Discrete-Time Network and State Equation Methods Applied to Computational Electromagnetics

Johannes A. Russer[1], Yury Kuznetsov[2], Peter Russer[1]

Abstract - **The representation of electromagnetic structures by lumped element circuits is revisited. Network models can be established by a subsequent application of system identification and circuit synthesis methods to data obtained by numerical simulation or from measurement. Network models provide a compact description of electromagnetic structures and can contribute significantly to the formulation of electromagnetic field problems and their efficient solution. On the field level network methods are introduced by segmentation of the electromagnetic structures and application of the field form of Tellegen's theorem. Methods for synthesis of lumped element models for lossless as well as lossy linear reciprocal multiports and for radiating structures are discussed. The state equation method as a general framework for lumped element network description is presented. Discrete time representations on the basis of Richards transformation and wave digital filter formulation are introduced.**

I. Introduction

The design of modern high-speed analog and digital electronics makes use of distributed passive circuit structures. The modeling of distributed circuits requires full-wave electromagnetic analysis. Usually the whole circuitry contains lumped as well as distributed subcircuits connected via interconnects or transmission lines such that each interconnect or connecting transmission line carries a single transverse mode only. This allows the segmentation of the circuits by cutting through the connecting transmission lines. The circuit segments obtained in this way, exhibiting a number of n open transmission lines each of them carrying a single transverse mode only in the considered frequency band, is called a multiport or n-port, respectively [1]. Whereas lumped element multiports can be treated by methods of network theory [2] distributed circuits require electromagnetic full-wave modeling [1, p. 413].

According to Guillemin [3, p. 73] we are concerned with three things in network theory: an excitation, a response and a network. When any two of them are known the third can be determined. The transfer function is defined as the ratio of response spectrum to excitation spectrum. When the network is given we can either compute the response from the excitation or the required excitation from the known response. In both of these cases we have an analysis problem. The third problem, i.e. to determine the network yielding a given response for a given excitation, is the *synthesis problem.* This problem will have solutions only if it is well-posed. If the synthesis problem can be solved it usually has various solutions, some of them are canonical. Furthermore the solution will depend on the frequency band in which the synthesized network should exhibit the required transfer function, and the tolerated error of the solution.

Distributed circuits can be modeled with arbitrary accuracy using lumped element network models. A general way to establish network models is based on modal analysis and similar techniques [4].

Since distributed circuits can be modeled in principle by lumped element equivalent circuits this raises the issue to apply lumped element synthesis methods to realize the transfer function of a distributed circuit. In general, the exact solution of this problem would yield a lumped element equivalent circuit with an infinite number of circuit elements. However, if the transfer function of the lumped element equivalent circuit needs to approximate the transfer function of the given distributed circuit only within a certain frequency range and also there only within a certain accuracy margin in many cases a lumped element equivalent circuit with a quite limited number of circuit elements can be found. Lumped element models provide a compact description of the distributed circuits. Especially when modeling complex circuits containing also nonlinear and active lumped elements it is advantageous to describe the distributed circuit parts by lumped element models. The whole circuit then can be modeled with a lumped element circuit simulator which is much more efficient than field oriented simulation.

The system identification (SI) of microwave structures and subsequent lumped element model synthesis can be performed by full-wave simulation or measuring of the input and output signals of the device in time or in frequency domain [5–16]. To establish the network model of a distributed circuit we may apply a three-step procedure:

1. Computation of the transfer functions by numerical electromagnetic full-wave analysis,
2. Determination of the rational functions representing

[1]Johannes A. Russer and Peter Russer are with the Institute for Nanoelectronics, Technische Universität München, Arcisstrasse 21, Munich, Germany, email: jrusser@tum.de, russer@tum.de

[2]Yury Kuznetsov is with the Theoretical Radio Engineering Department, Moscow Aviation Institute, Volokolamskoe shosse, 4, GSP-3, Moscow, 125993, Russia, email: kuznetsov@mai-trt.ru

the transfer functions by system identification or vector fitting methods,

3. Synthesis of a lumped element equivalent circuit realizing the transfer function.

The simulation or the measurement data can be represented as a table of generalized voltages and currents or incident and reflected waves in time or frequency domain [16–20]. High-Q resonant microwave circuits exhibit long impulse response times and therefore require a long simulation time. Application of SI allows constructing a lumped element model using a short sequence of the simulated response. As soon as the model parameters can be estimated with sufficient accuracy, the numerical simulation process can be terminated.

Furthermore, network oriented modeling can also be applied at the field level. In network theory lumped element circuits are separated into the circuit elements and the connection circuit containing only connections and ideal transformers. This methodology can also be applied to electromagnetic structures. We can divide the electromagnetic structure into substructures separated by boundary surfaces. The segmentation of the problem in subdomains establishes substructures which define the pertinent circuit elements and boundary surfaces between the substructures. Lossless structures in these subdomains can be represented by canonical Foster equivalent circuits. Description of radiation modes is provided by canonical Cauer networks. Analytic methods, e.g. Green's function or mode matching approaches, or numerical methods in connection with system identification techniques allow the synthesis of lumped element models. By this way electromagnetic field problems can be solved by means of network methods which provide for a systematic framework to formulate the problem and facilitate the development of efficient solution methodologies [1, 4, 21–27].

Electromagnetic structures can be modeled efficiently in the time-domain using a time-discrete *transmission line segment circuit* (TLSC) algorithm upon time discretization using Richards transformation [28]. A special case of the TLSC scheme is the scheme of the Transmission Line Matrix (TLM) method [1, 4, 29, 30]. It can be easily incorporated into the TLSC scheme yielding a powerful hybrid method. Furthermore, we discuss the application of wave digital filter (WDF) methods [31,32] for time-discrete modeling and their relation to TLSC and TLM schemes.

This paper is organized as follows: In Section II. we discuss the state equation description as a general framework for the lumped element representation of circuits. Section III. deals with the system identification method in frequency or time domain that allows to extract the rational function description of transfer functions obtained numerically from simulation or measurement. In Section IV. the connection circuit and Tellegen's theorem are discussed for electromagnetic structures. It is shown that in a segmented electromagnetic structure the subdomains represent the circuit elements and the complete set of boundary surfaces defines the connection circuit. In Section V. the Foster equivalent circuit realizations for reciprocal linear lossless multiports is presented and in Section VI. it is outlined how lumped element equivalent circuits could be synthesized for reciprocal linear lossy multiports. Section VII. presents the Cauer lumped element equivalent circuits for radiating modes. In Section VIII. the discrete time state equation representation is introduced which gives a general framework for the discrete time solution of electromagnetic circuits. In Section IX. it is shown that the discrete time state equation representation of electromagnetic structures is equivalent to the wave digital filter (WDF) representation. WDF theory provides a powerful theoretical framework for the treatment of time-discrete network models.

II. State Equation Representation

The lumped element equivalent circuit for electromagnetic structures can be also described by the state equations. The appropriate description in time-domain is given by a system of first order ordinary differential equations [2, 33]

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\,, \tag{1a}$$

$$\boldsymbol{w}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)\,, \tag{1b}$$

where the n-dimensional vector $\boldsymbol{x}(t)$ summarizes the state variables, $\boldsymbol{u}(t)$ is the input m-dimensional vector, $\boldsymbol{W}(t)$ is the output p-dimensional vector, $\boldsymbol{A}$ is called the system $n \times n$ matrix, $\boldsymbol{B}$ the input $n \times m$ matrix, and $\boldsymbol{C}$ the output $p \times n$ matrix [33]. The $p \times m$ matrix $\boldsymbol{D}$ is the transmission matrix which occurs only if there is a direct connection between input and output variables [34]. In the Laplace domain the state equations are given by

$$s\boldsymbol{X}(s) = \boldsymbol{A}\boldsymbol{X}(s) + \boldsymbol{B}\boldsymbol{U}(s)\,, \tag{2a}$$

$$\boldsymbol{W}(s) = \boldsymbol{C}\boldsymbol{X}(s) + \boldsymbol{D}\boldsymbol{U}(s)\,. \tag{2b}$$

Let $\boldsymbol{P}$ be an n-th order nonsingular square matrix which diagonalizes the system matrix $\boldsymbol{A}$, then

$$\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{L} = \begin{bmatrix} s_1 & 0 & \dots & 0 \\ 0 & s_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & s_n \end{bmatrix}, \tag{3}$$

where s_i, $i = 1, \dots, n$ are the distinct poles of the dynamical system. The matrix $\boldsymbol{P}$ can be used for the transformation of the system (2) to the normal form

$$s\boldsymbol{V}(s) = \boldsymbol{L}\boldsymbol{V}(s) + \boldsymbol{G}\boldsymbol{U}(s)\,, \tag{4a}$$

$$\boldsymbol{W}(s) = \boldsymbol{H}\boldsymbol{V}(s) + \boldsymbol{D}\boldsymbol{U}(s)\,, \tag{4b}$$

where $\boldsymbol{X}(s) = \boldsymbol{P}\boldsymbol{V}(s)$, $\boldsymbol{G} = \boldsymbol{P}^{-1}\boldsymbol{B}$ is a normal form input matrix, and $\boldsymbol{H} = \boldsymbol{C}\boldsymbol{P}$ is a normal form output matrix.

The dynamical system is stable if $\Re\{s_i\} < 0$ for all i. The rank r_u of the input matrix is the effective number of inputs which can influence the state vector. The rank

r_w of the output matrix defines the effective number of outputs available for observing the state of the system. It is possible without loss of generality to reduce the number of ineffectual inputs and outputs of the system.

A dynamical system is *controllable* if the normal form input matrix $\boldsymbol{G}$ exhibits no row where all elements are zero [34, 35]. The state variables $V_i(s)$ corresponding to rows of $\boldsymbol{G}$ containing only zeros are called uncontrollable. The evolution of the state of an uncontrollable system depends only on the initial conditions and not on the input signal $\boldsymbol{u}(s)$.

A dynamical system is *observable* if the normal form output matrix $\boldsymbol{H}$ has no columns which contain only zeros. The state variables $V_j(s)$ corresponding to columns of $\boldsymbol{H}$ exhibiting only zeros are called unobservable. These state variables cannot be detected in the output signal $\boldsymbol{W}(s)$. Thus a system which is unobservable has dynamic modes of behavior which cannot be ascertained from measurement of the output variables $\boldsymbol{W}(s)$.

The *admittance* $p \times m$ matrix $\boldsymbol{Y}(s)$ of the linear dynamical system is the 'black-box' input-output representation of the observable and controllable part of the system for zero initial conditions

$$\boldsymbol{W}(s) = \boldsymbol{Y}(s)\boldsymbol{U}(s)\,, \tag{5}$$

This matrix can be obtained from the state equations (2) and (4) by the following relation [35]. Substituting (2) and (4) into (5) we will obtain

$$\boldsymbol{C}\boldsymbol{X}(s) + \boldsymbol{D}\boldsymbol{U}(s) = \boldsymbol{Y}(s)\boldsymbol{U}(s)\,, \tag{6a}$$

$$\boldsymbol{C}(s\mathbf{1} - \boldsymbol{A})^{-1}\boldsymbol{B}\boldsymbol{U}(s) + \boldsymbol{D}\boldsymbol{U}(s) = \boldsymbol{Y}(s)\boldsymbol{U}(s)\,, \tag{6b}$$

$$\begin{aligned}\boldsymbol{Y}(s) &= \boldsymbol{C}(s\mathbf{1} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \\ &= \boldsymbol{H}(s\mathbf{1} - \boldsymbol{L})^{-1}\boldsymbol{G} + \boldsymbol{D} = \sum_{i=1}^{n} \frac{\boldsymbol{K}_i}{s - p_i} + \boldsymbol{D}\,,\end{aligned} \tag{6c}$$

where $\mathbf{1}$ is a unitary matrix.

If the admittance matrix $\boldsymbol{Y}(s)$ of the microwave system is given by measurements or numerical simulation, it is generally impossible to derive the corresponding differential equation representation. This is because the state variable choice is not unique and all information concerning the 'invisible' part of the system is missing. It is possible, however, to find a set of state equations (2) or (4) which yield the prescribed $\boldsymbol{Y}(s)$.

The partial expansion of the admittance matrix $\boldsymbol{Y}(s)$ whose elements are rational functions, can be given as

$$\boldsymbol{Y}(s) = \sum_{i=1}^{n} \frac{\boldsymbol{K}_i}{s - s_i} + \boldsymbol{D}\,, \tag{7}$$

where $\boldsymbol{K}_i$ are the residue matrices

$$\boldsymbol{K}_i = \lim_{s \to s_i} (s - s_i)\boldsymbol{Z}(s)\,, \tag{8}$$

$$\boldsymbol{D} = \lim_{s \to \infty} \boldsymbol{Z}(s)\,. \tag{9}$$

The rank of the i-th pole is defined as the rank of the corresponding matrix $\boldsymbol{K}_i$ with frequency independent constant coefficients.

III. System Identification

The SI procedure in the frequency domain starts from the approximation of the obtained discrete data by complex frequency rational functions describing the model of the microwave structure. The vector fitting (VF) method originally introduced in [36] provides a least mean-square approximation of the sampled data in frequency domain by rational fractions with a minimum possible order in the form

$$Y(s) = Es + D + \sum_{k=1}^{K} \frac{B_k}{s - s_k}\,, \tag{10}$$

where E and D are constant coefficients, s_k are the poles and B_k are their residues. The internal procedure for the approximation is an iterative procedure of relocating the positions of poles s_k in the complex s-plane. The model order K should be predefined in advance so the method will try to find such positions of K poles that give a minimum deviation between the given samples and the modeled one. The VF method was significantly improved in [37] by increasing the number of degrees of freedom that led to its relaxed modification.

One of the disadvantages inherent to the VF method is a possibility to obtain an unrealizable system function as a result of the fitting procedure. It means that a good fitting in the mean square error sense does not guarantee the realizability of the lumped element network.

Time domain SI procedure assumes the splitting of the model of the passive microwave circuit into two parts: the dynamic linear system and the delay lines linear system. This splitting corresponds to the scattering model represented in [38] and is known as a singularity expansion method [39, 40]. The boundary between the late-time part where the scattered field is described only by the dynamic system and the early-time part where both systems contribute is called the late-time border. This allows distinguishing the time interval where the SI procedure gives only parameters of the exponentially decaying sinusoids describing the impulse response of the dynamical part of the model

$$h_{LT}(t) = \sum_{n=1}^{N} C_n \exp\{s_n t\}\,. \tag{11}$$

The early-time part of the impulse response can be determined by the subtraction of the obtained late-time part $h_{LT}(t)$ convolved with the input waveform $a(t)$, from the output waveform $b(t)$ of the whole microwave system

$$b_{ET}(t) = b(t) - b_{LT} = b(t) - h_{LT} \star a(t) = \sum_{m=1}^{M} B_k a(t - T_m)\,, \tag{12}$$

where $a(t - T_m)$ is the partial delayed input waveform in the predefined frequency band [41]. As a result of the full-wave numerical simulation or measurements in time domain the discrete time domain waveform can be represented by its M samples expressed through the discrete convolution as follows

$$b[n] = h[n] \star a[n] = \sum_{m=0}^{M-1} h[m] \star a[n-m] = \sum_{m=0}^{M-1} a[m] \star h[n-m]\,, \qquad (13)$$

It also can be written in the matrix form

$$\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{M-1} \end{bmatrix} = \begin{bmatrix} a_0 & 0 & \dots & 0 \\ a_1 & a_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ a_{M-1} & a_{M-2} & \dots & a_0 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{M-1} \end{bmatrix} \qquad (14)$$

or more compactly

$$\boldsymbol{b} = \boldsymbol{A}\boldsymbol{h}\,. \qquad (15)$$

To solve the equation (15) the inversion of the matrix can be used

$$\boldsymbol{h} = \boldsymbol{A}^{-1}\boldsymbol{b}\,, \qquad (16)$$

but this way leads to an ill-conditioned problem because the determinant of the initial matrix $\boldsymbol{A}$ could be very close to zero and even small errors could result in significant inaccuracy of the calculated impulse response $\boldsymbol{h}$. This inconvenience can be solved by using the Moore-Penrose pseudo inversion of the matrix $\boldsymbol{A}$

$$\boldsymbol{A}^{+} = (\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H\,, \qquad (17)$$

where $\boldsymbol{A}^H$ is the Hermitian conjugated matrix. So the impulse response of the microwave system under investigation can be obtained by

$$\boldsymbol{h} = \boldsymbol{A}^{+}\boldsymbol{b}\,. \qquad (18)$$

Another efficient procedure for the deconvolution problem is a singular value decomposition of the initial matrix $\boldsymbol{A}$ for the defined number of the largest singular values λ_n, $n = 1, \dots D$

$$\boldsymbol{A}_D = \sum_{n=1}^{D} \sqrt{\lambda_n}\boldsymbol{u}_n\boldsymbol{v}_n^H = \boldsymbol{U}_D\boldsymbol{\Sigma}_D\boldsymbol{V}_D^H\,, \qquad (19)$$

where $\boldsymbol{U}_D$, $\boldsymbol{\Sigma}_D$, $\boldsymbol{V}_D$ are the truncated matrices of the standard singular value decomposition of the initial matrix $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$, and the matrices $\boldsymbol{U}_D$ and $\boldsymbol{V}_D$ are composed from the defined left-handed and right-handed eigenvectors $\boldsymbol{u}_n$ and $\boldsymbol{v}_n$ correspondingly. $\boldsymbol{\Sigma}_D$ is a diagonal matrix comprised of the singular values corresponding to the defined eigenvectors. Than the pseudo inversion of the matrix (19) can be expressed by

$$\boldsymbol{A}_D^{+} = \boldsymbol{V}_D\boldsymbol{\Sigma}_D^{-1}\boldsymbol{U}_D^H \qquad (20)$$

and the impulse response of the microwave system is

$$\boldsymbol{h} = \boldsymbol{V}_D\boldsymbol{\Sigma}_D^{-1}\boldsymbol{U}_D^H\boldsymbol{b}\,, \qquad (21)$$

Prony	$Y_{ij}(\mathsf{z}) = \sum_{r=1}^{p} \frac{v_r}{1-\mathsf{z}_r\mathsf{z}^{-1}}$
AR	$Y_{ij}(\mathsf{z}) = \frac{1}{1+\sum_{r=1}^{p} c_r\mathsf{z}^{-r}}$
ARMA	$Y_{ij}(\mathsf{z}) = \frac{1+\sum_{k=1}^{d} a_k\mathsf{z}^{-k}}{1+\sum_{l=1}^{q} b_l\mathsf{z}^{-l}}$

Table 1: Approximations for the Elements of the Admittance Matrix.

where $\boldsymbol{\Sigma}_D^{-1}$ is a diagonal matrix

$$\boldsymbol{\Sigma}_D^{-1} = \mathrm{diag}\left[\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_D}\right]. \qquad (22)$$

The most popular pole extraction method for time domain signals is the Matrix Pencil Method [42] which is a modification of the well known Prony's method [43]. This method provides the approximation of the time domain signal by the predefined number of exponentially damped sinusoids as in (11). The number of poles to be chosen depends on the specified limit for the mean square approximation error.

The definition of the output response late-time border can be obtained by the application of the stability criterion introduced in [44]. The realization of the stability criterion needs a specific tool which allows evaluating the closeness of one pole set to another one. One of the possible choices is the distance between the signatures of the pole sets in the normalized multidimensional space [45].

In the following the z-domain transfer function $\tilde{H}(\mathsf{z})$ represents a matrix element of the z-domain impedance matrix $\tilde{\boldsymbol{Z}}(\mathsf{z})$ or admittance matrix $\tilde{\mathbf{Y}}(\mathsf{z})$, respectively.

Applying the singularity expansion method [39, 40], one can express the electromagnetic system response function $\tilde{H}(\mathsf{z})$ in the form

$$\tilde{H}(\mathsf{z}) = \tilde{H}'(\mathsf{z}) + \tilde{H}''(\mathsf{z}) = \sum_{n=1}^{N} \frac{A_m(\mathsf{z})}{\mathsf{z}-\mathsf{z}_m} + \tilde{H}''(\mathsf{z})\,, \qquad (23)$$

where $\tilde{H}'(\mathsf{z})$ and $\tilde{H}''(\mathsf{z})$ denote the transient and driven parts, respectively, of the system response. $\tilde{H}''(\mathsf{z})$ is formed when the excitation wavefront is interacting with the system. Rational function approximations for the elements $\tilde{Y}_{ij}(\mathsf{z})$ of a multiport admittance matrix $\tilde{\mathbf{Y}}(\mathsf{z})$ relating the output currents $\tilde{I}_i(\mathsf{z})$ to the input voltages $\tilde{V}_j(\mathsf{z})$ by Prony's, AR and ARMA methods, respectively, are listed in Table 1 [43]. TLM modeling and SI of distributed microwave circuits and antennas is presented in [16–20, 41, 45]. System identification has been combined with model order reduction for TLM analysis in [46].

IV. The Connection Circuit

In an electromagnetic structure subdivided into subdomains the electric and magnetic fields $\mathcal{E}(\boldsymbol{x}, t)$ and $\mathcal{H}(\boldsymbol{x}, t)$

fulfill the field form of Tellegen's theorem [1, 47],

$$\oint_{\partial V} \mathcal{E}'(\boldsymbol{x}, t') \wedge \mathcal{H}''(\boldsymbol{x}, t'') = 0\,, \tag{24}$$

where the integration is performed over both sides of every boundary surface. Different time, and also different material fill of the subdomains are indicated by ' and ".

Expanding the transverse fields $\mathcal{E}_t(\boldsymbol{x}, t)$ and $\mathcal{H}_t((\boldsymbol{x}, t)$, on both sides α and β of all boundary surfaces into $2M$ biorthonormal basis differential forms e_m^ξ and h_m^ξ, respectively, with $\xi = \alpha, \beta$ yields

$$\mathcal{E}_t = \sum_m^M \sum_{\xi=\alpha,\beta} V_m^\xi e_m^\xi\,, \quad \mathcal{H}_t = \sum_m^N \sum_{\xi=\alpha,\beta} I_m^\xi h_m^\xi\,, \tag{25}$$

where the expansion coefficients V_m^ξ and I_m^ξ may be considered as generalized voltages and currents, respectively. We summarize the generalized voltages and currents into

$$\boldsymbol{V^\alpha}(t) = [V_1^\alpha(t), V_2^\alpha(t), \ldots V_M^\alpha(t)]^T\,, \tag{26a}$$

$$\boldsymbol{V^\beta}(t) = \left[V_1^\beta(t), V_2^\beta(t), \ldots V_M^\beta(t)\right]^T\,, \tag{26b}$$

$$\boldsymbol{I^\alpha}(t) = [I_1^\alpha(t), I_2^\alpha(t), \ldots I_M^\alpha(t)]^T\,, \tag{26c}$$

$$\boldsymbol{I^\beta}(t) = \left[I_1^\beta(t), I_2^\beta(t), \ldots I_M^\beta(t)\right]^T\,, \tag{26d}$$

and define $\boldsymbol{V} = [\boldsymbol{V^\alpha}\, \boldsymbol{V^\beta}]^T$ and $\boldsymbol{I} = [\boldsymbol{I^\alpha}\, \boldsymbol{I^\beta}]^T$.

Inserting (25) to (26d) into (24) and applying the method of moments [1, 4] yields the general network form of Tellegen's theorem

$$\boldsymbol{V}'^T(t')\, \boldsymbol{I}''(t'') = 0\,, \tag{27}$$

where the vectors $\boldsymbol{V}'(t')$ and $\boldsymbol{I}''(t'')$ summarize all voltages and currents on all surfaces [47, 48]. The superscripts ' and " denote that $\boldsymbol{V}'(t')$ and $\boldsymbol{I}''(t'')$ may be taken at different times and also for different material fills of the subregions.

In wave amplitude representation Tellegen's Theorem is given by [49–51]

$$\boldsymbol{a}'^T(t')\, \boldsymbol{b}''(t'') = \boldsymbol{b}'^T(t')\, \boldsymbol{a}''(t'')\,, \tag{28a}$$

$$\boldsymbol{a}'^T(t')\, \boldsymbol{a}''(t'') = \boldsymbol{b}'^T(t')\, \boldsymbol{b}''(t'')\,, \tag{28b}$$

where the vectors $\boldsymbol{a}(t)$ and $\boldsymbol{b}(t)$, respectively, summarize the wave amplitudes incident into and scattered from the connection network. The connection network exhibits the interconnect structure and ideal transformers. Its scattering matrix $\mathbf{\Gamma}$ is symmetric, real, Hermitian, unitary and orthogonal,

$$\mathbf{\Gamma} = \mathbf{\Gamma}^T = \mathbf{\Gamma}^* = \mathbf{\Gamma}^\dagger = \mathbf{\Gamma}^{-1}\,; \tag{29}$$

and exhibits the eigenvalues ± 1 only [52].

For a specific segmentation of the electromagnetic structure a connection network of the canonical form shown in Figure 1 is obtained, where the n_{ij} are the turns ratios

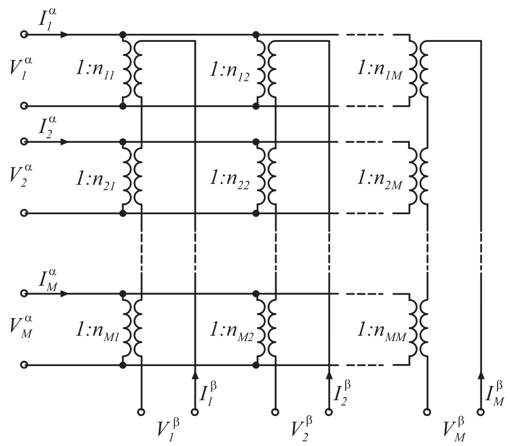

Figure 1: Canonical Form of the Connection Network.

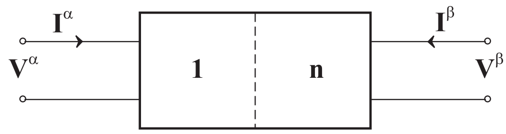

Figure 2: Representation of the canonical connection network.

of the ideal transformers. The voltages and currents are related by

$$\left[\begin{array}{cc|cc} -\boldsymbol{n}^T & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \boldsymbol{n} \end{array}\right] \left[\begin{array}{c} \boldsymbol{V}^\xi \\ \boldsymbol{I}^\xi \end{array}\right] = \mathbf{0}\,, \tag{30}$$

where $\xi = \alpha, \beta$, the matrix

$$\boldsymbol{n} = \begin{bmatrix} n_{11} & n_{12} & \ldots & n_{1M} \\ n_{21} & n_{22} & \ldots & n_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ n_{M1} & n_{M1} & \ldots & n_{MM} \end{bmatrix}, \tag{31}$$

summarizes the turns ratios of the M^2 transformers, and $\mathbf{1}$ is the $M \times M$ diagonal unit matrix.

The transformer network can be also represented by two matrix equations

$$\boldsymbol{V}^\beta = \boldsymbol{n}^T \boldsymbol{V}^\alpha\,, \tag{32a}$$

$$(\boldsymbol{I}^\alpha)^T + \boldsymbol{n}\boldsymbol{I}^\beta = \mathbf{0}\,, \tag{32b}$$

where $\boldsymbol{V}^\alpha$ is the input terminal voltages vector, $\boldsymbol{V}^\beta$ is the output terminal voltages vector, $\boldsymbol{I}^\alpha$ is the input terminal currents vector, $\boldsymbol{I}^\beta$ is the output terminal currents vector. The black box representation of the $2M$-port connection network defined by (32) is shown in the Figure 2.

A Gyrator [1, 53] satisfies (29) and should be considered as part of the connection circuit. Distributed gyrator surfaces have been discussed in [54].

The complete *transmission line segment circuit* (TLSC) can be divided into a connection circuit consisting only of interconnects and ideal transformers, open stubs, shorted stubs, reflection free terminations and sources exciting

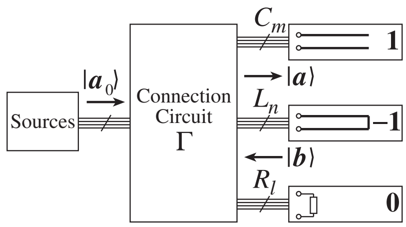

Figure 3: The TLSC Scheme

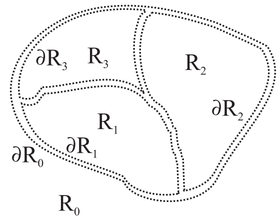

Figure 4: Partitioning of an electromagnetic structure.

the circuit. Figure 3 shows the a schematic representation of the TLSC. The connection circuit is described by the scattering matrix $\boldsymbol{\Gamma}$ with eigenvalues ± 1 [52]. All capacitors are represented by open stubs and all inductors by shorted stubs formed by transmission line segments of length $\Delta l = \tau/2c$ with equal characteristic impedance Z_0. This means no restriction, since we can include ideal transformers into the connection circuit to fulfill this condition. In z-domain the scattering matrix representing the stubs and the matched terminations exhibits the diagonal form

$$\tilde{\boldsymbol{S}}(\mathsf{z}) = \mathsf{z}^{-1}\boldsymbol{S} \quad \text{with} \quad \boldsymbol{S} = \operatorname{diag}[\mathbf{1}, -\mathbf{1}, \mathbf{0}] \,, \tag{33}$$

where the diagonal submatrices $\mathbf{1}$, $-\mathbf{1}$ and $\mathbf{0}$ of dimension $M \times M$, $N \times N$, and $L \times L$, respectively represent M open stubs, N shorted stubs, and L matched terminations. Using the notation of [1] for the vector $|\tilde{\boldsymbol{a}}\rangle$ summarizing all wave pulses incident in $\boldsymbol{S}$, and $|\tilde{\boldsymbol{b}}\rangle$ for the wave pulses scattered from $\boldsymbol{S}$, $|\tilde{\boldsymbol{a}}\rangle_s$ for the wave pulses exciting the structure, and $|\tilde{\boldsymbol{b}}\rangle_r$ for the port responses we obtain the TLSC scheme represented by

$$|\tilde{\boldsymbol{b}}\rangle = \mathsf{z}^{-1}\,\boldsymbol{S}\,|\tilde{\boldsymbol{a}}\rangle \,, \tag{34a}$$

$$|\tilde{\boldsymbol{a}}\rangle = [\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_s] \begin{bmatrix} |\tilde{\boldsymbol{b}}\rangle \\ |\tilde{\boldsymbol{a}}\rangle_s \end{bmatrix} , \tag{34b}$$

$$|\tilde{\boldsymbol{b}}\rangle_r = \boldsymbol{\Gamma}_Q |\tilde{\boldsymbol{a}}\rangle \,, \tag{34c}$$

where the connection matrix $\boldsymbol{\Gamma}$ has been split into the parts $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_{\boldsymbol{s}}$, and $\boldsymbol{\Gamma}_Q$ is the output matrix. This is formally identical with the transmission line matrix (TLM) scheme [1,4,27].

From (34a) to (34c) we obtain the response function

$$\tilde{\boldsymbol{H}} = \boldsymbol{\Gamma}_Q(\mathsf{z}\boldsymbol{I} - \boldsymbol{\Gamma}_0\,\boldsymbol{S})^{-1}\boldsymbol{\Gamma}_R = \sum_{k=1}^{\infty} \mathsf{z}^{-k}\boldsymbol{\Gamma}_Q\,(\boldsymbol{\Gamma}_0\boldsymbol{S})^{k-1}\,\boldsymbol{\Gamma}_R \tag{35}$$

relating the port response vector to the excitation vector:

$$|\tilde{a}\rangle_r = \tilde{\boldsymbol{H}}|\tilde{a}\rangle_s \,. \tag{36}$$

V. Lumped Element Equivalent Circuits for Reactance Multiports

The network oriented modeling assumes the realization of the obtained scattering matrix of the dynamic part of the model by the lumped elements multiport containing ideal transformer connection network and the lossy reactance one-ports, representing the evaluated set of poles and zeros. The scattering matrix of passive multiport microwave circuits is an analytic, real and unity bounded function of the complex frequency $s = \sigma + j\omega$ if

$$\boldsymbol{I} - \boldsymbol{S}(j\omega)\boldsymbol{S}^H \geq 0 \tag{37}$$

is a non-negative matrix for all ω in $-\infty < \omega < \infty$ [55].

Equivalently the impedance or admittance matrix $\boldsymbol{Y}(s)$ is an analytic and positive real matrix if its elements are rational functions of s and if

$$2\boldsymbol{R}(s) = \boldsymbol{Y}(s) + \boldsymbol{Y}^H \geq 0 \tag{38}$$

is positive definite for $\Re\{s\} \geq 0$.

The electromagnetic response of the subregion R_i (Figure 4) filled with a structure consisting of linear lossless reciprocal media may be characterized by the relation between tangential electric and magnetic fields on ∂R_l. Covering the closed boundary surface ∂R_l either by a perfect electric conductor or a perfect magnetic conductor makes the complete structure a lossless resonator. Its electromagnetic field can be expanded into orthogonal modal functions [56–59].

The relation of electric and magnetic field on the boundary surface ∂R_i can be expressed by the integral

$$\underline{\mathcal{E}}_t^l(\boldsymbol{x}, s) = \oint_{\partial R_l}' \mathcal{Z}^l(\boldsymbol{x}, \boldsymbol{x}', \omega) \wedge \underline{\mathcal{H}}_t^l(\boldsymbol{x}', \omega) \,, \tag{39}$$

where $\mathcal{Z}^l(\boldsymbol{x}, \boldsymbol{x}', \omega)$ is the dyadic Green's form. Its spectral representation is given by

$$\mathcal{Z}^l(\boldsymbol{x}, \boldsymbol{x}', \omega) = \frac{1}{\mathrm{j}\omega}\mathcal{L}_0^l(\boldsymbol{x}, \boldsymbol{x}') + \sum_p \frac{\mathcal{L}_p^l(\boldsymbol{x}, \boldsymbol{x}')}{\mathrm{j}(\omega - \omega_p)} \,, \tag{40}$$

where $\mathcal{L}_0^l(\boldsymbol{x}, \boldsymbol{x}')$ and the $\mathcal{L}_p^l(\boldsymbol{x}, \boldsymbol{x}')$ are frequency-independent dyadic double-one-forms [1].

Expanding the field as in (25) into basis functions the voltages and currents are related by an impedance matrix

$$\boldsymbol{Z}_\lambda(s) = \frac{1}{sC_0}\boldsymbol{B}_0 + \sum_{\lambda=1}^{N} \frac{1}{sC_\lambda}\,\frac{s^2}{s^2 + \omega_\lambda^2}\tilde{\boldsymbol{B}}_\lambda \,, \tag{41}$$

where s is the complex frequency, C_0 and C_λ are capacitances and $\boldsymbol{B}_0$ and $\boldsymbol{B}_\lambda$ are real frequency independent matrices of rank 1 given by

$$\boldsymbol{B}_\lambda = \begin{bmatrix} n_{\lambda 1}^2 & n_{\lambda 1}n_{\lambda 2} & \dots & n_{\lambda 1}n_{\lambda M} \\ n_{\lambda 2}n_{\lambda 1} & n_{\lambda 2}^2 & \dots & n_{\lambda 2}n_{\lambda M} \\ \vdots & \vdots & \ddots & \vdots \\ n_{\lambda M}n_{\lambda 1} & n_{\lambda M}n_{\lambda 2} & \dots & n_{\lambda M}^2 \end{bmatrix} . \tag{42}$$

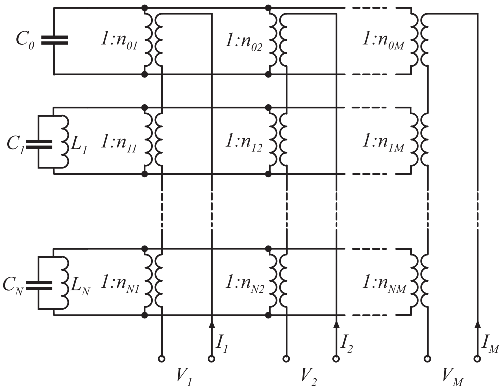

Figure 5: Canonical Foster Impedance Multiport.

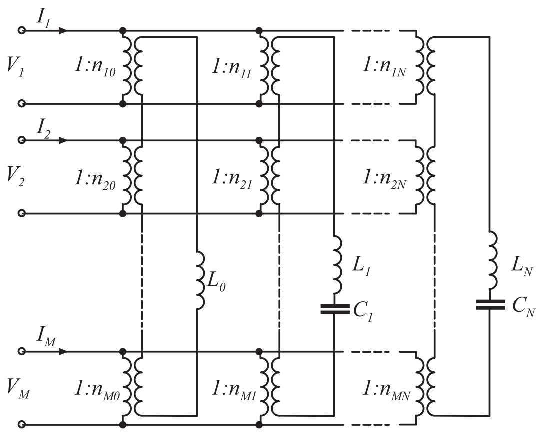

Figure 6: Canonical Foster Admittance Multiport.

This impedance matrix describes a canonical Foster impedance multiport as shown in Figure 5 [1, 4, 23].

The dual procedure of derivation yields the canonical Foster admittance multiport realization described by the admittance matrix

$$\boldsymbol{Y}_{\lambda}(s) = \frac{1}{sL_0}\,\boldsymbol{A}_0 + \sum_{\lambda=1}^{N} \frac{1}{sL_{\lambda}}\,\frac{s^2}{s^2+\omega_{\lambda}^2}\,\tilde{\boldsymbol{A}}_{\lambda} \qquad (43)$$

with the inductances L_0 and L_λ and the real frequency independent rank 1 matrices

$$\boldsymbol{A}_{\lambda} = \begin{bmatrix} n_{\lambda 1}^2 & n_{\lambda 1}n_{\lambda 2} & \dots & n_{\lambda 1}n_{\lambda M} \\ n_{\lambda 2}n_{\lambda 1} & n_{\lambda 2}^2 & \dots & n_{\lambda 2}n_{\lambda M} \\ \vdots & \vdots & \ddots & \vdots \\ n_{\lambda M}n_{\lambda 1} & n_{\lambda M}n_{\lambda 2} & \dots & n_{\lambda M}^2 \end{bmatrix}. \qquad (44)$$

The Foster impedance circuit is shown in Figure 6. The analytic computation of the circuit parameters of waveguide junctions and other distributed microwave circuits has been treated in literature [60–65]. It is also possible to find an equivalent Foster representation from admittance parameters calculated by numerical field analysis [66, 67]. This has been done by numerical Laplace transformation, and in a more efficient way by applying system identification methods which have been discussed in Section III..

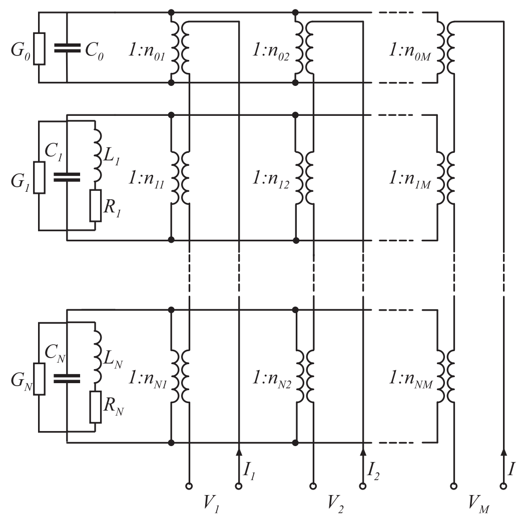

Figure 7: Canonical Foster Impedance Multiport for the lossy case.

VI. Lumped Element Equivalent Circuits for Linear Lossy Reciprocal Multiports

The transient response of linear lossy circuits can be described by a small number of pairs of conjugate complex frequency poles [38, 68]. Difficulties arise from the circumstance that lossy electromagnetic structures are described by partial differential equations exhibiting non-self adjoint operators. This usually does not allow to find orthogonal modal eigenfunctions. In case of weak losses we can seek the modal eigenfunctions of the lossless structure obtained by neglecting the losses and than compute the complex poles applying the power loss method [69].

A further difficulty arises to find an equivalent lumped element circuit realization for lossy structures. The canonical Foster realizations are only defined for reactance multiports. In the lossy case equivalent lumped element circuits can be found by including also resistors in the equivalent circuits, however, special care has to be taken to maintain stability of the equivalent circuits [66, 67]. Figures 7 and 8, respectively show the extension of the Foster multiport equivalent circuits according to Figures 5 and 6 for the lossy case. The circuit elements can be determined from poles and residues of the impedance and admittance functions. However, in the lossy case the computation may yield negative values of the lumped element circuit parameters. Even if the admittance matrix is fulfilling the realizability criteria it cannot be in general realized by a network containing only positive resistors, inductors and capacitors. The problems of synthesis of RLC impedance functions is discussed in [3, p. 330]. This is a serious drawback of the Foster-like synthesis of lossy multiports. It can be explained by the fact that the partial-fraction expansion method for the breakdown of a given impedance into a sum of simple components, represented by equation (10), cannot guarantee the realizability of components. This can be explained by a possible cancellation of positive and neg-

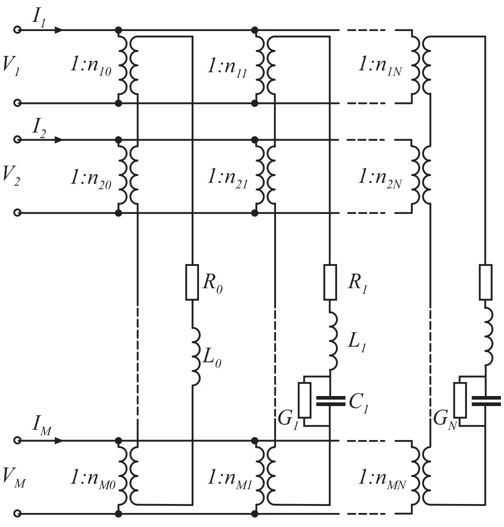

Figure 8: Canonical Foster Admittance Multiport for the lossy case.

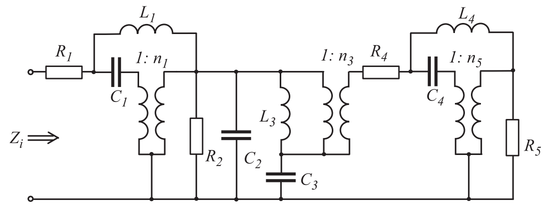

Figure 9: The equivalent circuit of the driving-point admittance.

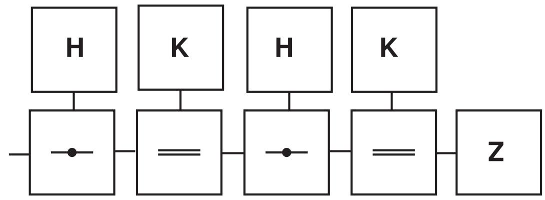

Figure 10: Brune n-port iteration step.

ative real-part values of the components in (10) for obtaining the positive real part of the whole admittance.

In principle we can proceed in the synthesis of lumped element equivalent circuits for linear lossy reciprocal multiports in a similar way as in the case of lossless multiports. We start with the matrix of positive rational functions describing the impedance or admittance matrix of the multiport under consideration and try to decompose this matrix into a sum of two or more positive real function matrices, for one or more of which we can give circuit realizations. The remaining parts we can invert and by this way changing between impedance and admittance representations. A sum of impedance matrices always represents a series connection of multiports whereas a sum of admittance matrices represents parallel circuited multiports.

If the Foster-like procedure fails for the network synthesis by positive lumped components, the more complicated but more versatile Brune's synthesis procedure [70] can be applied. It realizes an admittance or impedance matrix of order N as a lossless two-port terminated by an admittance or impedance of the order $N-2$. It also consists of a resistance extraction producing a zero of the real part of the initial admittance at some point $s = j\omega_0$ of the imaginary axis. The lossless two-port of the network needs to have a parallel or series resonance at this frequency point. The example of the Brune's synthesis realization for the one of the driving-point impedances of the four-port microwave structure can be found in [71]. Figure 9 gives an example of Brune's realization of the driving-point impedance of a one-port. It can be seen that a passive equivalent circuit contains not only R, L, C elements but ideal transformers.

The original Brune's method was extended in [72] to the case of realizable and possibly nonreciprocal multiports. The implementation of the proposed extension assumes some new kinds of elements: ideal transformers with complex turn ratios, real ideal gyrators and imaginary resistors. In the following we only will give a short outline of the generalized Brune process for the synthesis of general linear reciprocal lossy n-ports. The procedure comprises:

1. Extraction of a series resistance multiport $\boldsymbol{R}$,
2. Extraction of a reactive shunt multport $\boldsymbol{H}_i$,
3. Extraction of a series resistance multiport $(1/(s - s_i))\boldsymbol{K}_i$,

The operations 1) and 2) do not enter the degree of the multiport while operation 3) decreases the degree by the rank of $\boldsymbol{K}_i$ so that the process stops after a finite number of iterations. Figure 10 shows schematically this Brune n-port realization procedure. The adaptors for parallel and series connection of the multiports are shown in Figure 11.

VII. Radiating Structures

Consider the complete electromagnetic structure being embedded in a sphere of radius r_0 as shown in Figure 12. The wave impedances $Z_{mn}^{+\text{TM}}(s)$ and $Z_{mn}^{+\text{TE}}(s)$ for the outward propagating TM and TE modes in free space $r \geq r_0$ outside the sphere are [1, 4, 73]

$$\frac{Z_{mn}^{+\text{TM}}(s)}{Z_{F0}} = \frac{Z_{F0}}{Z_{mn}^{+\text{TE}}(s)} = j\frac{\frac{d}{dr}\left[rh_n^{(2)}\left(-\frac{jsr}{c_0}\right)\right]}{rh_n^{(2)}\left(-\frac{jsr}{c_0}\right)} \equiv z(s)\,, \quad (45)$$

where c_0 is the free space speed of light and $h_n^{(j)}(kr)$ are the spherical Hankel functions. The normalized impedance $z(r)$ can be expressed by the continued fraction expansion

$$z(s) = \frac{n}{s\tau_r} + \cfrac{1}{\frac{2n-1}{s\tau_r} + \cfrac{1}{\frac{2n-3}{s\tau_r} + \cfrac{}{\ddots \; \cfrac{1}{\frac{1}{s\tau_r}+1}}}}\,, \quad (46)$$

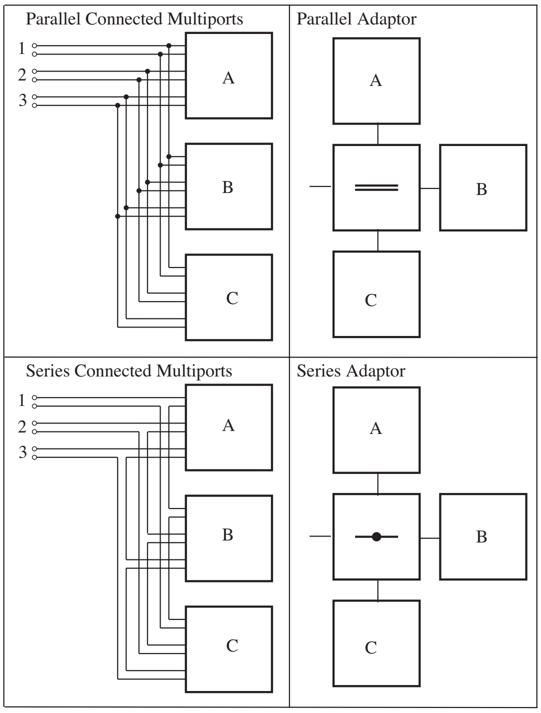

Figure 11: Adaptors for parallel and series connection of the multiports.

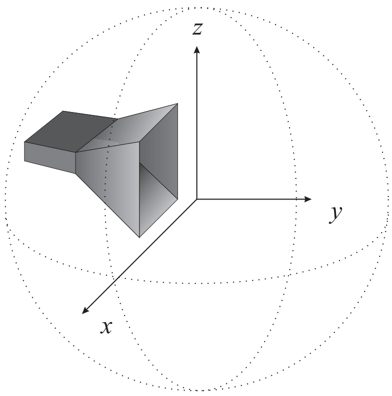

Figure 12: Horn antenna.

where $\tau_r = r/c_0$. The corresponding lumped element equivalent circuits representing $Z_{mn}^{+\text{TM}}(s)$ and $Z_{mn}^{+\text{TE}}(s)$ are shown in Figure 13. We can establish lumped element circuit models of complex electromagnetic structures by representing substructures by Foster equivalent circuits, their connection by connection circuits and the radiation modes by Cauer equivalent circuits. The block diagram of such a model is shown in Figure 14.

VIII. Discrete-Time State Equation Representation

To discretize a variable $x(t)$ in time we take samples at integer multiples $n\tau$ of a chosen sampling time interval τ.

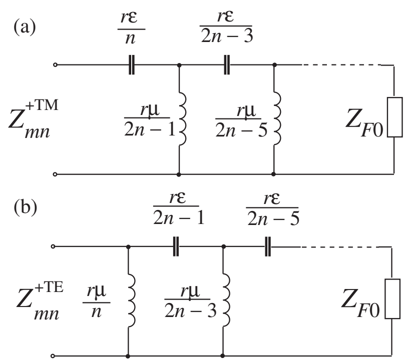

Figure 13: Equivalent Circuits of Spherical Waves.

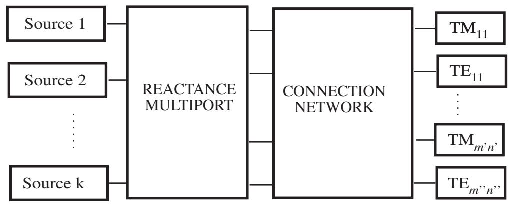

Figure 14: Equivalent Circuit of the Radiating Electromagnetic Structure.

The z-transform

$$\tilde{X}(\mathsf{z}) \equiv \mathsf{Z}\{x(t)\} = \sum_{k=0}^{\infty} x_k \mathsf{z}^{-k} \text{ with } x_k \equiv x(k\tau) \tag{47}$$

defines the sequence of sampled values of $x(t)$ [33]. Time discretization of the state equations describing the time continuous system can be performed by replacing the differential quotient by the difference quotient, applying in (2a) and (2b) the following transformation

$$s \quad \rightarrow \quad \overset{\text{Richards transform}}{\frac{2}{\tau}\frac{e^{s\tau}-1}{e^{s\tau}+1}} \qquad \overset{\mathsf{z}\text{-transform}}{\frac{2}{\tau}\frac{\mathsf{z}-1}{\mathsf{z}+1}} \tag{48}$$

to the complex frequency s. This corresponds to the Richards transformation [28] in frequency domain or to the z-domain representation of the time discretized system [33]. Although both representations are formally equivalent they are obtained in a different way. The z-transform representation is obtained in a formal way by replacing differential quotients by difference quotients, whereas in the case of the Richards transform inductors and capacitors are replaced according to Figure 15 by short and open stub lines, all of them exhibit an equal forth-and-back delay time τ equal to the time discretization interval. The circuit obtained by this way is still time-continuous. When the circuit is excited by delta pulses with spacing τ also the output signal will be a sequence of delta pulses with spacing τ since all delay times occurring in this circuit are integer multiples of the *fundamental* delay time τ.

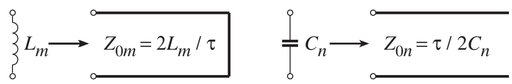

Figure 15: The Richards Transformation.

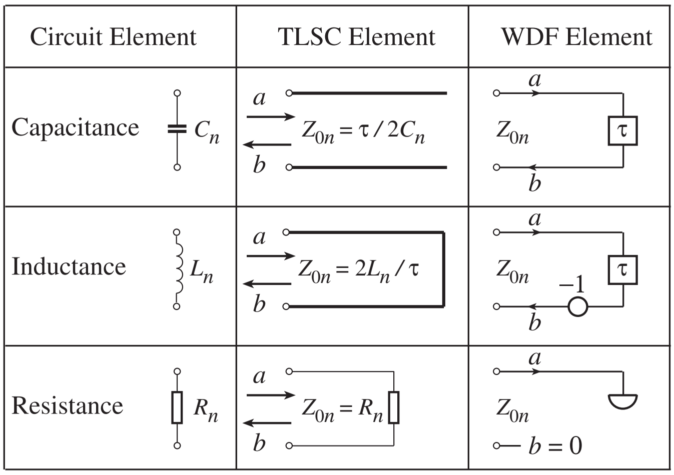

Figure 16: One-Port Elements and their TLS- and WDF Representations.

IX. Wave Digital Filter Methods

The wave digital filter (WDF) concept introduced by Alfred Fettweis in 1971 [31, 32] has proven to be a powerful tool for time-discrete wave-based modeling of physical systems [74, 75]. The application of WDF structures for electromagnetic field simulation already has been discussed in detail by S. Bilbao [76, 77].

There is a one-to-one correspondence between a TLSC model as introduced in Section II. and a WDF model. The difference, however, is that the TLSC model is an equivalent circuit model based on transmission line segments, whereas the WDF model deals with signal flow graphs and is closer to the software implementation of the model. The

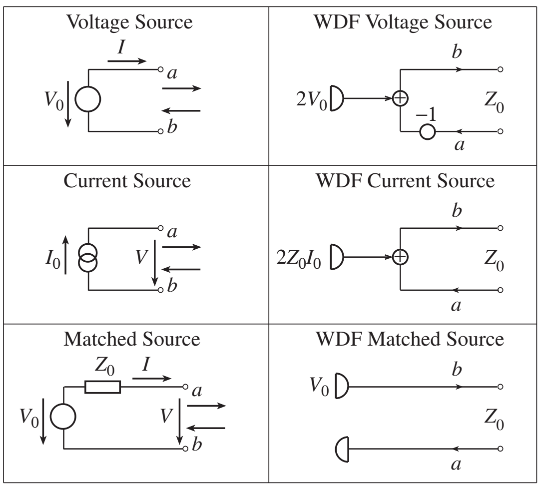

Figure 17: Sources and their WDF Representations.

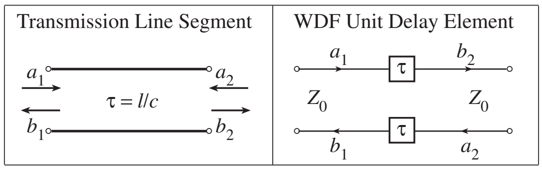

Figure 18: Transmission Line Segment and WDF Unit Element.

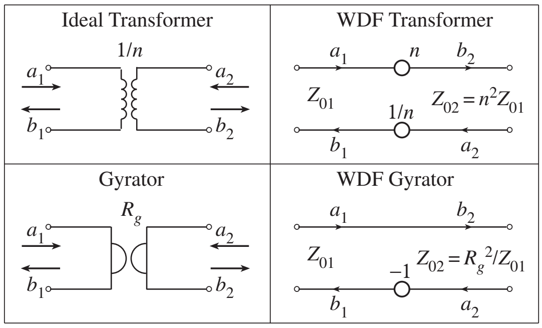

Figure 19: WDF Representation of Connection Two-Ports.

inherent properties of WDFs like stability, passivity and reciprocity guarantee corresponding properties of WDF models [78, 79].

In the following we only can give a brief outline of the application of WDF methods for electromagnetic field modeling. To implement a TLSC model according to Figure 3 in WDF we can follow the TLSC topology. The WDF elements representing C, L and R are summarized in Figure 16. Figure 17 shows the representation of sources as WDF elements. The transmission line segment with unit delay time $\tau = l/c$ is represented by the WDF unit delay element shown in Figure 18.

The WDF elements representing ideal transformer and gyrator two-ports are shown in Figure 19. In WDF adaptors are used as the connection elements. A. Fettweis has introduced elementary parallel and series adaptors representing parallel and series connections of ports [31, 32]. Figure 20 shows four-port parallel and series adaptors representing parallel and series connections of ports. For the lumped element equivalent circuit of the TM_{mn} radiating mode shown in Figure 13a the WDF representation is given in Figure 21.

The 2D–TLM network, established by a mesh of transmission line segments with equal propagation time τ connected via 4–port parallel interconnects [80,81] can be represented by a WDF–network depicted in Figure 22 [77]. The WDF implementation of 3D–TLM can be done by arranging 12–port adaptors in a three-dimensional mesh. These adaptors realize the scattering matrix given in [1, p. 616, eq. (14.51)]. Every 12–port adaptor is connected via two WDF unit delay elements with each of its six neighbors.

In [82, 83] WDF techniques have been used to combine the 3D–TLM with the Cauer representation of the radiating modes. In WDF structures macro–adaptors can be

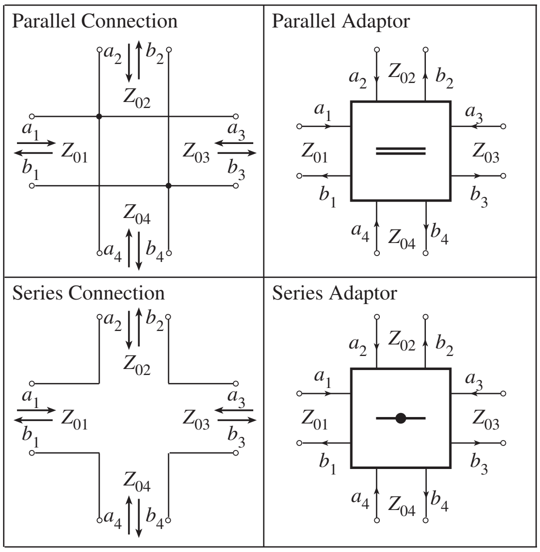

Figure 20: WDF Adaptors.

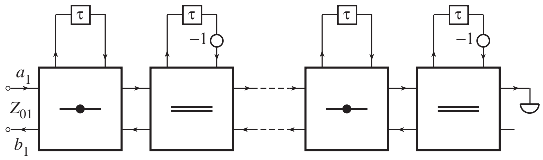

Figure 21: WDF Scheme of a Cauer Equivalent Circuit.

introduced by arbitrarily interconnecting together elementary adaptors, transformers and gyrators [74, 75, 84]. Concentrating all adaptors into a single macro-adaptor yields the WDF scheme shown in Figure 23 which is equivalent to the TLSC scheme presented in Figure 3.

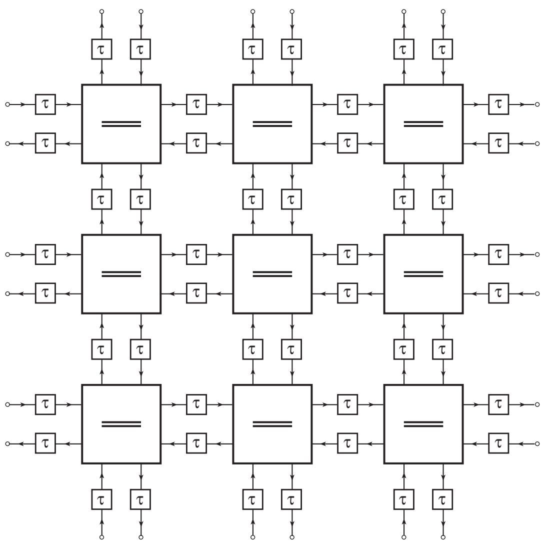

Figure 22: WDF Representation of the 2D-TLM Scheme.

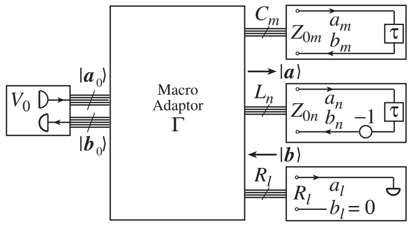

Figure 23: The WDF Scheme

X. Conclusion

Analytic and numerical methods and examples of their application have been discussed. Network methods are applicable in connection with the main analytic and numerical methods for electromagnetic field modeling and provide a large variety of tools for an efficient modeling of complex electromagnetic structures.

XI. Acknowledgments

This work was supported by the Deutsche Forschungsgemeinschaft.

References

[1] P. Russer, *Electromagnetics, Microwave Circuit and Antenna Design for Communications Engineering*, 2nd ed. Boston: Artech House, 2006.

[2] L. O. Chua, C. A. Desoer, and E. S. Kuh, *Linear and Nonlinear Circuits.* New York: Mc Graw Hill, 1987.

[3] E. A. Guillemin, *Synthesis of Passive Networks.* New York: Wiley, 1957.

[4] L. B. Felsen, M. Mongiardo, and P. Russer, *Electromagnetic Field Computation by Network Methods.* Berlin, Germany: Springer, Mar. 2009.

[5] L. Ljung, *System Identification. Theory for the User.* Upper Saddle River, NJ: Prentice Hall PTR, 1999.

[6] S. L. Marple, *Digital Spectral Analysis.* Englewood Cliffs, NJ: Prentice Hall, 1997.

[7] J. L. Dubard, D. Pompei, J. L. Roux, and A. Papiernik, “Characterization of microstrip antennas using the TLM simulation associated with a Prony-Pisarenko method,” *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 3, no. 4, pp. 269–285, 1990.

[8] J. D. Wills, “Spectral estimation for the transmission line matrix method,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 38, no. 4, pp. 448–451, 1990.

[9] W. L. Ko and R. Mittra, “A combination of FD-TD and Prony’s methods for analyzing microwaveintegrated circuits,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 39, no. 12, pp. 2176–2181, 1991.

[10] Zhiquiang Bi, Ying Shen, Keli Wu, and John Litva, “Enhancing finite-difference time-domain analysis of dielectricresonators using spectrum estimation techniques,” in *IEEE MTT-S International Microwave Symposium Digest, 1992.*, 1992, pp. 869–872.

[11] W. Kuempel and I. Wolff, “Digital signal processing of time domain field simulation results using the system identification method,” in *Microwave Symposium Digest, 1992., IEEE MTT-S International*, 1992, pp. 793–796 vol.2.

[12] T. Huang, B. Houshmand, and T. Itoh, “Application of system identification technique to FDTD and FDTD diakoptics method,” in *European Microwave Conference, 1993. 23rd*, 1993, pp. 278–280.

[13] C. Eswarappa and W. Hoefer, "Autoregressive (AR) and autoregressive moving average (ARMA) spectral estimation techniques for faster TLM analysis of microwave structures," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 42, no. 12, pp. 2407–2411, 1994.

[14] P. Russer, M. Righi, C. Eswarappa, and W. Hoefer, "Lumped element equivalent circuit parameter extraction of distributed microwave circuits via tlm simulation," in *Microwave Symposium Digest, 1994., IEEE MTT-S International*, 1994, pp. 887–890 vol.2.

[15] M. Righi, C. Eswarappa, W. Hoefer, and P. Russer, "An alternative way of computing S–parameters via impulsive TLM analysis without using absorbing boundary conditions," *1995 Int. Microwave Symposium Digest, Orlando*, pp. 1203–1206, May 1995.

[16] C. Eswarappa and W. J. R. Hoefer, "Fast s–parameter computation of a microstrip interdigital filter using TLM, Prony's and digital filtering techniques," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 9, no. 3, pp. 237–248, 1996.

[17] V. Chtchekatourov, F. Coccetti, and P. Russer, "Full-wave analysis and model-based parameter estimation approaches for y-matrix computation of microwave distributed rf circuits," in *Microwave Symposium Digest, 2001 IEEE MTT-S International*, vol. 2, 2001, pp. 1037–1040 vol.2.

[18] F. Coccetti and P. Russer, "A Prony's model based signal prediction (PSP) algorithm for systematic extraction of Y-parameters from TD transient responses of electromagnetic structures," in *Microwaves, Radar and Wireless Communications, 2004. MIKON-2004. 15th International Conference on*, vol. 3, 2004, pp. 791–794 Vol.3.

[19] Y. Kuznetsov, A. Baev, F. Coccetti, and P. Russer, "The ultra wideband transfer function representation of complex three-dimensional electromagnetic structures," in *34th European Microwave Conference, Amsterdam, The Netherlands, 11.-15.10.2004*, Oct. 2004, pp. 455–458.

[20] U. Siart, K. Fichtner, Y. Kuznetsov, A. Baev, and P. Russer, "TLM modeling and system identification of distributed microwave circuits and antennas," in *Electromagnetics in Advanced Applications, 2007. ICEAA 2007. International Conference on*, 2007, pp. 352–355.

[21] L. B. Felsen, M. Mongiardo, and P. Russer, "Electromagnetic field representations and computations in complex structures I: Complexity architecture and generalized network formulation," *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields*, vol. 15, pp. 93–107, 2002. [Online]. Available: http://dx.doi.org/10.1002/jnm.433

[22] ——, "Electromagnetic field representations and computations in complex structures II: Alternative Green's functions," *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields*, vol. 15, pp. 109–125, 2002. [Online]. Available: http://dx.doi.org/10.1002/jnm.434

[23] P. Russer, M. Mongiardo, and L. B. Felsen, "Electromagnetic field representations and computations in complex structures III: Network representations of the connection and subdomain circuits," *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields*, vol. 15, pp. 127–145, 2002. [Online]. Available: http://dx.doi.org/10.1002/jnm.435

[24] P. Russer and A. C. Cangellaris, "Network–oriented modeling, complexity reduction and system identification techniques for electromagnetic systems," *Proceedings of the 4th International Workshop on Computational Electromagnetics in the Time–Domain: TLM/FDTD and Related Techniques, 17–19 September 2001 Nottingham*, pp. 105–122, Sep. 2001.

[25] P. Russer, "Electromagnetic field computation by network methods," in *Proceedings of the 25th Annual Review of Progress in Applied Computational Electromagnetics ACES*, Monterey, California, USA, Mar. 8 – 12 2009.

[26] P. Russer, D. Bajon, S. Wane, and N. Fichtner, "Overview and status of numerical electromagnetic field simulation methods applied to integrated circuits," in *Silicon Monolithic Integrated Circuits in RF Systems, 2009. SiRF 2009. IEEE Topical Meeting on*, Jan. 2009, pp. 100–107.

[27] P. Russer, "Overview over network methods applied to electromagnetic field computation," in *ICEAA2009, Proc. Int. Conf. on Electromagnetics in Advanced Applications, Sept. 14-18, 2009, Torino*, Torino, Italy, Sept. 2009.

[28] P. I. Richards, "Resistor-transmission-line circuits," *Proceedings of the IRE*, vol. 36, no. 2, pp. 217–220, 1948.

[29] W. Hoefer, "The transmission line matrix method-theory and applications," *IEEE Trans. Microwave Theory Techn.*, vol. 33, no. 10, pp. 882–893, Oct. 1985. [Online]. Available: http://dx.doi.org/10.1109/TMTT.1985.1133146

[30] C. Christopoulos, *The Transmission-Line Modeling Method TLM.* New York, NY: IEEE Press, 1995.

[31] A. Fettweis, "Digital filters structures related to classical filter networks," *AEU: Archive fur Elektronik und Ubertragungstechnik*, vol. 25, no. 2, pp. 79–89, 1971.

[32] ——, "Wave digital filters: Theory and practice," *Proceedings of the IEEE*, vol. 74, no. 2, pp. 270–327, 1986.

[33] J. M. Smith, *Mathematical Modeling and Digital Simulation for Engineers and Scientists*, 2nd ed. New York: John Wiley & Sons, 1987.

[34] R. E. Kalman, "Mathematical description of linear dynamical systems," *Journal of the Society for Industrial and Applied Mathematics, Series A: Control*, vol. 1, no. 2, pp. 152–192, 1963.

[35] E. G. Gilbert, "Controllability and observability in multivariable control systems," *Journal of the Society for Industrial and Applied Mathematics, Series A: Control*, vol. 1, no. 2, pp. 128–151, 1963.

[36] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting," *Power Delivery, IEEE Transactions on*, vol. 14, no. 3, pp. 1052 –1061, July 1999.

[37] B. Gustavsen, "Improving the pole relocating properties of vector fitting," *Power Delivery, IEEE Transactions on*, vol. 21, no. 3, pp. 1587 –1592, July 2006.

[38] C. E. Baum, "Emerging technology for transient and broad-band analysis and synthesis of antennas and scatterers," *Proceedings of the IEEE*, vol. 64, no. 11, pp. 1598–1616, 1976.

[39] C. Baum, "The singularity expansion method in transient electro–magnetic fields," in *Transient Electromagnetic Fields*, L. B. Felsen, Ed. Berlin: Springer, 1976.

[40] E. Miller and T. Sarkar, "An introduction to the use of model–based parameter estimation in electromagnetics," *Review of Radio Science 1996–1999, ed. by R. Stone, Oxford University Press*, pp. 139–174, Aug. 1999.

[41] N. Fichtner, U. Siart, Y. Kuznetsov, A. Baev, and P. Russer, *Bandwidth Optimization using Transmission Line Matrix Modeling and System Identification*, ser. Springer Proceedings in Physics. Berlin: Springer, 2008, no. 121, pp. 147–171.

[42] Y. Hua and T. Sarkar, "Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 38, no. 5, pp. 814 –824, may 1990.

[43] M. H. Hayes, *Statistical Digital Signal Processing and Modelling.* New York: John Wiley & Sons, Inc., 1996.

[44] Y. Kuznetsov, A. Baev, T. Shevgunov, M. Zedler, and P. Russer, "Transfer function representation of passive electromagnetic structures," in *Microwave Symposium Digest, 2005 IEEE MTT-S International*, 12-17 2005, p. 4 pp.

[45] Y. Kuznetsov, A. Baev, T. Shevgunov, P. Lorenz, and P. Russer, "Application of the stability criterion to the passive electromagnetic structures modeling," in *Microwave Conference, 2006. 36th European*, 2006, pp. 13–16.

[46] D. Lukashevich, F. Coccetti, and P. Russer, "System identification and model order reduction for tlm analysis," *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields*, vol. 20, no. 1–2, pp. 75–92, Jan. 2007.

[47] P. Penfield, R. Spence, and S. Duinker, *Tellegen's theorem and electrical networks.* Cambridge, Massachusetts: MIT Press, 1970.

[48] B. D. H. Tellegen, "A general network theorem with applications," *Philips Research Reports*, vol. 7, pp. 259–269, 1952.

[49] K. C. Gupta, R. Garg, and R. Chadha, *Computer-Aided Design of Microwave Circuits.* Boston: Artech House, 1981.

[50] J. A. Dobrowolski, *Introduction to Computer Methods for Microwave Circuit Analysis and Design.* Boston: Artech House, 1991.

[51] P. Russer, *Electromagnetics, Microwave Circuit and Antenna Design for Communications Engineering*, 2nd ed. Boston: Artech House, 2006.

[52] ——, "Noise analysis of linear microwave circuits with general topology," *The review of radio science 1993–1996, Oxford, England,*, pp. 887–890, 1996.

[53] B. D. H. Tellegen, "The gyrator, a new electric network element," *Philips Res. Rep.*, vol. 3, pp. 81–101, 1948.

[54] P. Russer, "Electromagnetic properties and realisability of gyrator surfaces," in *Electromagnetics in Advanced Applications, 2007. ICEAA 2007. International Conference on*, 2007, pp. 320–323.

[55] D. Youla, L. Castriota, and H. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," *Circuit Theory, IRE Transactions on*, vol. 6, no. 1, pp. 102 – 124, mar 1959.

[56] J. C. Slater, "Microwave electronics," *Rev. Mod. Phys.*, vol. 18, no. 4, pp. 441–512, Oct 1946.

[57] T. Teichmann, "Completeness relations for Loss-Free microwave junctions," *J. Appl. Physics*, vol. 23, no. 7, pp. 701–710, Jul. 1952.

[58] S. A. Schelkunoff, "On representation of electromagnetic fields in cavities in terms of natural modes of oscillation," *J. Appl. Physics*, vol. 26, no. 10, pp. 1231–1234, Oct. 1955.

[59] K. Kurokawa, "The expansions of electromagnetic fields in cavities," *IEEE Trans. Microwave Theory Techn.*, vol. 6, no. 2, pp. 178– 187, Apr. 1958.

[60] T. E. Rozzi and W. F. G. Mecklenbrauker, "Wide-Band network modeling of interacting inductive irises and steps," *IEEE Trans. Microwave Theory Techn.*, vol. 23, no. 2, pp. 235–245, 1975.

[61] P. Arcioni, M. Bressan, G. Conciauro, and L. Perregrini, "Wideband modeling of arbitrarily shaped e-plane waveguide components by the "boundary integral-resonant mode expansion method"," *IEEE Trans. Microwave Theory Techn.*, vol. 44, no. 11, pp. 2083–2092, 1996.

[62] G. Conciauro, P. Arcioni, M. Bressan, and L. Perregrini, "Wideband modeling of arbitrarily shaped h-plane waveguide components by the "boundary integral-resonant mode expansion method"," *IEEE Trans. Microwave Theory Techn.*, vol. 44, no. 7, pp. 1057–1066, 1996.

[63] P. Arcioni and G. Conciauro, "Combination of generalized admittance matrices in the form of pole expansions," *IEEE Trans. Microwave Theory Techn.*, vol. 47, no. 10, pp. 1990–1996, 1999.

[64] M. Guglielmi, F. Montauti, L. Pellegrini, and P. Arcioni, "Implementing transmission zeros in inductive-window bandpass filters," *IEEE Trans. Microwave Theory Techn.*, vol. 43, no. 8, pp. 1911–1915, 1995.

[65] G. Conciauro, M. Guglielmi, and R. Sorrentino, *Advanced Modal Analysis.* New York: John Wiley & Sons, 2000.

[66] T. Mangold and P. Russer, "Generation of lumped element equivalent circuits for distributed multiport structures via TLM simulation," *Second International Workshop on Transmission Line Matrix (TLM) Modeling – Theory and Applications, Munich, Germany, 29.-31.10.1997, pp. 236–245, Oct. 1997.*, pp. 236–245, Oct. 1997.

[67] ——, "Full-wave modeling and automatic equivalent-circuit generation of millimeter-wave planar and multilayer structures," *IEEE Trans. Microwave Theory Techn.*, vol. 47, pp. 851–858, Jun. 1999.

[68] C. E. Baum, *On the singularity expansion method for the solution of electromagnetic interaction problems.* Storming Media, 1971.

[69] J. Gustincic, "A general power loss method for attenuation of cavities and waveguides," *IEEE Trans. Microwave Theory Techn.*, vol. 11, no. 1, pp. 83–87, 1963.

[70] O. Brune, "Synthesis of a finite two terminal network whose driving-point impedance is a prescribed function of frequency," *J. Math. and Phys.*, vol. 10, no. 3, pp. 191–236, 1931.

[71] Y. Kuznetsov, A. Baev, T. Shevgunov, U. Siart, H. Yordanov, and P. Russen, "Generation of network models for planar microwave circuits by system identification methods," in *Electromagnetics in Advanced Applications, 2009. ICEAA '09. International Conference on*, 14-18 2009, pp. 966 –969.

[72] V. Belevitch, "On the brune process for n-ports," *Circuit Theory, IRE Transactions on*, vol. 7, no. 3, pp. 280 – 296, sep 1960.

[73] R. F. Harrington, *Time Harmonic Electromagnetic Fields.* New York: McGraw-Hill, 1961.

[74] F. Pedersini, A. Sarti, and S. Tubaro, "Object-based sound synthesis for virtual environments-using musicalacoustics," *IEEE Signal Processing Magazine*, vol. 17, no. 6, pp. 37–51, 2000.

[75] R. Rabenstein, S. Petrausch, A. Sarti, G. D. Sanctis, C. Erkut, and M. Karjalainen, "Block-based physical modeling for digital sound synthesis," *IEEE Signal Processing Magazine*, vol. 24, no. 2, p. 42, 2007.

[76] S. Bilbao and J. O. S. Smith, "Finite difference schemes and digital waveguide networks for the wave equation: stability, passivity, and numerical dispersion," *IEEE Transactions on Speech and Audio Processing*, vol. 11, no. 3, pp. 255–266, 2003.

[77] S. Bilbao, *Wave and Scattering Methods for Numerical Simulation.* Hoboken, New Jersey: Wiley, 2004.

[78] A. Fettweis, "Pseudo-passivity, sensitivity, and stability of wave digital filters," *IEEE Transactions on Circuit Theory*, vol. 19, no. 6, pp. 668–673, 1972.

[79] ——, "Reciprocity, inter-reciprocity, and transposition in wave digital filters," *International Journal of Circuit Theory and Applications*, vol. 1, no. 4, 1973.

[80] M. Krumpholz, P. Russer, Q. Zhang, and J. Hoefer, "Field-theoretic foundation of two-dimensional TLM based on a rectangular mesh," in *Microwave Symposium Digest, 1994., IEEE MTT-S International*, 1994, pp. 333–336 vol.1.

[81] M. Krumpholz and P. Russer, "Two-dimensional FDTD and TLM," *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields*, vol. 7, pp. 141–153, Apr. 1994.

[82] P. Lorenz and P. Russer, "Discrete and modal source modeling with connection networks for the transmission line matrix (TLM) method," in *Microwave Symposium, 2007. IEEE/MTT-S International*, 2007, pp. 1975–1978.

[83] ——, "Connection subnetworks for the transmission line matrix (TLM) method," in *Time-Domain Methods in Modern Engineering Electromagnetics*, ser. Springer Proceedings in Physics, P. Russer and U. Siart, Eds. Berlin: Springer, 2008, vol. 121, pp. 263–281.

[84] A. Sarti and G. D. Sanctis, "Structural equivalences in wave digital systems based on dynamic scattering cells," in *14th European Signal Processing Conference (EUSIPCO-06)*, 2006.