Using *z*-variable Functions for the Analysis of Wave-based Model of Microstrip Stub-line Structure

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Abstract – **An efficient method based on transfer parameter approach is proposed to obtain the scattering parameters in *z*-domain of planar microstrip stub-line structures. A wave digital network (WDN) represents a digital model of the structure. WDN is composed of cascaded uniform segments and three-port adaptors with open stubs connected to their dependent ports. Uniform segments (transmission lines and stubs) are represented by several cascaded unit elements. Also, a simple modeling approach of the T-junction discontinuity which involves changing line lengths is given here. A simulation validation of the proposed modeling and analysis approach is provided by means of an open T-resonator circuit realized in microstrip technique.**

Keywords – **Wave digital approach, wave digital networks, microstrip circuits, stub-line structure, T-resonator circuit, *z*-domain functions, scattering parameters**

I. Introduction

Wave digital filters (WDFs) represent a class of digital filters with a particular interest. WDFs were developed initially by Alfred Fettweis [1-2] in the late 1960s for digitizing lumped electrical circuits composed of inductors, capacitors, resistors, and other elements of classical network theory. The WDF approach is based on the traveling-wave formulation of lumped electrical elements. A detailed review of WDF theory is given in references [1-5]. Well known theory of WDFs is used for modeling of the planar structures. Recently, planar microstrip stepped-impedance structures are modeled and analyzed by use of the one-dimensional (1D) wave digital approach [5-8]. The wave digital networks (WDNs) are the models of the microstrip structures modeled by wave digital elements (delay, adder, multiplier and adaptors).

After an extensive search in the literature it was found an application of Advanced Design System (ADS [9]) to simulations of different microstrip structures based on their wave digital network representations [10]. The equivalent circuit model can be solved through a circuit simulator. In the case of the stub-line structures, the Kelly-Lochbaum implemetation of adaptors (with many multiply elements) is used.

The main emphasis of this paper is to give an original and general method to characterize the behaviour of microstrip stub-line structures by use of the wave digital networks in MATLAB environment [11]. Wave digital approach into MATLAB provides easy simulation of microwave layouts such as microstrip structures of different geometries. Also, allows the user to get the fastest return out of 3D electomagnetic tool investments. Two-multiply models of the three-port adaptors are going to be used here. The equivalent representation of this structure in MATLAB is much simple to operate than the one shown in [10].

Till now, the wave-based models of stub-line structures are analyzed directly by use of block-based Simulink model [12]. A short summary of the 1D wave digital approach is given in Section II.

A wave transfer matrix approach for response calculation is depicted in Section III. Response can be calculated either in the frequency or in the time domain directly from known network function in *z*-domain [13]. The scattering parameters of WDN, S_{21} and S_{11}, are derived as rational functions of *z*-variable.

The microstrip discontinuities associated with the T-pattern resonator, such as open-end and T-junction, are considered in Section IV.

In Section V, a simulation validation of the proposed modeling and analysis approaches is provided by means of two examples of one open T-resonator circuit.

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II. A Short Summary of 1D Wave Digital Approach

This section is intended to review the most important definitions of the wave digital approach in analysis of the microstrip stub-line structures [12].

Many of the important results in modeling and analysis of microstrip stepped-impedance structures are summarized in the paper [8] in order to emphasize their significance. If the complex structure comprises several uniform segments, each of them has to be represented as a cascade connection of a certain number of unit elements (UEs). This is because of their delays which vary from one another. A way of determination a minimal number of UEs in WDN of complex structure is based on the given relative error as described in the papers [6-8]. A problem of appropriate choice of a minimal section number in the model needs to be carefully addressed, because of its direct influence on the sampling frequency of that digital model, and on accuracy of the desired response.

The same conclusions and procedures (shortly described above) can obviously also be applied to the modeling of the stub-line structures. The basic idea is to treat the stub-line structure as a connection of separated uniform transmission lines (UTL segments). The simulation of connections between the three UTL segments is achieved by three-port parallel adaptor with one port being dependent. An open stub is connected to its dependent port. The WDNs derived from stub-line structures have first been defined in the paper [12].

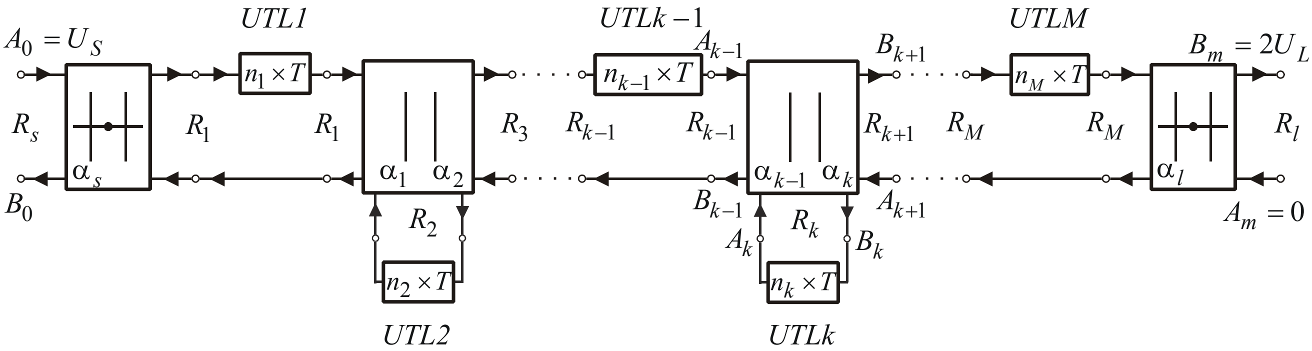

Fig. 1. Wave digital network of a planar microstrip stub-line structure

This leads to an universal and effective procedure capable of solving a wide range of practical problems.

III. WAVE-BASED MODEL AND ITS TRANSFER WAVE MATRIX

This section provides complete theory of the transfer wave matrix approach. The calculating of the scattering parameters of the known WDN is discussed here [13].

A planar microstrip stub-line structure can be represented as M uniform segments connected in a typical way. The wave-based model (i.e. wave digital network) of the planar stub-line structure is composed of three types of building blocks: uniform segments (contain several cascaded UEs), two-port adaptors and three-port adaptors with one port being dependent.

A WDN that contains n_t UEs and $(M-1)/2$ three-port adaptors, pictured in Fig. 1, is analyzed here. This WDN is a two-port circuit having at each port an input and an output wave variable. Each UE is associated with its delay T, and port resistances R_k at either port, where $k = 1,2,...,M$. The simulation of connections between the three UTL segments (one of them is an open stub connected at dependent port) is achieved by three-port parallel adaptor. Each three-port adaptor is associated with its coefficients α_{k-1} and α_k, $k = 2,4,...,2(M-1)$, and three incident A_{k-1}, A_k and A_{k+1}, and three reflected B_{k-1}, B_k and B_{k+1} waves, where $k = 2,5,...,3(M-1)-1$. The incident wave A_0 is equal to voltage U_S of the source, and the reflected wave B_m is equal to voltage $2U_L$ on the load. The first and the last two-port series adaptors are used for matching source and load resistances to the rest of the WDN.

The port impedances of UEs in $n_k \times T$ blocks are equal, which means that they can be directly cascade connected (coefficients of two-port adaptors are zeros). The transfer wave matrix for one uniform segment which is modeled by n_k cascaded UEs is

$$\boldsymbol{T}_{UE}^{n_k} = \underbrace{\boldsymbol{T}_{UE} \times \boldsymbol{T}_{UE} \times \cdots \times \boldsymbol{T}_{UE}}_{n_k} = \frac{1}{z^{-n_k}} \cdot \begin{bmatrix} z^{-n_k} & 0 \\ 0 & 1 \end{bmatrix}, \quad (1)$$

where the transfer wave matrix for a single UE is

$$\boldsymbol{T}_{UE} = \frac{1}{z^{-1}} \cdot \begin{bmatrix} z^{-1} & 0 \\ 0 & 1 \end{bmatrix}.$$

The k^{th} three-port parallel adaptor with port 2 chosen as dependent port (Fig. 1) is described by set of equations

$$\begin{aligned} B_{k-1} &= B_k + A_k - A_{k-1}, \\ B_{k+1} &= B_k + A_k - A_{k+1}, \\ B_k &= A_k + \alpha_{k-1} \cdot (A_{k-1} - A_k) + \alpha_k \cdot (A_{k+1} - A_k), \end{aligned} \quad (2)$$

where the multiplier coefficients are

$$\alpha_{k-1} = \frac{2G_{k-1}}{G_{k-1} + G_k + G_{k+1}}, \text{ and } \alpha_k = \frac{2G_{k+1}}{G_{k-1} + G_k + G_{k+1}}, \quad (3)$$

and G_{k-1}, G_k and G_{k+1} are the port conductances.

An open stub is connected to the port 2 being dependent and for this port the wave variables can be written

$$A_k = z^{-n_k} \cdot B_k. \quad (4)$$

The wave transfer matrix for the k^{th} three-port adaptor, with an open stub connected on the dependent port 2, is obtained by use of relations (2)-(4) and it is

$$\boldsymbol{T}_{\alpha_{k-1}\alpha_k}^{n_k} = \begin{bmatrix} T_{11}^{n_k} & T_{12}^{n_k} \\ T_{21}^{n_k} & T_{22}^{n_k} \end{bmatrix}, \quad (5a)$$

where the matrix elements are

$$T_{11}^{n_k} = \frac{(\alpha_{k-1} + \alpha_k - 1) + z^{-n_k}}{\alpha_{k-1} \cdot (1 + z^{-n_k})}, \quad (5b)$$

$$T_{12}^{n_k} = \frac{(\alpha_{k-1} - 1) - (\alpha_k - 1) \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 + z^{-n_k})}, \quad (5c)$$

$$T_{21}^{n_k} = \frac{(1 - \alpha_k) + (\alpha_{k-1} - 1) \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 + z^{-n_k})} \quad (5d)$$

and

$$T_{22}^{n_k} = \frac{(\alpha_{k-1} + \alpha_k - 1) \cdot z^{-n_k} + 1}{\alpha_{k-1} \cdot (1 + z^{-n_k})}. \quad (5e)$$

The complete transfer wave matrix $\boldsymbol{T}$ corresponding to the analyzed WDN is a product of the wave matrix of network building blocks as

$$\boldsymbol{T} = \boldsymbol{T}_{\alpha_s} \times \boldsymbol{T}_{UE}^{n_1} \times \boldsymbol{T}_{\alpha_1\alpha_2}^{n_2} \times \boldsymbol{T}_{UE}^{n_3} \times \ldots \times \boldsymbol{T}_{UE}^{n_{k-1}} \times \boldsymbol{T}_{\alpha_{k-1}\alpha_k}^{n_k} \times$$
$$\times \boldsymbol{T}_{UE}^{n_{k+1}} \times \ldots \times \boldsymbol{T}_{\alpha_l} . \quad (6)$$

Consider now the matrices of two-port series adaptors as follows

$$\boldsymbol{T}_{\alpha_s} = \frac{1}{1-\alpha_s} \cdot \begin{bmatrix} -1 & \alpha_s \\ \alpha_s & -1 \end{bmatrix} = \frac{1}{1-\alpha_s} \cdot \boldsymbol{T}_s \quad (7)$$

and

$$\boldsymbol{T}_{\alpha_l} = \frac{1}{1-\alpha_l} \cdot \begin{bmatrix} -1 & \alpha_l \\ \alpha_l & -1 \end{bmatrix} = \frac{1}{1-\alpha_l} \cdot \boldsymbol{T}_l , \quad (8)$$

where adaptors' coefficients are

$$\alpha_s = \frac{R_s - R_1}{R_s + R_1}, \text{ and } \alpha_l = \frac{R_M - R_l}{R_M + R_l}, \quad (9)$$

with the port resistances R_s, R_1, R_M, and R_l assigned as shown in Fig. 1.

The relation (1) dictate that the matrix of one uniform segment modeled with n_{2j-1} UEs can be written in the form

$$\boldsymbol{T}_{UE}^{n_{2j-1}} = \frac{1}{z^{-n_{2j-1}}} \cdot \begin{bmatrix} z^{-n_{2j-1}} & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{z^{-n_{2j-1}}} \cdot \boldsymbol{T}_{n_{2j-1}} , \quad (10)$$

where $j = 1,2,...,N_1$. The number N_1 depends on the number of segments in structure as

$$N_1 = \begin{cases} M/2 & \text{for } M \text{ being even,} \\ (M-1)/2 & \text{for } M \text{ being odd.} \end{cases} \quad (11)$$

According to relation (5), the matrix of three-port parallel adaptor with an stub on dependent port 2 which is modeled by n_{2j} UEs is given in the form

$$\boldsymbol{T}_{\alpha_{2j-1}\alpha_{2j}}^{n_{2j}} = \frac{1}{\alpha_{2j-1} \cdot (1 + z^{-n_{2j}})} \cdot \boldsymbol{T}_{n_{2j}} =$$
$$= \frac{1}{\alpha_{2j-1} \cdot (1 + z^{-n_{2j}})} \cdot \begin{bmatrix} T_{11}^{n_{2j}} & T_{12}^{n_{2j}} \\ T_{21}^{n_{2j}} & T_{22}^{n_{2j}} \end{bmatrix}, \quad (12a)$$

where the elements of $\boldsymbol{T}_{n_{2j}}$ matrix are

$$T_{11}^{n_{2j}} = (\alpha_{2j-1} + \alpha_{2j} - 1) + z^{-n_{2j}} , \quad (12b)$$

$$T_{12}^{n_{2j}} = (\alpha_{2j-1} - 1) - (\alpha_{2j} - 1) \cdot z^{-n_{2j}} , \quad (12c)$$

$$T_{21}^{n_{2j}} = (1 - \alpha_{2j}) + (\alpha_{2j-1} - 1) \cdot z^{-n_{2j}} , \quad (12d)$$

$$T_{22}^{n_{2j}} = (\alpha_{2j-1} + \alpha_{2j} - 1) \cdot z^{-n_{2j}} + 1 , \quad (12e)$$

and $j = 1,2,...,N_1$.

According to the relations (6)-(12), and due to the total number of uniform segments M, the polynomials can be written in the form

$$W_e(z) = (1-\alpha_s) \cdot (1-\alpha_l) \cdot \prod_{j=1}^{N_1} \left(\alpha_{2j-1} \cdot (1 + z^{-n_{2j}}) \cdot z^{-n_{2j-1}} \right)$$
$$\text{for even } M , \quad (13)$$

or

$$W_o(z) = W_e(z) \cdot z^{-n_M} , \text{ for odd } M . \quad (14)$$

The complete matrix can be represented in the form

$$\boldsymbol{T}_e(z) = \boldsymbol{T}_s \times \prod_{j=1}^{N_1} \left(\boldsymbol{T}_{n_{2j-1}} \times \boldsymbol{T}_{n_{2j}} \right) \times \boldsymbol{T}_l , \text{ for even } M , \quad (15)$$

or

$$\boldsymbol{T}_o(z) = \boldsymbol{T}_s \times \prod_{j=1}^{N_1} \left(\boldsymbol{T}_{n_{2j-1}} \times \boldsymbol{T}_{n_{2j}} \right) \times \boldsymbol{T}_{n_M} \times \boldsymbol{T}_l , \text{ for odd } M \quad (16)$$

where the matrix $\boldsymbol{T}_{n_M}$ corresponds to the last uniform segment in the series branch.

Finally, the complete wave transfer matrix $\boldsymbol{T}$ due to the number of segments in the structure can be written in one of two forms

$$\boldsymbol{T} = \frac{1}{W_e(z)} \cdot \boldsymbol{T}_e(z) , \text{ for even } M , \quad (17)$$

or

$$\boldsymbol{T} = \frac{1}{W_o(z)} \cdot \boldsymbol{T}_o(z) , \text{ for odd } M . \quad (18)$$

In other words, the elements of complete wave transfer matrix can be written in the form of polynomials

$$\boldsymbol{T} = \frac{1}{W_{e/o}(z)} \cdot \boldsymbol{T}_{e/o}(z) = \frac{1}{W_{e/o}(z)} \cdot \begin{bmatrix} T_{e/o11}(z) & T_{e/o12}(z) \\ T_{e/o21}(z) & T_{e/o22}(z) \end{bmatrix} =$$
$$= \begin{bmatrix} T_{11}(z) & T_{12}(z) \\ T_{21}(z) & T_{22}(z) \end{bmatrix}, \quad (19)$$

where index e/o corresponds to even (e) or odd (o) number M. The wave matrix elements are the rational polynomial functions of z^{-1}, and they are

$$T_{11}(z) = \frac{T_{e/o11}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o22}(m-k) \right), \quad (20)$$

$$T_{12}(z) = \frac{T_{e/o12}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o12}(k+1) \right), \quad (21)$$

$$T_{21}(z) = \frac{T_{e/o21}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o12}(m-k) \right), \quad (22)$$

$$T_{22}(z) = \frac{T_{e/o22}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o22}(k+1) \right). \quad (23)$$

The polynomial coefficients of the wave matrix elements depend only on adaptors' coefficients. According to relations (20)-(23), only two elements, $T_{12}(z)$, and $T_{22}(z)$, have to be calculated. If necessary, two other elements, $T_{11}(z)$, and $T_{21}(z)$, can be derived from relations (20) and (22).

Providing $A_m = 0$, the output response (forward voltage transmission coefficient) is

$$S_{21} = \left.\frac{B_m}{A_0}\right|_{A_m=0} = \frac{W_{e/o}(z)}{T_{e/o22}(z)}, \quad (24)$$

and the input response (input reflection coefficient) is

$$\Gamma_0 = S_{11} = \left.\frac{B_0}{A_0}\right|_{A_m=0} = \frac{T_{e/o12}(z)}{T_{e/o22}(z)}. \quad (25)$$

IV. MODELING OF THE T-JUNCTION DISCONTINUITY

In this section, only discontinuities associated with the T-pattern resonator, such as open-end and T-junction, are considered. Since discontinuity dimensions are usually much smaller than the wavelength in a microstrip, they can be modeled by lumped-element equivalent ciruits.

An open-end discontinuity occurs frequently in a number of circuits such as resonators, matching stubs, filters, and microstrip antennas. A closed form expressions for calculating the excess length of transmission line [14] are used here.

One of the most important and frequently utilized discontinuities in microstrip structures is the three-port T-junction. It is a very offten used in the microstrip circuits, such as microstrip stubs and power dividers, [14]. This discontinuity is most obvious in the resonant frequency of the structure as a shift downward in frequency.

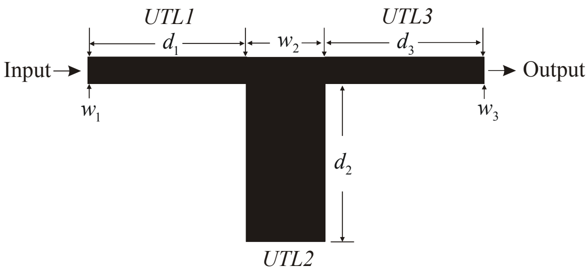

Fig. 2. Layout of a stub-line microstrip structure

Consider now a T-pattern resonator depicted in Fig. 2. T-junction discontinuity must be considered and compensated. A new modeling procedure of the T-junction discontinuity is based on decreasing line lengths. The corrections for T-junction effects are done as follows: the physical length of the series line which length is d_1 and width w_1 is decreased by the value $w_2/2$, i.e. it is decreased by the value of one half of width of the stub line. The physical length of the stub line with length d_2 and width w_2 is decreased by the value $w_1/2$, i.e. it is decreased by the value of one half of width of the series line.

This approach of the modeling effects of this discontinuity which involves changing line lengths is a very simple. Also, it is very acceptable because there are not new blocks here. The proposed method reduces drastically the computation time while giving acceptable accuracy.

V. RESULTS AND DISCUSSION

In this section, a simple example of stub-line microstrip structures is analyzed by use of suggested 1D approach.

A. Frequency Response of T-resonator Circuit - Example I

A microstrip stub-line structure with one stub, so-called T-resonator circuit, depicted in Fig. 2, is used for verification of the proposed method. The substrate dielectric constant is $\varepsilon_r = 2.32$ and the board thickness $h = 1.58\,mm$.

T-resonator circuit is approximated by connection of three uniform segments with parameters given in the Tables I and II. In order to absorb discontinuity effects new line lengths are counted. A T-junction discontinuity is modeled by decreasing lengths of the line in the junction. The effect of the open stub is compensated by increasing length of the segment UTL2. According to the parameters given in the Tables, it can be concluded that the number of transmission line is the same in both cases, but their physical lengths differ one another and beacuse of that their delays differ either.

TABLE I
TRANSMISSION LINE PARAMETERS WITHOUT MODELED DISCONTINUITIES

nv	*d* [*mm*]	*w* [*mm*]	*Zc* [Ohm]	*Tv* [*ps*]
1	**30.0000**	4.7100	50.2540	**139.8003**
2	**30.0000**	15.7600	20.0016	**145.0746**
3	**30.0000**	4.7100	50.2540	**139.8003**

TABLE II
TRANSMISSION LINE PARAMETERS WITH MODELED DISCONTINUITIES

nv	*d* [*mm*]	*w* [*mm*]	*Zc* [Ohm]	*Tv* [*ps*]
1	**22.1200**	4.7100	50.2540	**103.0794**
2	**28.4681**	15.7600	20.0016	**137.6665**
3	**22.1200**	4.7100	50.2540	**103.0794**

A minimal number of sections for the given error can be found as described in the papers [13]-[14]. In the case when the discontinuities were modeled, for the given error $n\_er = 0.01\,\%$, a total minimal number of sections in the observed WDN is $n_t = \sum_{k=1}^{3} n_k = 447$. For segments UTL1 and UTL3, a number of sections n_k is 134, and for segment UTL2 is 179. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min} / q = 343.8254\,ps$, where $q = 134$ is a multiple factor and $T_{\min} = \min\{T_1, T_2, T_3\} = 103.0794\,ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{3} T_k = 343.8544\,ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 1299.9687\,GHz$. In this case, a relative error of delay is $er = -0.008457\,\%$. Adaptor coefficients are $\alpha_s = -\alpha_l = -0.0025$ and $\alpha_1 = \alpha_2 = 0.4432$.

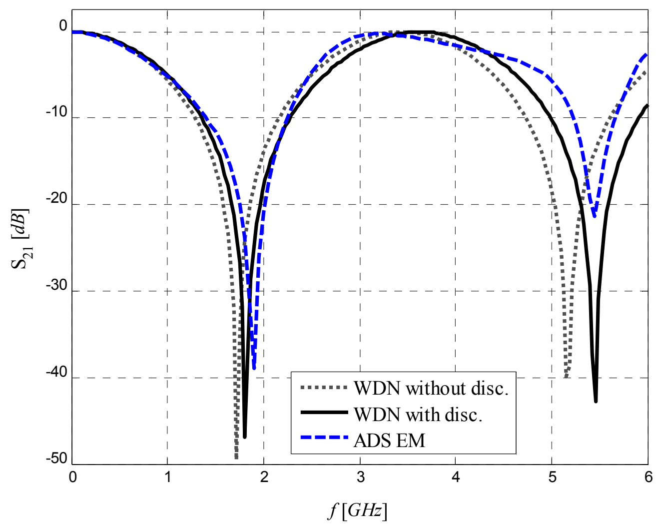

Fig. 3. Frequency response

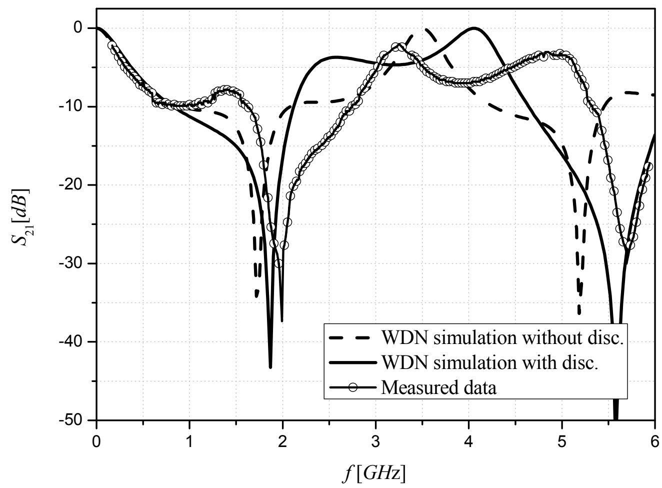

Fig. 4. The measured and simulated transmision parameters of the open T-resonator circuit

Fig. 3 shows responses of T-resonator both simulated in MATLAB by a new proposed approach and obtained in ADS simulator [12]. The $S_{21}[dB]$ results are similar in general. The WDN result shows only a small difference of the resonance frequency in comparison with the result of electromagnetic simulation in ADS.

In this structure, a number of uniform segments is odd ($M = 3$), and according to that the forward voltage transmission coefficient in z-domain is

$$S_{21}(z) = W_o(z)/T_{o22}(z),$$

and, the output reflection coefficient

$$S_{22}(z) = -T_{o21}(z)/T_{o22}(z).$$

These coefficients are the rational functions, where the polynomials in numerators are

$$W_o(z) = 0.4432 + 0.4432 \cdot z^{-179},$$

$$T_{o21}(z) = -0.0025 + 0.5568 \cdot z^{-134} + 0.0003 \cdot z^{-179} - \\ - 0.0003 \cdot z^{-268} - 0.5568 \cdot z^{-313} + 0.0025 \cdot z^{-447},$$

and the polynomial in denominator is

$$T_{o22}(z) = 1 - 0.0028 \cdot z^{-134} - 0.1136 \cdot z^{-179} + \\ + 7.2890 \cdot 10^{-7} \cdot z^{-268} + 0.0028 \cdot z^{-313} - 6.4180 \cdot 10^{-6} \cdot z^{-447}.$$

As previously stated in the Section III, it is quite enough to calculate the scattering parameters on the network output, $S_{21}(z)$, and $S_{22}(z)$. For the magnitudes of the parameters can be written $|S_{12}(z)| = |S_{21}(z)|$, and $|S_{11}(z)| = |S_{22}(z)|$. The phase characteristics differ from one another.

B. Frequency Response of T-resonator Circuit - Example II

The T-resonator circuit, depicted in Fig. 2, is used for verification of the proposed method. The structure is fabricated on CuFlon substrate with dielectric constant $\varepsilon_r = 2.17$ and width $h = 0.508\,mm$.

The S_{21} parameters of the structure at the frequencies from $300\,MHz$ to $6\,GHz$ are shown in Fig. 4. The transmission parameter was measured by a network analyser. It is important to notice that the measured data show a small difference of the resonance frequency in comparison with the WDN simulation results (WDN simulation with disc. curve). This difference is caused by the microstrip T-junction and the open-end, which are not described accurately enough by the models.

According to the previous discussion, a simple wave digital approach can produce results that are similar to the much more sophisticated methods. The simulation showed that the discontinuities of the T-resonator structure had a great effect on the resonance frequencies.

C. Comparison of Analysis Methods

The observed stub-line structure has been analyzed in this paper directly in both frequency and time domains. The proposed approach based on wave transfer parameters is implemented using MATLAB on a Pentium V personal computer operated on $2.4\,GHz$. The purpose of this part is to compare the duration of two response calculation ways based on the results of their simulation runs.

In the paper [12], a very simple block-diagram method of analysis of WDN is used. Block-based WDN is formed directly in the Simulink toolbox of the MATLAB environment. Frequency response is obtained by direct analysis of formed block-based network. In that case, results are obtained in $2.1265\,s$. If the network is analyzed here directly in the time domain, much more simulation time is required, and a memory problem is very frequent. A time for drawing WDN in the Simulink toolbox depends only on user's skills and it is not given here.

Frequency response from the known network function in z-domain can be found in two different ways. The first case starts with the response calculation directly in the time domain using differential equations. On the final stage Fourier transformation is used for frequency response calculation. A time of $4.2544\,s$ is needed for response calculation. In the second case, a time for a response calculation in the frequency domain directly is $0.0061\,s$. The results are showing significant difference between these two times. A great advantage of the second response calculation way is its computational efficiency, because all necessary calculations take place directly in the frequency domain.

This study focuses on a comparison of simulation times. As seen here, the simplest way for response calculation is direct analysis of WDN in the frequency domain and use of network transfer function in *z*-domain. This function can easily be obtained from the wave transfer matrix. In that case, there is no high memory request and a very short time for response calculation is required.

VI. Conclusion

This paper presents some of the current developments as well as challenges in applying *z*-variable functions for analysis of the wave-based model of microstrip stub-line structure.

There are several main conclusions that can be drawn here:

1. The purpose of this paper has been to apply the theory of the wave digital filters for the transfer characteristics of planar microstrip stub-line structures. The wave digital principles combined with transfer parameter approach can be used advantageously to determine response of WDN. This combination is a very efficient way for evaluating the scattering parameters of WDNs. A simple algorithm for coefficient calculation was derived. The described approach can be implemented very effectively in any matrix oriented mathematical programs. Also, the study of discontinuities associated with the T-pattern resonator is a very important. The simplest modeling approach for T-junction discontinuity involves changing line lengths.

2. Complete wave transfer matrix procedure make the design easily applicable to a board variety of stub-line structures (different number of uniform segments in wave-based model).

3. One of the main emphases is on the automatic analysis of wave-based models, which is inevitable when structures with larger numbers of building blocks are to be dealt with.

4. As has been told previously, response can be calculated either in the frequency or in the time domain directly from known network function in *z*-domain. Known network functions can be used as input data in some other simulations.

5. In order to prove the broadband accuracy of the proposed modeling and analyzing approach, two examples realized in the microstrip line technique, such as open T-resonator circuit are observed. The results of the analysis obtained by WDN have shown a very good agreement with those obtained by other programs mentioned above. The results of one T-resonator are compared with measurements performed in the frequency range $300\,MHz$ - $6\,GHz$. A much better agreement is achieved when the discontinuities in the observed structure are modeled.

6. A systematic comparison of the duration of two response calculation ways based on the results of their simulation runs is made. By comparison with the block-diagram method, the network function method provides faster structure simulation. Generally, these methods provide the fast simulations versus complex and time consuming 3D models. Also, a great advantage of the suggested network function method is its computational efficiency.

7. Implementation of WDN in analysis of microwave structures can be used by microwave engineers because of the associated simplicity and accuracy. It can be used for analyzing the transmission lines of various nonuniform shapes present in practice.

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