Simple DOA and ABF Methods for Analysis of Circular Smart Antennas with Omnidirectional Elements

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***Abstract* – – In this paper, the direction-of-arrival (DOA) and adaptive beamforming (ABF) estimation principles of uniform circular array (UCA) based on narrowband radio-frequency signals are introduced. An UCA is composed of a number of uniformly distributed identical omnidirectional antenna elements. Plane wave excitation of UCA is considered using UCA-ESPRIT. LMS algorithm is applied to antenna beamforming in UCA. Limited numerical examples and simulation results are presented to illustrate the proposed methods.**

***Keywords* – smart antennas, direction of arrival, adaptive beamforming, wireless communications.**

I. INTRODUCTION

Smart antennas have become popular among researchers during recent years. Spatial processing is the central idea of these adaptive antennas. Wireless operators are currently looking for new technologies that would be implemented into the existing wireless communications infrastructures to provide broader bandwidth per user channel, better quality, and new improved services [1]. These research efforts will enable wireless carriers to maximize the spectral efficiency of their networks so as to meet the explosive growth of the wireless communication industry, and so to take advantage of the huge market opportunity.

Deployed at the base station of the existing infrastructure, smart antennas can provide a substantial capacity improvement (very important in urban and densely populated areas) in the frequency-resource-limited radio-communication system by an efficient frequency-reuse scheme.

Until now, the investigation of smart antennas suitable for wireless communication systems has involved primary uniform linear arrays (ULA) and uniform rectangular arrays (URA). It is obvious that one can take advantage from the symmetry of *uniform circular array* (UCA).

II. SMART ANTENNA WITH UNIFORM CIRCULAR ARRAY STRUCTURE

The UCA with radius *a* consisting *N* equally distributed identical omnidirectional antenna elements, as illustrated in Fig. 1 is located on *x-y* plane.

Let us assume that an incoming narrowband signal (plane wave) arrives at the array from elevation angle θ and azimuth angle ϕ. A spherical coordinate system is utilize to denote the arrival direction for incoming plane wave with wavelength λ. The origin of coordinate system is located at the center of the array.

As demonstrated in Fig. 1, the array factor *(AF)* of UCA is given by [2]

$$AF(\theta,\phi) = \sum_{n=1}^{N} w_n e^{-jka\sin\theta\cos(\phi-\phi_n)} \tag{1}$$

where w_n, $\theta = \pi/2$ and ϕ_n are the estimated weights and angular positions of the *n*th element, respectively, *a* is the radius of the UCA, and *k* is the wave number (k=2π/ λ).

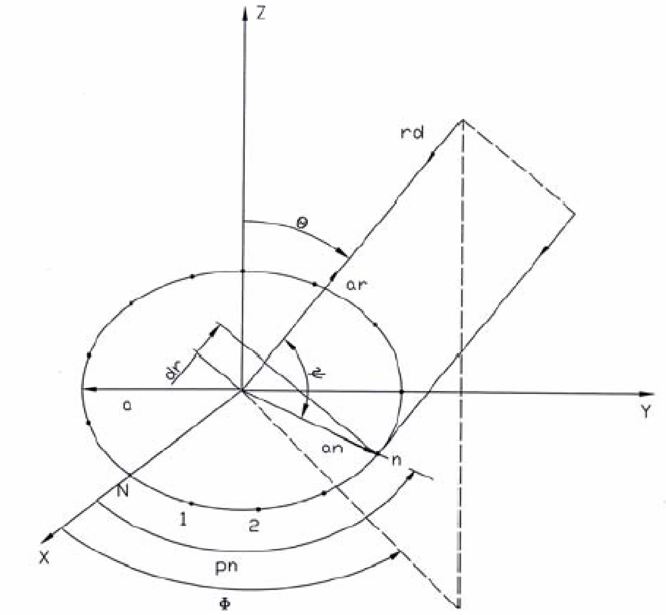

Fig. 1. Geometry of *N*-element UCA, along with an incoming plane wave.

III. DIRECTION OF ARRIVAL ESTIMATION

After that the UCA receives all incoming signals from directions of arrival, the DOA algorithm determines the directions of these signals based on the time delays. The DOA estimation involves a correlation analysis followed by signal/noise subspace formation and eigenstructure analysis.

Let us assume that a narrowband plane wave impinges at an angle (θ,ϕ), on the UCA. It produces time delays relative to the other array elements. These time delays depend on array geometry, number of elements, and interelement spacing. For the UCA of Fig. 1, the time delay of the narrowband signal at the *n*th element with respect to the origin, is written as [3]

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$$\tau_n = (a/c)\sin\theta\cos(\phi - \phi_n), \quad n = 1,2,...,N \qquad (2)$$

where *c* is speed of light in free space.

For the significant improvement in smart antenna resolution a several subspace-based methods have been considered in the literature. Two algorithms that fall into category for the azimuth and elevation estimation are MUSIC (Multiple Signal Classification) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique). In the paper the latter one is presented and used.

A. Classical ESPRIT – advantages in comparison with MUSIC

Classical ESPRIT is a robust method of DOA estimation. It uses two identical arrays in sense that elements need to form matched pairs with an identical displacement vector (the second element of each pair ought to be displaced by the same distance and the same direction relative to the first element). Furthermore, this does not mean that one has to have two separate antenna arrays but this method exploits this subarray structure for DOA estimation [4].

ESPRIT has become the method of choice because it has ability to offer a number of *advantages* over MUSIC, such as: a) computationally less intensive and more efficient; b) does not require calibration of the antenna array; c) does not involve search through all possible steering vectors to estimate the DOA.

B. The UCA-ESPRIT algorithm for DOA estimation

The UCA-ESPRIT algorithm is unique different from the classical ESPRIT, first of them provides closed-form automatically paired two dimensional estimation as long as the elevation and azimuth of each narrowband signal arrives at the UCA. This closed-form algorithm provides removal of computationally intensive spectral search procedures and iterative solutions. Applying this method for a UCA structure, the eigenvalues of each correlation array matrix have the form [3]

$$\lambda_i = \sin\theta_i e^{j\phi_i} \qquad (3)$$

where (θ_i, ϕ_i) are respectively elevation and azimuth angles of incoming plane wave of *i*th signal source ($i = 1,2,...,M$), where M is the number of the narrowband sources. Now the three basic steps of real valued estimation are briefly described in terms of UCA [5]:

1. The signal eigenvectors estimation.
2. The equation system solution derived from computed in step 1eigenvectors.
3. The eigenvalues estimation of the solution to the system worked out in step 2.

This algorithm gives several *advantages* in comparison with classical ESPRIT, such as: a) reduced computational complexity; b) lower SNR (signal-to-noise ratio) resolution thresholds, and c) very accurate finds simultaneously both the elevation and azimuth angles of arrival for impinging signals at the antenna array [6].

IV. METHODS FOR ADAPTIVE BEAMFORMIG ESTIMATION

Two classes of adaptive beamforming (ABF) algorithms are represented in literature : 1) DOA-based adaptive beamformig algorithms that utilizes information for angles of arrival of incoming signals to ideally steer the maximum of the antenna radiation pattern toward the desired signal and place nulls toward the unwanted signals or interferences, as depicted in Fig. 2; 2) reference-based ABF algorithms does not need DOA information but instead uses the reference signal to adjust weights of correlation array matrix to match the created time delays. In this section, we consider one of the most popular *reference-based* ABF algorithms applicable to UCA:
LMS (least mean squares) algorithm. It is a method that uses previous samples when estimating the gradient at the nth iteration.

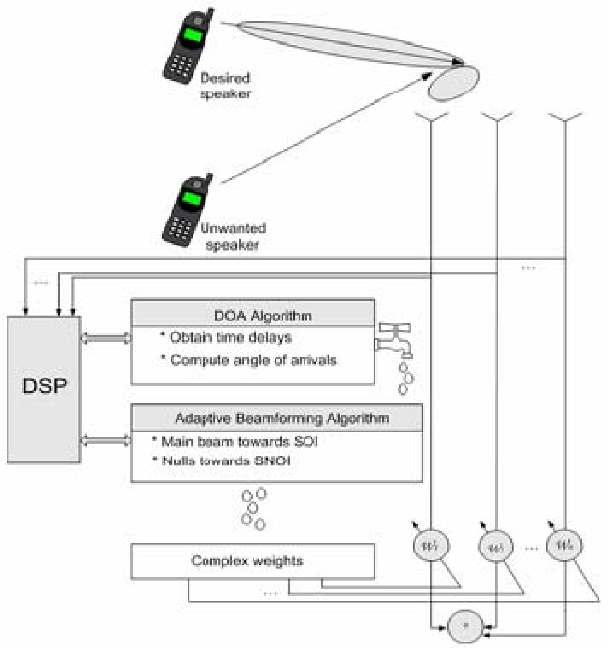

Fig. 2. A block scheme of a smart antenna.

The LMS algorithm is one of the simplest methods applicable to estimate optimal weights of an antenna array. It is applicable mainly when weights are updated utilizing reference signal. This algorithm uses an estimator of the gradient instead of the real value of the gradient because the real value estimation requires knowledge of incoming signals (DOA information).

The expression of optimal weights is given by [7]

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu g(\mathbf{w}(n)) \qquad (4)$$

where $\mathbf{w}(n+1)$ denotes a *new computed weights vector* at the (n+1)th iteration, μ is the *gradient step size*, and the array output is given by

$$y(\mathbf{w}(n)) = \mathbf{w}^H(n)\mathbf{x}(n+1) \qquad (5)$$

where $\mathbf{x}(n+1)$ is *array signal vector* computed at the (n+1)th iteration, and $y(\mathbf{w}(n))$ is *output signal*.

In its standard form it uses an estimate of the gradient by replacing *array correlation matrix* **R** and correlation between array signals and *reference signal* **r** by their noisy estimates at the (n+1)th iteration [7]

$$g(\mathbf{w}(n)) = 2\mathbf{x}(n+1)\mathbf{x}^H(n+1)\mathbf{w}(n) - 2\mathbf{x}(n+1)\mathbf{r}^*(n+1) \quad (6)$$

where **g** is the gradient vector.

The error between array output and the reference signal is given by [7]

$$\varepsilon(\mathbf{w}(n)) = r(n+1) - \mathbf{w}^H(n)\mathbf{x}(n+1) \quad (7)$$

and

$$g(\mathbf{w}(n)) = -2\mathbf{x}(n+1)\,\varepsilon^*(\mathbf{w}(n)) \quad (8)$$

The estimated gradient is a product of the error between the reference signal and the output of the array and the signals after the *n*th iteration.

Apparently, this algorithm provides several advantages: the low complexity, the gradient estimate is unbiased, but the main disadvantage of the LMS is that it tends to *convergent slowly.*

V. Numerical Examples and Simulation Results

Simulation results, utilizing the UCA-ESPRIT algorithm gave precise results when estimate simultaneously both the elevation and azimuth angles of arrival for incoming narrowband signals toward the UCA. In Table I are presented results from simulations. The UCA with *N=8* elements, and radius *a*=0.8λ is examined about two scenarios: a) when the number of incoming signals *M=2* in the presence of the interfering signal (SNOI), and Additive White Gaussian Noise (AWGN) with the mean, and variance 0.1; b) in these terms when *M=4.* The results demonstrate its great performance, accurate estimation ability, and robustness.

To illustrate the ABF algorithm applicability for UCA, we considered the two cases where LMS algorithm is used:

a) the UCA with *N=8* elements and radius *a*=0.8λ when angles of arrival of the SOI are $\theta = 45°, \phi = 90°$,

b) the UCA *N=12* elements, and radius *a*=0.8λ when angles of arrival of the SOI are $\theta = 30°, \phi = 90°$. Fig. 3 and Fig. 4 illustrate the resulting radiation pattern with respect to $\phi_0 = 0°$.

TABLE I
THE DOA ESTIMATIONS OBTAINED UTILIZING UCA-ESPRIT

	Case 1	Case 2
Number of elements	N=8	N=8
Radius of the UCA	a=0.8λ	a=0.8λ
Number of incoming signals	M=2	M=2
Number of data samples	2000	2000
Actual		
SOI 1	$\theta_1=30^0$, $\varphi_1=60^0$	$\theta_1=10^0$, $\varphi_1=150^0$
SOI 2	$\theta_2=40^0$, $\varphi_2=150^0$	$\theta_2=15^0$, $\varphi_2=180^0$
SOI 3		$\theta_3=20^0$, $\varphi_3=210^0$
SOI 4		$\theta_4=70^0$, $\varphi_4=350^0$
SNOI	$\theta=45^0$, $\varphi=120^0$	$\theta=25^0$, $\varphi=200^0$
DOA Estimations		
SOI 1	$\theta_1=29.984^0$, $\varphi_1=59.975^0$	$\theta_1=9.995^0$, $\varphi_1=150.006^0$
SOI 2	$\theta_2=45.032^0$, $\varphi_2=119.995^0$	$\theta_2=15.002^0$, $\varphi_2=179.994^0$
SOI 3		$\theta_3=19.993^0$, $\varphi_3=210.004^0$
SOI 4		$\theta_4=70.035^0$, $\varphi_4=349.995^0$
SNOI	$\theta=44.988^0$, $\varphi=120.004^0$	$\theta=25.024^0$, $\varphi=349.987^0$

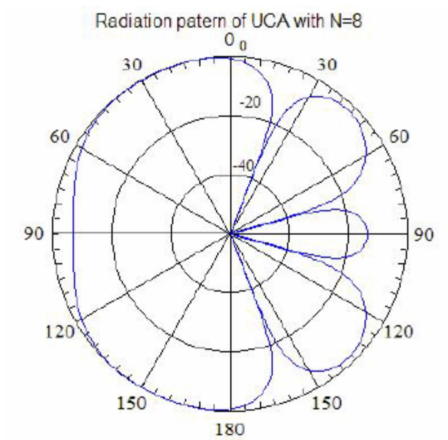

Fig. 3. The radiation pattern of the UCA with N=8 elements.

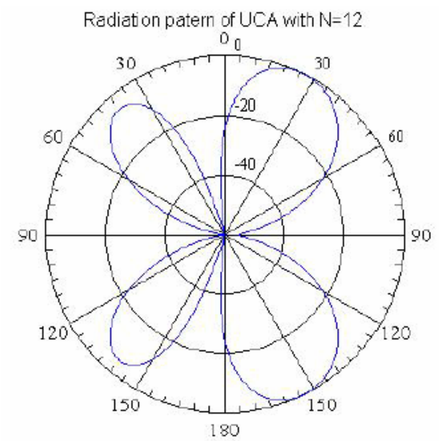

Fig. 4. The radiation pattern of the UCA with N=12 elements.

VI. Conclusion

This paper investigated uniform circular arrays (UCA) for smart antennas. Two main issues: estimation of direction of arrival (DOA) and adaptive beamforming (ABF) were examined. The main approach to DOA here was the algorithm ESPRIT. The technique for ABF used here was the LMS algorithm. The *symmetry* of the UCA antennas was exploited in order to obtain *more efficient* method for a calculation of accurate eigenvalues.

The UCA-ESPRIT is an algorithm that provides closedform automatically-paired source azimuth and elevation estimates. These results are proved to be accurate enough. Concerning beamforming the UCA has shown to be accurate and stable enough regarding both: SOI (maximum) and SNOI (deep nulls).

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