Synthesis and Analysis of Reduced-Size Branch-Line Hybrids

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Abstract: This paper proposes a method for synthesis and analysis of reduced size branch-line hybrids with arbitrary power division ratio. Based on dispersion equation of the capacitively loaded transmission line, closed form synthesis formulae are derived. Using the fourfold symmetry of the reduced size branch-line hybrid, analysis formulae are derived. The output ports' phase difference is analyzed for different synthesis configurations. The fractional bandwidth of the reduced size hybrid is about 10% narrower than the conventional branch-line hybrid. The results presented in the paper are applicable to uniplanar structure designs.

Keywords: Reduced size, branch line hybrid, capacitively loaded transmission line.

I. Introduction

Quadrature branch-line hybrids play important role as power divider, or power combiner, image rejection mixers, and balanced amplifiers. The classic form of the quadrature branch-line hybrid is shown on Fig.1.

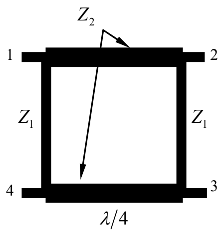

Fig.1 Conventional branch-line coupler

For a given input power at port 1, there will be 90° phase difference between the waves at ports 2 and 3 for the centre frequency. This phase difference varies over $\pm 5^{\circ}$ for a 10% fractional bandwidth. The usable bandwidth of the branch-line hybrid is constrained by the change in the isolation. Such branchline couplers can be designed for 3-10dB coupling.

Main arms of the coupler are of quarterwave length. This size is unacceptable large especially for the lower microwave bands and MMIC applications. The branch-line hybrid occupies large area on the printing circuit board or on the chip.

The use of lumped-element hybrid, which uses spiral inductors and lumped capacitors, is one possible solution to this problem [1-3]. However, the design of spiral inductors is requires precise empirical models based on measurement of test elements including parasitics.

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This design approach becomes difficult for frequencies above 20GHz [4,5]. Another method for the hybrid coupler size reduction is proposed in [2-5]. The branch line is realized as lumped element Π or T equivalent networks. Each lumped element value is determined by equating the ABCD matrices of both structures – transmission line and lumped element. The coupler involves shunt inductors, which are inconvenient for fabrication. However, this method eliminates the uncertainty caused by the lumped inductors.

The method presented in [1-5], the branchline hybrid utilizes high impedance lines loaded by shunt lumped capacitors. The authors present a special case for the theory of this coupler.

In this paper, we present a general theory of reduced size branch-line hybrid with arbitrary power division ratio, composed of capacitively loaded transmission lined. With the proposed method, a branch-line coupler can be designed for various impedance and capacitor values for a given power division ration. The reduction of the size is connected with increase of the characteristic impedance of the transmission line. The phase difference between the output ports for various characteristic impedances and capacitors is studied. Because of the slow-wave effect of the capacitive loaded line, the phase difference is not symmetrical along the 90^0 value.

II. Reduced Size hybrid

The main element in the conventional hybrid coupler is the quarter-wavelength transmission line. In order to obtain reduced-size coupler, shown on Fig.2, in [1-4,7] is proposed to use capacitive loaded transmission line instead of quarter-wavelength line. To assure equivalent electrical parameters, it is examined both circuits shown on Fig.3a and Fig.3b. The circuit on Fig.3b may be considered as a unit element of periodic slow wave structure [4-7].

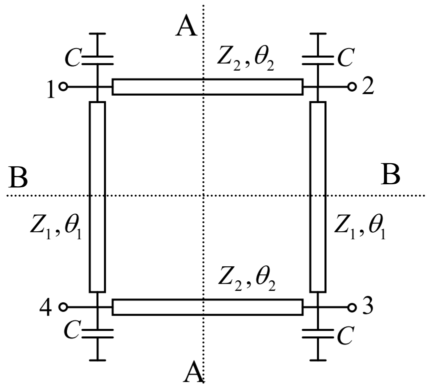


Fig.2 Reduced size branch-line coupler

The slow-wave line consists of a transmission line loaded on both sides by lumped capacitors C. The main parameters of the transmission line are the characteristic impedance Z_c, length of the line l, and the propagation constant k.

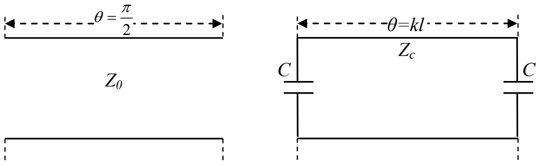


Fig.3 (a) Quarter wavelength line, (b) capacitive loaded line

The electric length of the unloaded line is $\theta = kl$. The electrical characteristics of the line are described by the ABCD matrix. The overall matrix of the resonator is a product of the three ABCD matrices – the lumped capacitors C and the unloaded transmission line.

$$[ABCD] = [ABCD]_1 [ABCD]_2 [ABCD]_3 \qquad (1),$$

where

$$[ABCD]_1 = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}, \ [ABCD]_3 = \begin{bmatrix} \cos\theta & jZ_c \sin\theta \\ j\dfrac{1}{Z_c}\sin\theta & \cos\theta \end{bmatrix},$$

$$[ABCD]_3 = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}.$$

After the matrix multiplication and equating the corresponding elements of the ABCD matrix of quarter wavelength line, Z_c and C may be found as follows [1-5]:

$$\begin{aligned} &\cos\theta - \omega C Z_c \sin\theta = 0 \\ &Z_c \sin\theta = Z_0 \end{aligned} \qquad (2)$$

It is obvious from Eq.2 that the characteristic impedance of the slow-wave line Z_c is greater than the characteristic impedance Z_0 of the conventional hybrid coupler.

At the center frequency the S-parameters of the conventional branchline hybrid (Fig.1) are as follows [5]:

$$[S] = \begin{bmatrix} 0 & -jZ_s/Z_0 & -Z_s/Z_p & 0 \\ -jZ_s/Z_0 & 0 & 0 & -Z_s/Z_p \\ -Z_s/Z_p & 0 & 0 & -jZ_s/Z_0 \\ 0 & -Z_s/Z_p & -Z_s/Z_0 & 0 \end{bmatrix} \qquad (3)$$

and $|S_{21}|^2 + |S_{31}|^2 = 1$.

For arbitrary power division the impedance of the serial and parallel arms of the branchline hybrid must be [5]:

$$Z_s = Z_0\sqrt{1-|S_{31}|^2} \ \text{and} \ Z_p = \frac{Z_0\sqrt{1-|S_{31}|^2}}{|S_{31}|} \qquad (4).$$

Equating (2) and (4) we derive the design equations for arbitrary power division reduced size hybrid:

$$Z_1 = \frac{Z_0\sqrt{1-|S_{31}|^2}}{|S_{31}|\sin\theta_1},$$

$$Z_2 = \frac{Z_0\sqrt{1-|S_{31}|^2}}{\sin\theta_2}, \qquad (5)$$

$$C = \frac{1}{\omega Z_0\sqrt{1-|S_{31}|^2}}\left(|S_{31}|\cos\theta_1 + \cos\theta_2\right).$$

For the equal power division $|S_{21}| = |S_{31}| = 3dB$, the derived equations (5) simplify to the equations derived in [1-4].

The analysis of the reduced size hybrid is carried out using the fourfold symmetry of the schematic shown on Fig.2. Using proper excitation of the structure both symmetry planes AA and BB can be either magnetic (open circuit) or electric walls (short circuit). The equivalent schematics for the four different excitations are shown on Fig.4.

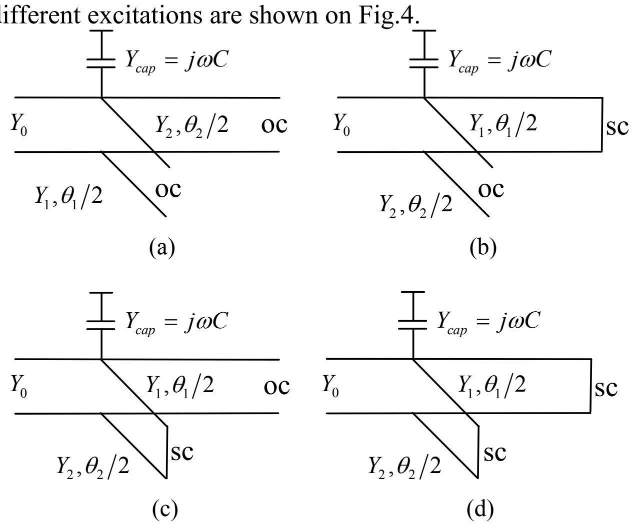


Fig.4. Equivalent schematics for one quarter section when (a) AA and BB are magnetic walls, (b) AA is electric wall, BB is magnetic wall, (c) AA is magnetic wall, BB is electric wall, (d) AA and BB are electric walls.

The reflection coefficients of each equivalent schematic are obtained as follows:

$$\Gamma_a = \frac{Y_c - j\omega C - jY_1 tg\frac{\theta_1}{2} - jY_2 tg\frac{\theta_2}{2}}{Y_c + j\omega C + jY_1 tg\frac{\theta_1}{2} + jY_2 tg\frac{\theta_2}{2}} \qquad (7a)$$

$$\Gamma_b = \frac{Y_c - j\omega C - jY_1 tg\frac{\theta_1}{2} + jY_2 cotg\frac{\theta_2}{2}}{Y_c + j\omega C + jY_1 tg\frac{\theta_1}{2} - jY_2 cotg\frac{\theta_2}{2}} \qquad (7b)$$

$$\Gamma_c = \frac{Y_c - j\omega C + jY_1 cotg\frac{\theta_1}{2} - jY_2 tg\frac{\theta_2}{2}}{Y_c + j\omega C - jY_1 cotg\frac{\theta_1}{2} + jY_2 tg\frac{\theta_2}{2}} \qquad (7c)$$

$$\Gamma_d = \frac{Y_c - j\omega C + jY_1 cotg\frac{\theta_1}{2} + jY_2 cotg\frac{\theta_2}{2}}{Y_c + j\omega C - jY_1 cotg\frac{\theta_1}{2} - jY_2 cotg\frac{\theta_2}{2}} \qquad (7d).$$

The lengths of the branches are shorter than quarter wavelength. Consequently the electrical lengths are less than $\pi/8\ rad$. The following approximations are assumed $tg\theta \approx \theta$ and $cotg\theta \approx 1/\theta$. The previously derived formulae for the reflection coefficients are reduced to:

$$\Gamma_a = \frac{Y_c - j\omega C - jY_1\dfrac{\theta_1}{2} - jY_2\dfrac{\theta_2}{2}}{Y_c + j\omega C + jY_1\dfrac{\theta_1}{2} + jY_2\dfrac{\theta_2}{2}} \tag{8a}$$

$$\Gamma_b = \frac{Y_c - j\omega C - jY_1\dfrac{\theta_1}{2} + jY_2\dfrac{2}{\theta_2}}{Y_c + j\omega C + jY_1\dfrac{\theta_1}{2} - jY_2\dfrac{2}{\theta_2}} \tag{8b}$$

$$\Gamma_c = \frac{Y_c - j\omega C + jY_1\dfrac{2}{\theta_1} - jY_2\dfrac{\theta_2}{2}}{Y_c + j\omega C - jY_1\dfrac{2}{\theta_1} + jY_2\dfrac{\theta_2}{2}} \tag{8c}$$

$$\Gamma_d = \frac{Y_c - j\omega C + jY_1\dfrac{2}{\theta_1} + jY_2\dfrac{2}{\theta_2}}{Y_c + j\omega C - jY_1\dfrac{2}{\theta_1} - jY_2\dfrac{2}{\theta_2}} \tag{8d}$$

The scattering matrix elements are derived as follows [8,9]:

$$\begin{aligned}
S_{11} &= \frac{1}{4}\left(\Gamma_a + \Gamma_b + \Gamma_c + \Gamma_d\right) \\
S_{12} = S_{21} &= \frac{1}{4}\left(\Gamma_a - \Gamma_b + \Gamma_c - \Gamma_d\right) \\
S_{13} = S_{31} &= \frac{1}{4}\left(\Gamma_a - \Gamma_b - \Gamma_c + \Gamma_d\right) \\
S_{14} = S_{41} &= \frac{1}{4}\left(\Gamma_a + \Gamma_b - \Gamma_c - \Gamma_d\right)
\end{aligned} \tag{9}$$

If the reduced size hybrid is designed according to Eq.5 or Eq.6, therefore the scattering matrix elements (Eq.9) will reduce to $S_{11} = S_{14} = 0$, $S_{13} = -1/\sqrt{2}$, $S_{12} = -j/\sqrt{2}$.

III. NUMERICAL RESULTS

In order to verify the applicability of the derived formulas (5) for design of reduced-size hybrid, it is examined the phase difference between the output ports for different power division, electrical lengths and characteristic impedances of the branches.

It is convenient in some design cases to fix the electrical length of the reduced-size hybrid branches. Utilizing Eq.(5) there are calculated the characteristic impedances and the lumped capacitances of the branches. The results are summarized in Table 1 for 5, 10 and 20dB.

TABLE 1 Arbitrary power division reduced size branchline hybrid

Power division, dB	Z_2, Ω	Z_1, Ω	θ, deg	$C, [pF]$
5	32.47	39.26	60	5.17
10	18.26	19.25	60	9.8
20	5.80	5.77	60	31.75
5	43.74	52.9	40	7.92
10	24.6	25.93	40	15
20	7.78	7.82	40	48.6
5	82.21	99.42	20	9.72
10	46.23	48.73	20	18.43
20	14.69	14.62	20	59.67

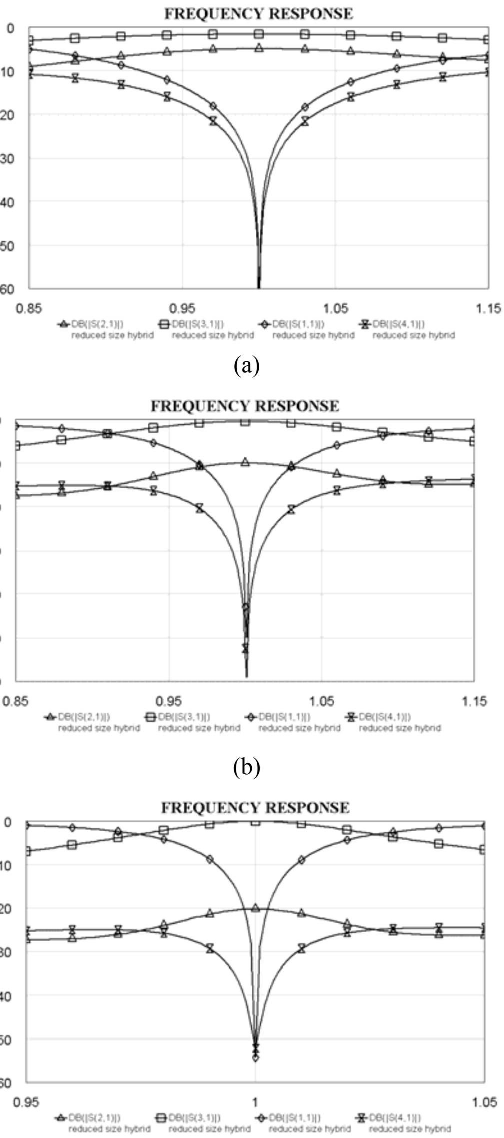
It is obvious that the increase in the characteristic impedance leads to shorter lines. The value of the lumped capacitance *C* is big enough and it could be realized in SMD as a trimmer for the low GHz range.

The corresponding characteristic impedances of the branches for the conventional hybrid shown on Fig.1 are summarized in Table2.

TABLE2. Conventional branchline hybrid

Power division, dB	Z_1, Ω	Z_2, Ω
5	34	28.18
10	16.66	15.81
20	5.03	5

The frequency responses of the designed branchline hybrids for 5, 10 and 20dB are shown on Fig.5.



(a)

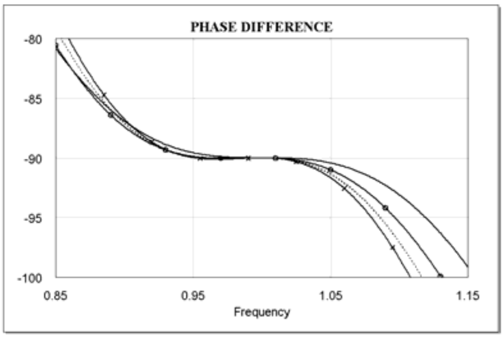
(b)

(c)

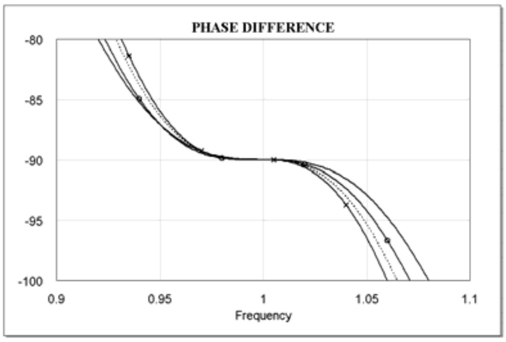
Fig.5 Frequency responses of reduced branchline hybrid for (a) 5dB, (b) 10dB, (c) 20dB

As it is clearly seen increasing the power division ratio leads to narrower fractional bandwidth of the branchline hybrid. This dependence is also well pronounced for the conventional hybrid. The fractional bandwidth for 20dB is lower than 3% and the characteristic impedances' values are not realizable in microstrip technology (Table2). On the other hand the 20dB hybrid may be realized if the electrical length of the branches is relatively small and the lumped capacitor is tenths of pF. The expense for this realization is the extremely low fractional bandwidth. Therefore it is better to use a directional coupler.

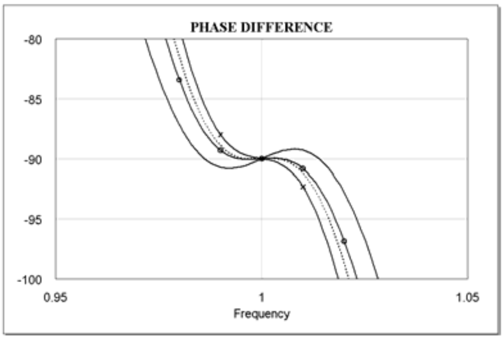
The phase difference between the output ports for three different power division ratios is shown on Fig.6



(a)



(b)



(c)

Fig.6 Phase difference between the output ports for (a) 5dB, (b) 10dB, (c) 20dB (solid line-conventional hybrid, circle- $\theta = 20\deg$, dashed line- $\theta = 40\deg$, cross- $\theta = 60\deg$)

In all cases for the power division ratio, the bandwidth of the conventional branchline hybrid is wider than the bandwidth of the reduced size hybrid. The phase difference of reduced size hybrid is asymmetric against the central frequency. For the frequencies upper than the center, the phase difference increases very rapidly from the reference point of -90° . For the frequencies lower than the center, the phase difference varies around -90° . In contrast to the conventional branch-line coupler the output phase difference of the reduced size hybrid equals twice -90° , while alternates around this value. This is due to the slow-wave effect observed in capacitively loaded transmission line. For lower frequencies the phase difference rises more rapidly than the conventional branch line coupler. For 20dB power division ratio, the phase difference of the conventional hybrid is equiripple around -90deg, while the phase difference of the reduce size hybrid is relatively constant in the passband.

V. CONCLUSION

The paper proposes a method for synthesis and analysis of reduced size branch line hybrids with ,arbitrary power division ratio utilizing slow wave effect of capacitively loaded line. Using the fourfold symmetry of the hybrid, analysis formulae are obtained. Closed form synthesis formulae are derived for fixed characteristic impedance or fixed electrical length of the branch lines. The numerical results show that the fractional bandwidth of the reduced size hybrid is narrower than the conventional hybrid. The phase difference between the output ports is no longer symmetrical around -90° due to the slow wave effect. The proposed synthesis method is suitable for low GHz range hybrids, balanced amplifiers on a microstrip technology.

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