Field Tunnelling and Losses in Narrow Waveguide Channel

Miranda Mitrović, Branka Jokanović

Abstract – **In this paper we investigate the field tunnelling through the narrow waveguide channel formed by reducing the height of a rectangular waveguide. Geometrical parameters of the channel and material properties strongly affect the energy tunnelling and field density in the channel. The role of material losses is systematically examined as an important practical issue limiting the maximum achievable tunnelling transmission level.**

Keywords – **ENZ metamaterial, narrow waveguide channel, energy tunnelling, increased field density, loss.**

I. Introduction

In the past two decades there has been great interest in metamaterials whose electromagnetic properties were first predicted in 1968 by Veselago [1]. Metamaterials show characteristics that are not available in ordinary materials found in nature and therefore can be used in designing novel microwave and optical devices with enhanced properties.

Until recently research in this area was focused on double negative (DNG) metamaterials with permittivity and permeability less than zero leading to negative index of refraction, but in the past few years there has been an increased interest in metamaterials whose relative dielectric permittivity is close to zero (epsilon-near-zero, ENZ). Pioneers in this area of study were Silveirinha and Engheta with their theoretical work on energy tunnelling through subwavelength channels filled with ENZ metamaterial in 2006 [2]. They used two sections of parallel plate waveguides connected by a very narrow channel and demonstrated that wavefront is reproduced on the second end of the channel with very small losses. They suggested that ENZ materials can be used in improving the transmission efficiency of waveguides with sharp bends or discontinuities and in concentrating energy in a small subwavelength cavity. Later on, several experiments were conducted to confirm this theory in microwave regime [3, 4].

It can be seen that there are two approaches for realization of ENZ metamaterials, one developed by Liu and Chang et al [3] using split-ring resonator media inserted in narrow channel connecting two parallel plate waveguide sections, and the other developed by Edwards et al [4] using dispersion characteristic of a rectangular waveguide near its cut-off frequency. Various applications have been suggested for these materials, such as cloaking devices [5], confinement of energy beyond diffraction limit [2], waveguide coupling with narrow channel [4], and design of high directivity small antenna [6].

Miranda Mitrović and Branka Jokanović are with the Institute of Physics Belgrade, Pregrevica 118, 11080 Beograd, Serbia and Montenegro, E-mail: miranda@ipb.ac.rs, brankaj@ipb.ac.rs

In this work, we systematically investigate how geometrical parameters of the channel and characteristics of materials filling waveguide and channel affect tunnelling of energy and field density inside ENZ channel. We also examine how material losses depend on the channel thickness. Our simulations reveal the strong impact of losses both in metal and dielectric when the channel thickness is very small.

II. Theoretical Analysis

Rectangular waveguide of width *a* and height *b* (*a*>*b*) supports travelling of TE and TM modes. Cut-off frequencies for both modes are given by Eq. (1).

$$f_{c(m,n)} = \frac{c}{2\pi\sqrt{\varepsilon_{rw}}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad (1)$$

Here ε_{rw} is the relative dielectric constant of the dielectric in rectangular waveguide, *m* and *n* are numbers assigned to different modes.

The sequence of these modes in the case of $b=a/2$ can be seen on Fig. 1. The frequency range between TE_{10} and TE_{20} represents the pass band of a rectangular waveguide. Reducing the height *b* of a rectangular waveguide shifts the cut-off frequencies of the higher modes toward higher frequencies, while its pass band stays unchanged.

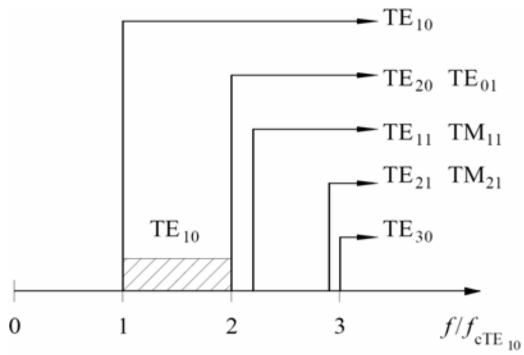


Fig. 1. The sequence of propagating modes in a rectangular waveguide for $b=a/2$

It is intuitively obvious that reducing the height of rectangular waveguide (Fig. 2) drastically affects the reflection (S_{11}) and transmission (S_{21}) coefficients of the waveguide, causing a poor transmission due to strong mismatching.

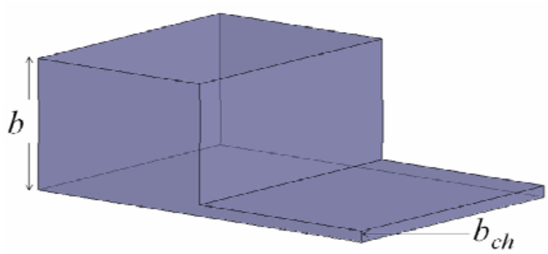


Fig. 2. Reducing height b of a rectangular waveguide

In a difference to the structure with a single step, Fig. 3 shows the structure with Π-channel (b_{ch}<<b) inserted between two waveguide sections, which can support transmission in a short frequency range.

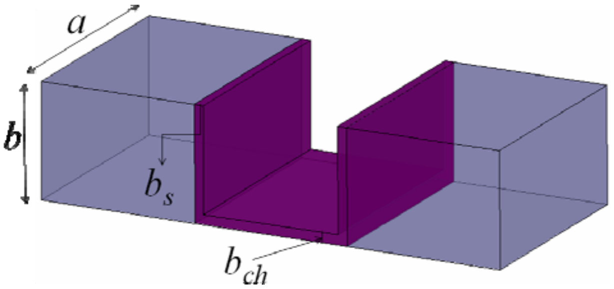


Fig. 3. Waveguide sections connected with a narrow Π-channel (b_{ch}<<b) for impute matching

In the case of b_{ch}<<b, it is possible to describe the propagation of TE_{10} mode in a rectangular waveguide as a propagation of TEM mode in parallel-plate waveguide with effective permittivity ε_{reff} [3]:

$$\beta_{TE_{10}}^{ch} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} \cong \beta_{TEM}^{ch} = \frac{2\pi f \sqrt{\varepsilon_{reff}}}{c}, \tag{2a}$$

where

$$k = \frac{2\pi f \sqrt{\varepsilon_{rch}}}{c} \tag{2b}$$

From previous equations we can derive an expression for ε_{reff}:

$$\varepsilon_{reff} \cong \varepsilon_{rch} - \frac{c^2}{4f^2a^2} \tag{3}$$

Here c is the speed of light in vacuum and ε_{rch} is a relative dielectric constant in the channel. It is seen that ε_{reff} equals zero at the cut-off frequency of the channel, which gives us the opportunity to consider this structure as an ENZ metamaterial near this frequency. This is the frequency where tunnelling of energy occurs (Eq. 4).

$$f_{tun} \cong f_{TE_{10}}^{ch} = \frac{c}{2a\sqrt{\varepsilon_{rch}}} \tag{4}$$

The frequency of tunnelling should be within the pass band of two waveguides (Eq. (5a)) and from this condition we can see that relative dielectric constant in the waveguide sections should be greater than in the channel (Eq. (5c)) to ensure transmission within the pass band.

$$f_{TE_{10}}^{w} < f_{tun} < f_{TE_{20}}^{w} \tag{5a}$$

$$\frac{c}{2a\sqrt{\varepsilon_{rw}}} < \frac{c}{2a\sqrt{\varepsilon_{rch}}} < \frac{c}{a\sqrt{\varepsilon_{rw}}} \tag{5b}$$

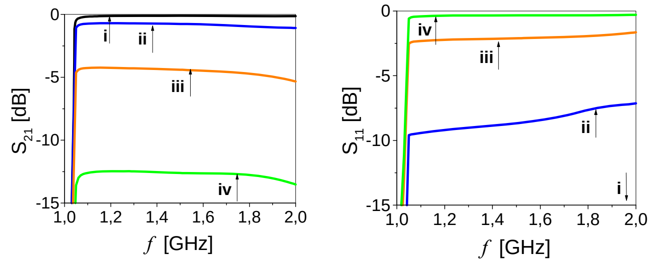
$$\frac{\varepsilon_{rw}}{4} < \varepsilon_{rch} < \varepsilon_{rw} \tag{5c}$$

Considering that ε_{reff} is near zero in the narrow channel around cut-off frequency, wave vector β is also near zero, which means that wavelength approaches infinity. This kind of behaviour is analogous to a balance case for LH (left-handed) metamaterials, or zeroth-order resonance (ZOR) [7]. As a consequence, energy is capable of tunnelling through the narrow channel at this frequency, and a great density of field proportional to b/b_{ch} which is constant along the channel is achieved.

III. Simulation Results

A. ENZ Channel

Here we consider the structure from Fig. 2. with following dimensions: a=101.6mm (waveguide width), b=a/2=50.8mm (waveguide height), and channel thickness b_{ch} with different values, b_{ch1}=b, b_{ch2}=b/2=25.4mm, b_{ch3}=b/8=6.35mm and b_{ch4}=b/64=0.8mm. Dielectric constant in both waveguide sections is ε_{rw}=2, and cut-off frequencies for TE_{10} and TE_{20} modes are 1.044GHz and 2.088GHz, respectively. The simulated transmission and reflection coefficients are given in Figs. 4 (a) and (b) respectively. Simulations are performed using Ansoft HFSS software.



(a) (b)

Fig. 4. Transmission (a) and reflection (b) coefficients for different values of b_{ch}: (i) b_{ch1}=b=50.8mm, (ii) b_{ch2}=25.4mm, (iii) b_{ch3}=6.35mm, (iv) b_{ch4}=0.8mm

As we can see in Fig. 4, even if the height of the rectangular waveguide is reduced to a half of its original value, the transmission with small attenuation is still possible. Considerable smaller transmission coefficient is noticeable only after waveguide height reduction of b_{ch}=b/8.

In order to compensate influence of discontinuity between waveguide and channel (which behaves as an equivalent capacitance), impedance matching should be accomplished. This can be achieved by introducing transition matching area between waveguide sections and ENZ channel that forms a Π-channel, (Fig. 3). Dimensions for a and b are the same as in the previous case, relative dielectric constant in the channel is ε_{rch}=1 (air) and in the waveguide sections ε_{rw}=2 (Teflon).

Transmission and reflection coefficients in case of $b_{ch}=b_s=0.8$mm is shown in Fig. 5. The first transmission peak occurs at frequency $f_{tun}=1.464$GHz which is assigned to the zeroth-order resonance (ZOR). Tunnelling of energy occurs at this frequency, since effective permittivity in the channel is equal to zero. The second transmission peak $f_{FP}=1.816$GHz is Fabry-Perot resonance which is highly dependent on length of ENZ channel. That is not the case with ZOR, as long as the condition $b_{ch}<<b$ is fulfilled [8].

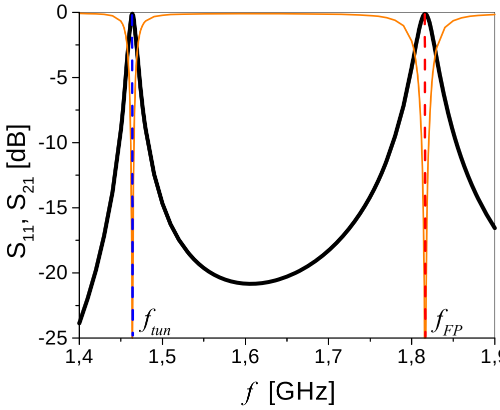


Fig. 5. Transmission coefficient (a=101.6mm, $b=a/2$=50.8mm, b_{ch}=0.8mm, $b_s=b_{ch}$, $\varepsilon_{rw}=2$, $\varepsilon_{rch}=1$)

Field distribution and a real part of Poynting vector in the channel are shown in Figs. 6 (a) and 6 (b) respectively. It can be seen from H-field bar that energy density is increased in channel by factor 63.88 which is very close to $b/b_{ch}=64$. The distribution of real part of Poynting vector shows the energy flow along the channel and is concentrated and greatly enhanced in the middle of the channel.

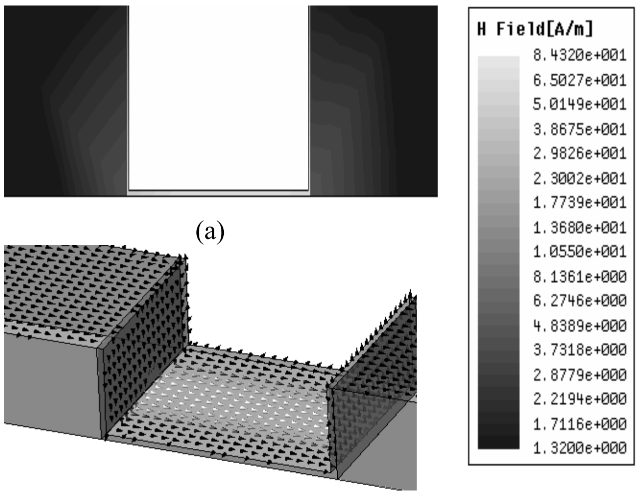


Fig. 6. (a) Field distribution in narrow channel shows H-field density inside the channel enhanced by factor b/b_{ch} in comparison to field in waveguide sections; field density is constant along the channel; (b) Real part of Poynting vector shows energy flow through the channel; energy is concentrated in in the middle of the channel and is gradually descending toward edges.

In approximation of perfectly conducting metallic walls and lossless dielectrics, field density in the channel is increased with reduction of channel height and transmission is perfect. In reality, field density in channel is restricted by break down voltage in dielectric and transmission is lowered due to finite conductivity of metallic walls and dielectric losses. Further discussion on this subject will be presented later on in this paper.

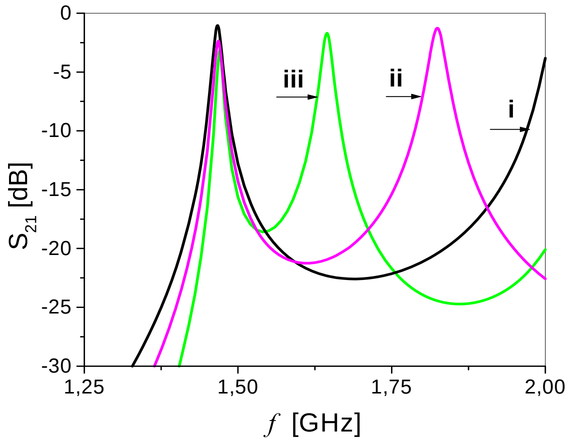


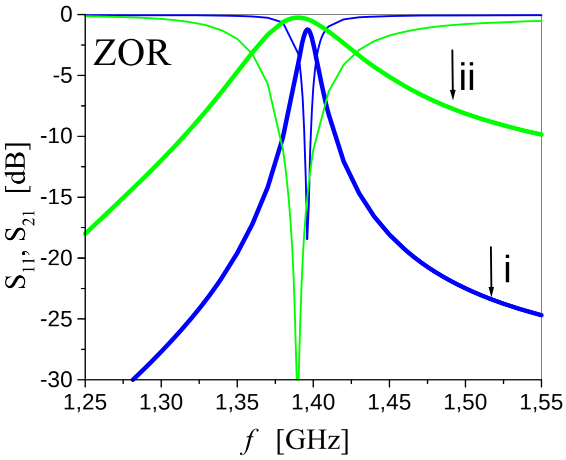
Fig. 7. Shifting of the second transmission peak (Fabry-Perot resonance) due to variation of channel length: (i) L_1=95.25mm, (ii) L_2=127mm, (iii) L_3=190.5mm

As it was pointed out before, the second transmission peak as a product of Fabry-Perot resonance is highly dependent on the channel length. This variation has no effect on the position of the zeroth-order resonance (ZOR) frequency at which tunnelling of energy is occurred. Change of Fabry-Perot resonance for various lengths of narrow channel ($b_{ch}=b/64$=0.8mm, L_1=95.25mm, L_2=127mm and L_3=190.5mm) can be seen in Fig. 7. This property can be used to manipulate second transmission peak in order to remove it or leave it within the pass band of the waveguide.

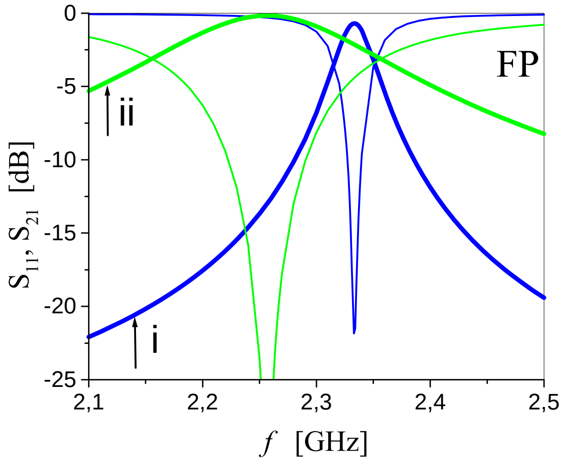
B. Losses in ENZ Channel

In order to study material losses we will consider structure with copper cladding (σ_{Cu}=58MS/m) and real laminated dielectrics: Plexiglas with properties $\varepsilon_r=3.4$, $tg\delta=10*10^{-4}$ in waveguide section), Rogers RT/duroid 5870 with $\varepsilon_r=2.33$, $tg\delta=12*10^{-4}$ and Arlon AD 250 with $\varepsilon_r=2.5$, $tg\delta=30*10^{-4}$ in channel area. To preserve the similar operating band, dimensions of the previous structure (Fig. 3) are changed: width a=70mm, height b=35mm, channel thickness b_{ch} varying between 0.5 and 4 mm, length of the channel L=49mm. Cut-off frequency in waveguide sections is f^w_{TE10}=1.162GHz, ZOR frequencies are 1.34÷1.35GHz while Fabry-Perot resonances vary in range 2.17÷2.27GHz with the change of channel thickness. In Figs. 8. (a) and (b) transmission and reflection coefficients for two values of channel thickness are given.

As it can be seen from Fig. 8, dielectric losses for both ZOR (a) and Fabry-Perot resonances (b) increase with decreasing the channel height. Also, change in resonant frequency is much greater for Fabry-Perot resonance in comparison to ZOR.



a)



b)

Fig. 8. Transmission and reflection coefficients for a) ZOR and b) Fabry-Perot resonance in the case of: (i) b_{ch}=0.5mm and (ii) b_{ch}=3mm. Materials used are: Plexiglas (ε=3.4, $tg\delta$=10*10^{-4}) in waveguide sections, Rogers RT/duroid 5870 (ε_r=2.33, $tg\delta$=12*10^{-4}) in the channel, and perfect conductor for metal plates.

In general, losses in narrow channel consist of two components: losses in metallic plates and losses in dielectrics filling the channel and waveguide sections. Firstly, losses in metallic plates are investigated using in simulations copper cladding with a real conductivity and dielectrics without losses. Tipical value for conductivity and rougheness of electrodeposited copper are σ_{Cu}=58MS/m and *RMS*=2.4. Dissipated power is calculated in two cases: perfectly smooth, and metallic walls with roughness *RMS*=2.4.

As it can be seen from Fig. 9, dissipated power in metallic plates with surface roughness is almost two times greater than in a case of perfectly smooth metal cladding. Also, reflection takes part in dissipated power smaller then 8% in all cases except for the minimum value of channel thickness at ZOR frequency. Dissipated power starts to grow rapidly above 10% for the channel thickness b_{ch}/b=0.043 (b_{ch}=1.5mm) for ZOR resonance and b_{ch}/b=0.029 (b_{ch}=1mm) for Fabry-Perot resonance. This is understandable having in mind that power density in channel takes much greater values at tunnelling frequency than at Fabry-Perot resonance.

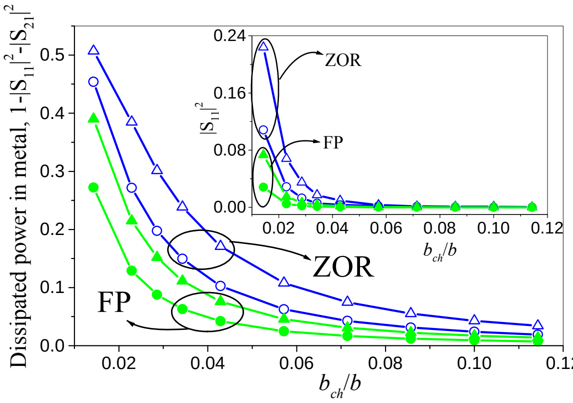


Fig. 9. Dissipated power in metal versus channel thickness - lines marked with circles represent perfectly smooth copper cladding and lines with triangles represent copper cladding with RMS=2.4; Results for $|S_{11}|^2$ versus channel thickness are given in right upper corner.

Comparative results for two dielectrics filling the channel with different loss tangents (Rogers RT/duroid 5870 (ε_r=2.33, $tg\delta$=12*10^{-4})) and Arlon AD 250 (ε_r=2.5, $tg\delta$=30*10^{-4})), while dielectric in waveguide sections is Plexiglas (ε_r=3.4, $tg\delta$=10*10^{-4}). Simulations are performed with perfect conductor used for metal cladding.

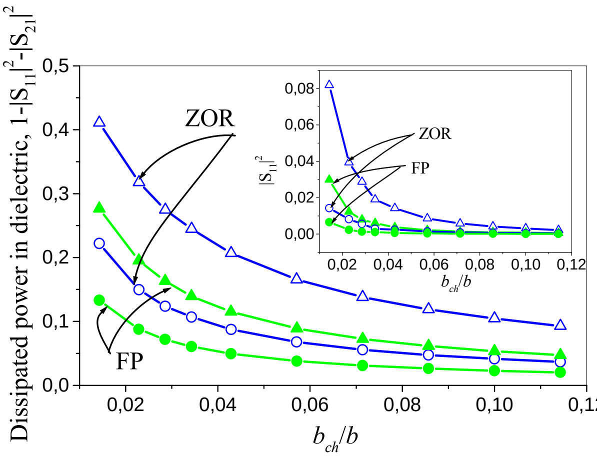
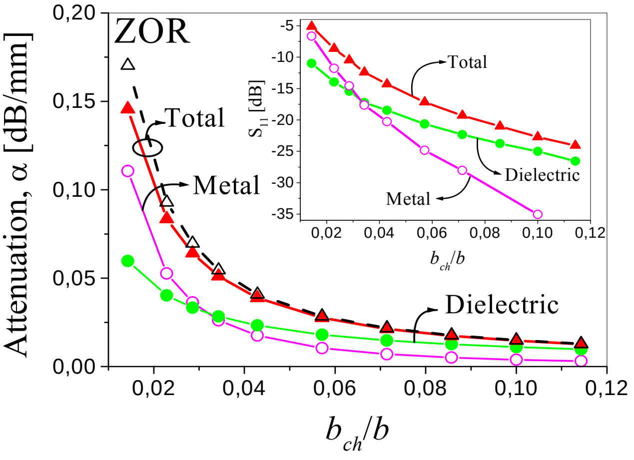


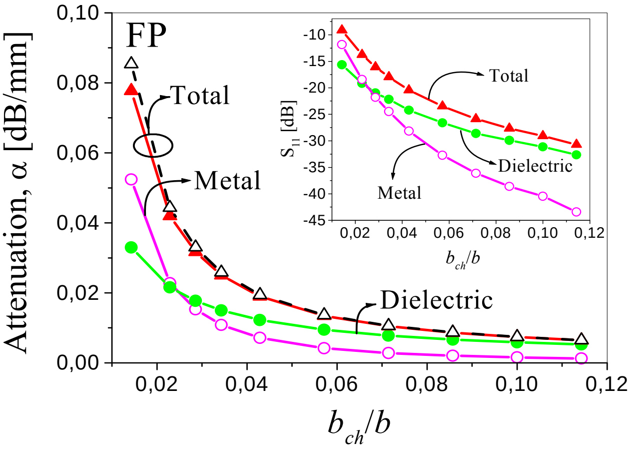
Fig. 10. Dissipated power in dielectric in dependence of channel thickness - lines marked with circles represent dielectric with $tg\delta$=12*10^{-4} and lines with triangles represent dielectric with $tg\delta$=30*10^{-4}; In the right upper corner the results for $|S_{11}|^2$ in dependence of channel thickness are given

According to the slope of the curves for the small channel thicknesses (Fig. 10), it is obvious that dissipated power in dielectrics is less sensitive to the change of channel thickness in comparison to dissipated power in metal. On the other hand, if we look at dissipated power in the whole range it can be seen that it is highly dependent on the choice of dielectric.

Next we want to show comparative results for attenuation in metal cladding and dielectrics. The worst scenario was used in simulations: copper cladding with *RMS*=2.4 and Arlon AD 250 (ε_r=2.5, $tg\delta$=30*10^{-4}) as dielectric in the channel. Waveguide sections are filled with Plexiglas (ε_r=3.4, $tg\delta$=10*10^{-4}). Diagrams for attenuation at ZOR and Fabry-Perot frequencies are displayed separately in Figs. 11 (a) and (b), respectively.



(a)



(b)

Fig. 11. Attenuation versus channel thickness for ZOR (a) and Fabry-Perot resonances (b); Losses in dielectric are dominant in channels greater than b_{ch}=1mm (b_{ch}/b=0.029) for ZOR and b_{ch}=0.8mm (b_{ch}/b=0.023) for Fabry-Perot resonance. Dashed line denotes added up attenuation for metal and dielectric and full line shows total simulated attenuation. In the right upper corner the results for S_{11} in dependence of channel thickness are given.

As it can be seen from Fig. 11, attenuation due to dielectric losses is dominant in channels with greater thickness values, while losses in metal are dominant for narrow channels. This is especially visible at ZOR frequency. For ZOR frequency attenuation in metal and dielectric becomes equal for the channel thickness b_{ch}=1mm (b_{ch}/b=0.029) and for Fabry-Perot resonance that happens for the narrower channel b_{ch}=0.8mm (b_{ch}/b=0.023). Along with attenuation in dielectric and metal, the total simulated attenuation is given and compared with the sum of dielectric and metal attenuation. Good agreement is observed between the last two. It can be seen that attenuation at ZOR frequency is approximately as twice as high as at Fabry-Perot resonance. As it was stated before, the reason for this is a greater power density in the channel at ZOR frequency which makes the channel more sensitive to losses.

At the end, the resonant frequencies along with Q-factors for total simulated losses from the previous example are given in Table I for different channel thicknesses.

TABLE 1
RESONANT FREQUENCIES AND LOADED Q-FACTORS

b_{ch} [mm]	f_{ZOR} [GHz]	f_{FP} [GHz]	Q_L^{ZOR}	Q_L^{FP}
0.5	1.344	2.249	61.1	43.25
0.8	1.349	2.271	51.9	33.9
1	1.349	2.264	48.2	28.7
1.2	1.346	2.253	43.4	25
1.5	1.347	2.244	36.4	21
2	1.345	2.222	31.3	16.2
2.5	1.344	2.205	24.4	13.4
3	1.344	2.189	21	11.2
3.5	1.343	2.175	18.15	9.75
4	1.343	2.169	16	8.5

It can be seen from Table I that loaded Q-factors have greater values for ZOR than for Fabry-Perot resonance.

IV. CONCLUSION

Metamaterials with relative dielectric permittivity close to zero (epsilon-near-zero, ENZ) can be designed by reducing the height of a rectangular waveguide. Field tunnelling in the channel occurs when the channel and waveguide thickness ratio becomes very low, and when dielectric permittivity in the channel is less than dielectric permittivity in waveguide sections. The frequency of tunnelling is assigned as the zeroth-order resonance (ZOR) since it does not depend on change of channel length. That is not the case with the second peak in transmission through the channel, which is Fabry-Perot resonance.

In this paper, we show that geometrical parameters of ENZ channel and characteristics of metal cladding and dielectric in the channel greatly affect the transmission at both resonances, ZOR and Fabry-Perot.

Dissipated power due to metal losses grows rapidly for channel thicknesses smaller than b_{ch}=1.5mm at ZOR resonance, and b_{ch}=1mm at Fabry-Perot resonance. The difference in channel thickness limitation between these two resonances occurs because of the higher power density in the channel at tunnelling frequency than at Fabry-Perot resonance.

Total simulated attenuation for the worst case which accounts a copper cladding with roughness RMS=2.4 and a lossy dielectric with $tg\delta=30*10^{-4}$ is in the range 0.013÷0.146dB/mm for ZOR, while at Fabry-Perot resonance total attenuation is about two times smaller than for ZOR, and in the range 0.007÷0.078dB/mm. Attenuation due to dielectric losses is dominant for the channel thicknesses greater than b_{ch}=1mm for ZOR, while for Fabry-Perot resonance channel thickness limitation is b_{ch}=0.8mm. It can be seen that waveguide channel with epsilon-near-zero exhibits the attenuation considerably greater than in standard rectangular waveguide (α=0.005dB/m).

ACKNOWLEDGEMENT

This work is supported by the Serbian Ministry of Science and Technological Development through the project TR-11009 – Dual-Band and Triple-Band Metamaterial-Based Devices and Antennas for Modern Communication Systems.

REFERENCES

[1] V.G. Veselago, “The electrodynamics of substances with simultaneously negative values of ε and μ“, *Soviet Physics Uspekhi*, vol. 10, no. 4, pp. 509-514, 1968.

[2] M. G. Silveirinha and N. Engheta, “Tunnelling of electromagnetic energy through subwavelength channels and bends using ε-near-zero materials“, *Physical Review Letters*, vol. 97, 157403, 2006.

[3] R. Liu, Q. Cheng, T. Hand, J. J. Mock, T. J. Cui, S. A. Cummer, and D. R. Smith, “Experimental demonstration of electromagnetic tunnelling through an epsilon-near-zero metamaterial at microwave frequencies“, *Physical Review Letters*, vol. 100, 023903, 2008.

[4] B. Edwards, A. Alù, M. E. Young, M. G. Silveirinha, and N. Engheta, “Experimental verification of epsilon-near-zero metamaterial coupling and energy squeezing using a microwave waveguide“, *Physical Review Letters*, vol. 100, 033903, 2008.

[5] M. G. Silveirinha, A. Alù, and N. Engheta, “Parallel-plate metamaterials for cloaking structures“, *Physical Review E*, vol. 75, 036603, 2007.

[6] S. Enoch, G. Tayeb, P. Sabouroux, N. Guérin, and P. Vincent, “A metamaterial for directive emission“, *Physical Review Letters*, vol. 89, 213902, 2002.

[7] C. Caloz and T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*, New Jersey, Wiley, 2006.

[8] M. G. Silveirinha and N. Engheta, “Theory of supercoupling, squeezing wave energy, and field confinement in narrow channels and tight bends using ε-near-zero metamaterials“, *Physical Review B*, vol. 76, 245109, 2007.