results in wave velocity oriented opposite the energy flow or backward-wave propagation. Among the extraordinary properties of the left-handed materials are the negative index of refraction (inversion of the Snell's law), inverted Doppler shift and opposite Cherenkov radiation.

Although the idea that a material with simultaneously negative permittivity and permeability can support wave propagation was introduced some 40 years ago [1], it did not attract much interest until 1999 when negative effective parameters were achieved in artificial periodical structures of split-ring resonators and metal cylinders or wires [2]. The split-ring resonator (SRR) element, excited by external coaxial time-varying magnetic field, acts like magnetic dipole with a dipole moment opposing the external field. The behaviour of an array of SRRs with sizes and period much smaller than the wavelength is strongly diamagnetic shortly over the first resonance of the particles resulting in negative effective magnetic permeability [2]. To achieve negative effective permittivity usually an array of metallic cylinders or wires is used. In the initial implementations the left-handed material comprised of bulky two- or three-dimensional arrangement of alternating layers of SRRs and wire strips etched on printed circuit boards. Measurements were usually performed in free space using horn antennas or in parallel-plate waveguides.

Further development aimed the implementation of the SRR structures for designing planar devices [3]. In coplanar waveguide implementations the SRRs, etched on the backside of the substrate, are combined with periodical metallic strips connecting the central conductor and the ground planes. The resonant elements give rise to negative effective permeability near the resonance frequency whereas negative effective permittivity is introduced by the periodical short circuits.

In the microstrip technology, similar effect is obtained by etching SRRs on the upper substrate side near the central conductor where they are excited by a magnetic field component, coaxial with the rings [4]. To obtain negative permittivity in microstrip technology, however, periodic shunt inductances are needed, usually implemented as metallic vias to the ground that make the structure more complex.

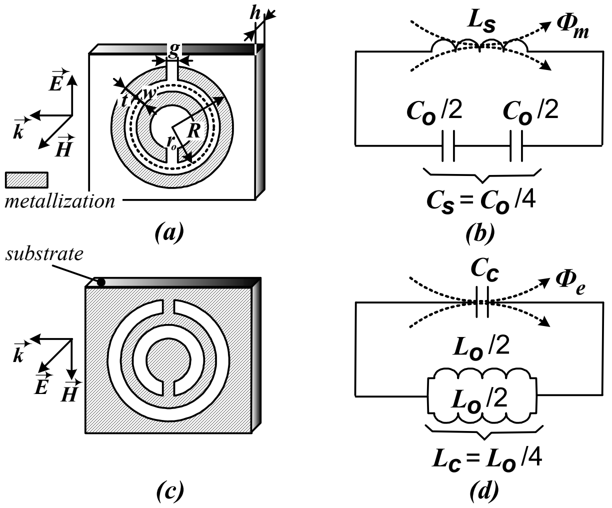

Fig.1. Split-ring resonator (a) and complementary split-ring resonator (c) topologies and equivalent circuits (b,d) [4]

The resonant diamagnetic behaviour of the SRRs is not affected when substituted by their dual complementary split-ring resonators – the metallic strips of the SRRs become apertures surrounded by metallization in the complementary SRR (CSRR). According to the duality principle, while the

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SRR acts as a magnetic dipole when excited by an external magnetic field, its dual counterpart – the CSRR – will act as an electric dipole under external electric field excitation [4]. In microstrip technology, to achieve proper excitation, the negative images of the SRRs are etched in the ground plane underneath the upper strip.

The purpose of the current work is to investigate the transmission behaviour of a microstrip line loaded by CSRRs and periodical capacitive elements etched on the upper strip. Firstly, a simple model for the CSRR is presented, followed by a transmission matrix analysis of a microstrip line periodically loaded by capacitors. Secondly, the experimental results for the transmission level measurements are given for a line loaded with CSRRs, capacitors and the combination of both elements. Finally, potential applications are discussed.

II. THEORETICAL CONSIDERATIONS

A. Electromagnetic behaviour of SRR and CSRR

In Fig. 1 the topology and equivalent circuits for the SRR and its dual counterpart – CSRR, are shown [4]. The SRR behaves as an *LC* resonator, excited by an external magnetic flux. SRRs also exhibit magnetoelectric coupling – properly polarized time-varying electric field can also induce a magnetic dipole moment [4]. In the equivalent circuit of Fig.1(a), the capacitance C_0 designates the total capacitance between the rings: $C_0 = 2\pi r_0 C_{pul}$, where C_{pul} is the per-unit-length capacitance between the rings. The resonance frequency is given by: $f_0 = (L_s C_o / 4)^{-0.5}/2\pi,$ where $C_0 /4$ is the series capacitance of the upper and lower parts of the SRR. The inductance L_s is approximately equal to that of a single ring with radius r_0 and width t [4].

To produce a CSRR the metal parts of the structure are replaced by apertures and vice versa. In this way the structure of Fig.1(c) is achieved. Provided that the metal thickness is negligible and the metal conductivity tends to infinity, the aperture can be considered a perfect magnetic conductor. In that case the original structure and the complementary one are effectively dual and, when the complementary SRR is excited by time-varying electric field parallel to the rings' axis, it provides a band of rejection around the resonance frequency of the particle. It is to be noticed that the CSRR in printed circuit board technology is not completely dual to the SRR of same dimensions because of the finite substrate thickness. However, the approximate duality can be used to estimate the resonance frequency of the CSRR as equal to that of the corresponding SRR. The approximation is acceptable for relatively thick substrates. Nevertheless, it can also be used for thin substrates at least to estimate the frequency band where the resonance occurs. Since the roles of *E* and *H* are changed for the CSRR because of duality, at the resonance frequency a strong electric dipole moment is induced by the external electric field in the direction opposite to this field. The macroscopic effect can be considered a negative effective permittivity.

In the equivalent circuit model of Fig.1(d) the inductance L_s of the SRR is replaced by the capacitance C_c of a disk with radius $r_0 - t/2$ surrounded by a ground plane at a distance t of its edge. The connection in series of the two $C_0/2$ in the SRR model is substituted by a parallel combination of two inductances $L_0/2$ connecting the disk to the ground. The inductance L_0 is calculated as $L_0 = 2\pi r_0 L_{pul}$ where L_{pul} is the per-unit-length inductance of the coplanar waveguides connecting the inner disk to the ground [4].

B. Periodically loaded microstrip transmission line

In microstrip implementation, proper excitation of the CSRRs can be achieved etching the resonators in the ground plane, under the upper strip as shown in Fig.2. However, to obtain a combination of simultaneously negative effective permittivity and permeability, a medium exhibiting negative effective permeability is needed as well. It can be achieved by periodically loading the microstrip line by capacitive elements formed by slots in the upper strip. An extended mathematical and physical description of how negative effective permeability emerges from periodical capacitive gaps in a microstrip line can be found in [5].

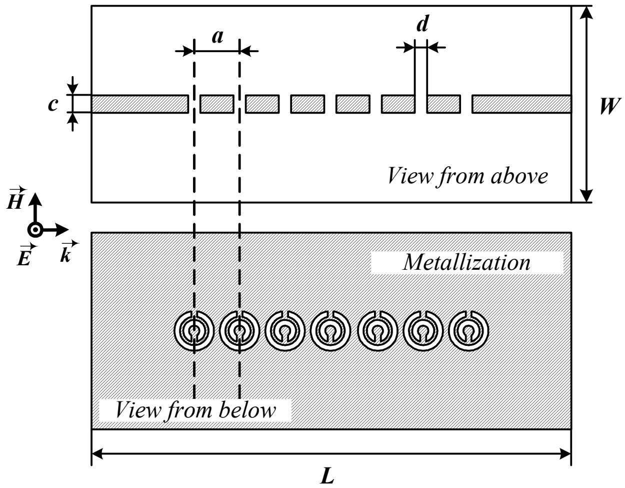

Fig.2. Microstrip line loaded by CSRRs and slots – front and back view

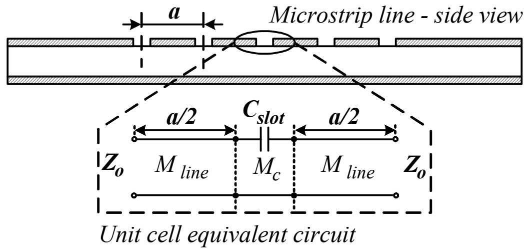

Fig.3. Unit cell's equivalent circuit and transmission matrices

The transmission characteristic of a microstrip line periodically loaded by capacitive slots can be analytically determined by the transmission matrix method. In Fig.3 the equivalent circuit for a unit cell of the microstrip line is given. The cell's length is equal to the period of the slots. The transmission matrix for the unit cell with a slot is represented by:

$$M_{u.c.} = M_{line} . M_C . M_{line} , \tag{1}$$

where

$$M_{line} = \begin{bmatrix} \cos(\beta a/2) & jZ_0 \sin(\beta a/2) \\ j\sin(\beta a/2)/Z_0 & \cos(\beta a/2) \end{bmatrix} \tag{2}$$

is the transmission matrix of a low-loss line with characteristic impedance Z_0, propagation constant β and length equal to half of the slots' period (a/2) and

$$M_C = \begin{bmatrix} 1 & Z_C \\ 0 & 1 \end{bmatrix}, \; Z_C = 1/(j\omega C_{slot}). \tag{3}$$

M_C is the transmission matrix of a capacitor with capacitance C_{slot} connected in series to the line. The slot capacitance can be estimated using the formulas in [6]. The substitution of (2) and (3) into (1) results in [7]:

$$\cos(\beta d) = \cos(\beta_0 d) + \frac{1}{2\omega C_{slot} Z_0} \sin(\beta_0 d). \tag{4}$$

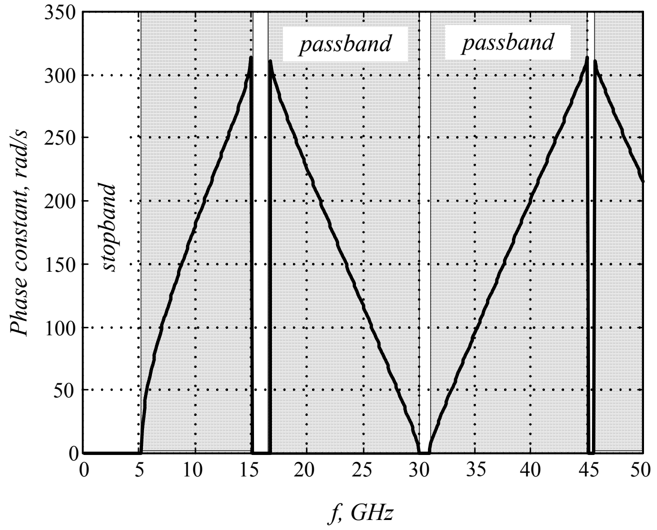

Fig.4. Phase constant versus frequency for a microstrip line periodically loaded by slots with capacitance 0.5 pF.

In Fig.4 the phase constant β is plotted versus frequency according to (4) for C_{slot} = 0.5 pF - the approximate value of the capacitance of slots of width 0.8 mm etched on the 2.9 mm wide upper conductor of a 50Ω microstrip line according to [7]. The above stated dimensions correspond to the prototype used for the experiments. The transmission characteristic comprises of alternating stop-bands and pass-bands. No wave propagation is supported in the stop-bands, which is described by negative effective permeability [5]. The first pass-band begins at 5.23 GHz, which is above the frequency range of the performed measurements.

III. PROTOTYPE AND MEASUREMENT SETUP

A schematic view of the fabricated prototype is given in Fig.2. The microstrip line used in the experiments is designed to have a characteristic impedance of 50Ω for a purpose of matching to the coaxial cables interfacing it with a signal generator and a spectral analyser. A standard 1.5 mm thick printed circuit board substrate is used to produce the microstrip line. The line's dimensions are:

- Upper strip width: c = 2.9 mm (Fig.2);
- Ground plane width: W = 70 mm;
- Line length: L = 150 mm;
- Substrate's relative permittivity: ε_r = 4.4;
- Substrate's loss tangent: tg δ = 0.025.

The calculated effective dielectric constant of the line is 3.33 and the guided wavelength is 4.99 cm for a frequency of 3.3 GHz, which is the central frequency for the frequency range of measurements performed.

The left-handed line is formed by a periodical arrangement of CSRR etched on the ground plane and slots etched above the CSRR on the upper strip (Fig. 2). Two types of structures with different dimensions have been investigated. The first type has the following dimensions:

- CSRR dimensions: $2R$ = 8 mm, t = w = 0.8 mm, g = 0.8 mm (Fig.1a,c);
- Capacitive slots width: d = 0.8 mm;
- CSRR and slots period: a = 8.6 mm.

The dimensions of the second structure are as follows:

- CSRR dimensions: $2R$ = 9.4 mm, t = w = 0.9 mm, g = 0.9 mm;
- Capacitive slots width: d = 0.8 mm;
- CSRR and slots period: a = 10 mm.

The dimensions were chosen to produce a resonance frequency in the frequency range of 2.5 ÷ 4 GHz, which was the range supported by the available generator.

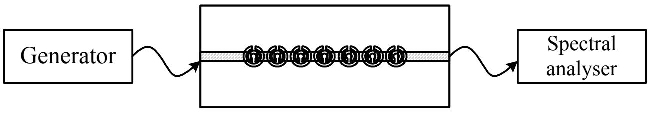

Fig.5. Measurement setup

In Fig.5 a sketch of the measurement setup is given. The operational frequency range of the signal generator is 2.506 ÷ 4.06 GHz. The level of the transmitted signal is measured in *dB* by a spectral analyser. The noise level on the screen of the analyser is at –68 dB. The transmitted signal level for the microstrip line without any additional elements is taken for a reference. 50Ω coaxial lines connect the generator output and the spectral analyser input to the line.

IV. RESULTS

The purpose of the measurements was, firstly, to investigate the transmission characteristic of a line loaded with CSRRs only, and secondly, to verify the existence of a transmission pass-band where effectively negative electrodynamic parameters co-exist.

In Fig.6 the measured transmission level for a structure with CSRR dimensions $2R$ = 8 mm, t = w = 0.8 mm, g = 0.8 mm and period 8.6 mm is shown. The resonance frequency, where a strong rejection is observed, is 3.44 GHz. The signal level in the rejection band is 55 dB below the line's normal transmission. It is in this band where a negative effective ε is supposed to arise.

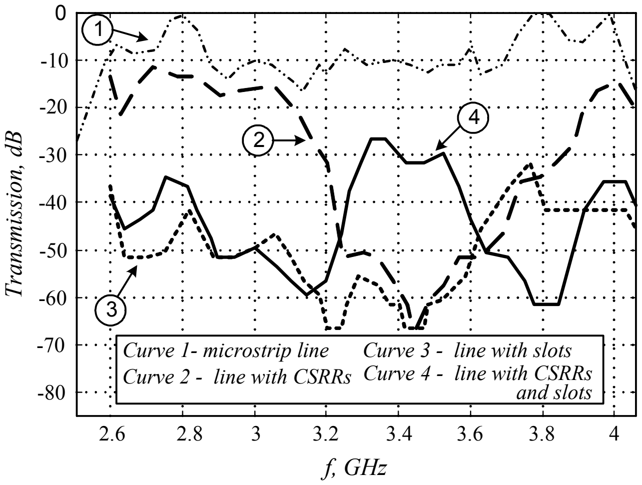

Fig.6. Measured transmission through CSRR-and-slot-loaded microstrip line, CSRR dimensions: $2R = 8$ mm, $t = w = 0.8$ mm

In the same figure the transmission through a periodically loaded by slots line is presented as well. As predicted by the analytical analysis given in section 2, this structure exhibits poor transmission of 40 dB on average below the reference level, which could be described by negative effective permeability [5].

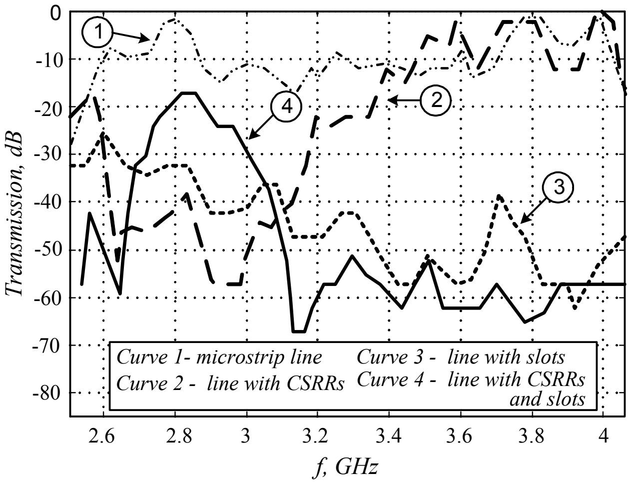

Fig.7. Measured transmission through CSRR-and-slot-loaded microstrip line, CSRR dimensions: $2R = 9.4$ mm, $t = w = 0.9$ mm

Curve 4 in Fig.6 presents the results for the measured transmission when CSRR are combined with slots to produce a metamaterial. The line's effective permittivity and permeability are simultaneously negative near the CSRR resonance frequency. As predicted by theory, a pass-band of 35 dB above the noise floor arises at the band of rejection for the CSRR-loaded line.

The results shown in Fig.7 correspond to the same measurements performed for a line with loading elements, arranged periodically by a period of 10 mm. Because of the larger CSRR dimensions, $2R = 9.4$ mm, $t = w = 0.9$ mm, the resonance frequency is shifted downwards to 2.83 GHz as compared to the resonance frequency of the first structure. The minimum attenuation in the pass-band was measured to be 12 dB. The transmission level can be enhanced by using low-loss dielectric substrate for microwave applications as well as taking measures to improve the matching between the line and the coaxial cables that interface it to the generator and the spectral analyser.

The 3 dB bandwidth for structure type 1 has a fractional width of 6% with a central frequency of 3.39 GHz. The fractional pass-band width for structure type 2 is 3.5% at a central frequency of 2.83 GHz.

V. CONCLUSION

The current paper presented the results for measured transmission through a left-handed microstrip line. Complementary split-ring resonators, introducing negative effective permittivity near the resonance frequency, and capacitive slots, producing negative effective permeability in certain frequency range, formed the left-handed line. The transmission through a line loaded by CSRRs only, slots only, and by a combination of CSRRs and slots, has been measured. The CSRR-loaded line exhibited a high rejection band near the particles' resonance frequency, corresponding to negative effective permittivity. This stop-band has been converted to a pass-band when adding capacitive slots in the upper conductor, introducing negative permeability. The observed pass-band verifies the theoretically predicted wave propagation through a medium having simultaneously negative constitutive parameters.

The prototypes used for measurements have been fabricated by etching the elements on a standard printed circuit board substrate. To reduce losses, a low-loss dielectric material for microwave applications should be used.

The performed experimental investigations demonstrated the eligibility of the concept of left-handed materials to filtering applications in microstrip technology.

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