Corona Discharge in Microwave Devices: A Comparison of Ionization Rate Models

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Abstract – **In this paper we compare the predicted Corona breakdown power threshold obtained by different ionization rate models for an infinite parallel-plate waveguide. We point out the lack of a unanimous model of the ionization rate for RF fields and study the differences in the current models.**

Keywords – **Corona breakdown, ionization rate, microwave discharge, weakly ionized plasma.**

I. INTRODUCTION

The study of low-pressure electrical discharges caught the attention of the space engineering community several years ago with the advent of the space exploration and the subsequent development of new onboard devices for satellite applications [1-3]. The exploration of planets and their moons, as well as the TTC (Telemetry, Tracking and Control) phase of spacecrafts launching, and the re-entry of vehicles in the earth atmosphere are subjected to the risk of plasma breakdown occurring either on the aperture of slot antennas or inside the satellite payloads.

The molecules of the planetary atmospheric gas remaining within the gaps of microwave devices may collide with the environmental space electrons. The electrons can either be constrained to the magnetic field lines of a planet, as it happens in the Van Allen belts around the earth, or come from external ionization processes as, for instance, the interaction of the galactic cosmic rays with the matter of the spacecraft. These electrons are accelerated by the RF electric field inside the microwave devices and can collide with the gas molecules, releasing new electrons from the external shells of them.

The electric field produces a collision rate between electrons and neutral molecules that results in an ionization rate provoking a local electron population growth. At the same time, the electrons tend to move away from the zones where there is a big concentration of electrons to level out the electron population density and attain the chemical equilibrium, process that is called diffusion. Above a certain value of RF power the local growth is produced fast enough to forbid the electrons to balance the local increase of the electron density. This lost of local equilibrium between ionization and diffusion initiates a bigger ionization process, inducing a glowing emission. The glowing is the result of the radiation of the free electrons accelerated by the RF field. This kind of discharge is usually called Corona Discharge and one fundamental property is the dependence of the breakdown power threshold on the gas pressure.

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The amount of collisions between electrons and neutral molecules increases with the pressure, but as the pressure increases, the flight time of the electrons between two collisions is reduced. This does not let the electrons reach the kinetic energy (i.e. the speed) necessary to ionize the gas molecules, so that the ionization rate decreases with increasing pressure. On the other hand, when the pressure is too low the electrons travel forth and back several times, under the action of the electric field, before impacting against a neutral molecule. This also decreases the collision rate, letting diffusion loss dominate the process, with the subsequent need of a higher microwave field to initiate the breakdown.

The pressure under which the ionizing process is optimum is called *critical pressure* and it corresponds to that one for which the electrons collide against the neutral molecules with a collision frequency that exactly equals the RF frequency. In this case electrons that have struck a molecule when the electric field was in a maximum, will reach the next molecule when the electric field will be maximal again, making the ionization process very efficient. For this pressure, the breakdown electric field threshold reaches a minimum, since the ionization rate will be maximal. This is the critical breakdown electric field.

Nowadays, an increasing interest from the satellite telecommunication industry as well as from planetary exploratory missions demands a technology for space applications that need microwave components dealing properly with larger bandwidths, higher component integration and better power handling capabilities. These requirements lead to higher electric field densities which increase the risk of electrical breakdown to occur within microwave devices on board of satellites.

The consequences of a Corona discharge are extremely harmful within a microwave device since the electron cloud formed by the breakdown onset reflects the RF signal, what can lead to the destruction of the power source. The discharge can severely increase the temperature, due to the high current density produced, resulting in the complete destruction of the device, with the subsequent danger of missing the complete mission if information between satellite and earth is lost.

In this work we compare the predicted Corona power threshold at the example of an infinitely long parallel-plate waveguide obtained by several ionization rate models that have been published so far and used in different papers to predict the breakdown power threshold for different geometries.

II. THEORY

A. Corona equation

We can consider the remaining gas within the microwave devices on spacecrafts as a system of neutral particles and a relatively small number of electrons before the build-up of a plasma due to the discharge. Some authors [4] call this system weakly ionized plasma which can be described as an electron gas coexisting in equilibrium with a neutral molecular gas provided the following condition is verified [4]:

$$e^6 N_e / T^3 \ll 1, \quad (1)$$

where e, N_e and T are the electron charge, the electron density and the electron temperature, respectively. The electron temperature is proportional to the mean kinetic energy of the electrons. Thus, a plasma whose parameters satisfy Eq. (1) is also known as *ideal plasma*, since the interaction between electrons and gas molecules is supposed to be weak enough to consider the electron population in thermal equilibrium with the gas molecule population. Under this assumption the electron temperature can be considered equal to the one of the gas molecules. This point is important since this has been assumed in the work of the different authors whose ionization rate models are going to be compared along this paper.

If we define the electron population density inside the gas as $n(\vec{r},t)$ we can recall the continuity equation for the electron population assuming an arbitrary volume within the gas cloud. Therefore, the change of the electron population with respect to the time inside this volume is given by the flow of electrons leaving this zone in order to preserve the conservation of charge principle; this produces a current density called $\vec{J}$. However some electrons are released from the shells of the molecules because of the ionization. This increases the *change* of the electron population density with respect to the time $(\partial n/\partial t)$, by an amount equal to the ionization rate v_i times the amount of electrons per unit volume enclosed in the imaginary volume $(\nu_i n)$. Therefore, the equation we have to deal with can be written as follows:

$$\frac{\partial n}{\partial t} = \nu_i n - \vec{\nabla} \cdot \vec{J} \,. \quad (2)$$

The electron flow through the boundaries of the considered volume is produced by the diffusion process, and it can be proved that the current density due to diffusion is given by [5]

$$\vec{J} = -\vec{\nabla} \cdot (Dn) \,, \quad (3)$$

where D is the diffusion coefficient. For this work we have chosen the diffusion model given by [6], where D is expressed for air as:

$$D = \left(29 + 0.9 \frac{E_e}{p} \right) \cdot 10^4 p \quad (4)$$

in the CGS system, preserving the form of the diffusion coefficient as given by the reference above. In this metric system of physical units p is the pressure given in Torr (mmHg) and distances are given in cm. The factor E_e is called the effective electric field.

The electron is accelerated by the RF field and experiences many collisions before it reaches enough energy to ionize a gas molecule. The average number of collisions per unit time is called the collision rate, ν_c, which depends on the gas pressure. As it was mentioned in the introduction, there is a pressure for which the efficiency in the gain of energy of the electrons, driven by the microwave field, is optimum. That pressure corresponds to the one for which the collision rate equals the microwave frequency ω. Many authors have measured the collision rate for air, most of them agreeing with the expression [8]:

$$\nu_c = 5.3 \cdot 10^9 p \,. \quad (5)$$

Since the ionization rate models are based on DC experiments the concept of the effective electric field is used. The effective electric field can be considered as the DC equivalent of the RF peak electric field (similar to effective and peak power). The relation is given by [5]:

$$E_e = \frac{E_0}{\sqrt{2\left(1 + \left(\frac{\omega}{\nu_c}\right)^2\right)}} \,, \quad (6)$$

where E_0 is the amplitude of the RF electric field. From Eq. (6) follows directly that for DC fields E_0 equals to $\sqrt{2}E_e$.

B. Ionization rate models

Several models for the ionization rate can be found in the literature. In this work we have used the models extracted from [7-10], what are based on semi-empirical arguments, to calculate the predicted Corona breakdown threshold for a parallel-plate waveguide. We have called each model according to the name of the first author of the aforementioned references.

In Fig. 1 the different ionization rates, that are going to be compared along this paper, are shown in a log-log plot for better comparison. Noticeable differences between the values of the ionization rate provided by the models can be observed. We will show how these discrepancies result in noticeably different predicted breakdown electric fields and, consequently, in different Corona discharge power thresholds.

The ionization rate in Fig.1 has been shown within the range of validity for E_e/p of the most restrictive models, i.e. Lupan and Woo [7,9]; Tang's model is valid for the widest range: from 30 to 1200 $\left(\mathrm{V \cdot cm^{-1} \cdot Torr^{-1}}\right)$; Mayhan does not say anything about the range of validity of his expressions.

Two main observations must be done about the models. Firstly, Mayhan's ionization rate model is developed in the

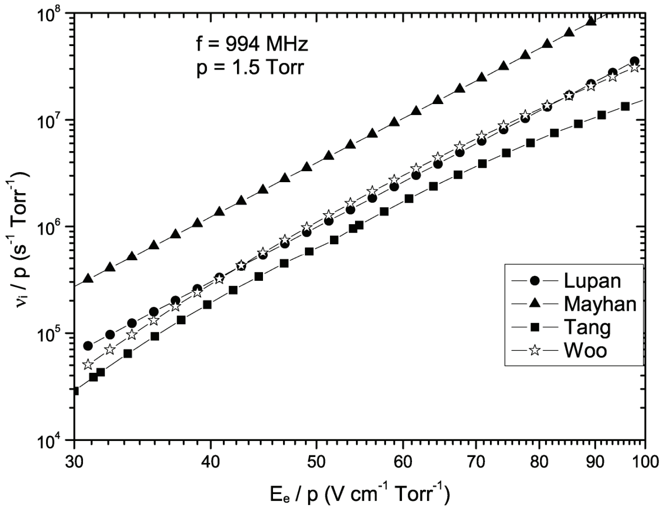

Fig. 1. Ionization rate of the considered models.

framework of re-entry vehicles in a high temperature environment, as it occurs in the ionosphere. It includes the environmental temperature as a parameter. We have chosen this temperature such that the ionization rate is as low as possible and, thus, as close as possible to the temperature conditions under which the other models have been developed. Secondly, Tang's model is developed such that both ionization coefficient and collision rates are piecewise functions. This can be observed in Fig. 1, where the ionization rate before and after $E_e/p = 54$ has different values, although this is almost not noticeable.

C. Parallel-plate waveguide

For our purposes it suffices to assume a TEM field distribution between parallel-plates subject to a voltage *V(t)* and separated by a distance *d* (see Fig. 2). In this case the electric field is $E_0(t) = V(t)/d$, i.e. constant in the region between the plates. Thus the ionization rate and the diffusion coefficient are also constant since they both depend on the electric field. Therefore we can solve Eq. (2) analytically because all the coefficients are constant. We can write the usually called Corona equation for RF fields as follows:

$$\frac{\partial n}{\partial t} = \nu_i n + D\nabla^2 n \; . \qquad (7)$$

The general breakdown criterion is found when the electron population density is just in equilibrium or when $\partial n/\partial t = 0$, that is, when

$$\nu_i n + D\nabla^2 n = 0 \qquad (8)$$

Furthermore, due to the symmetry of the problem we can consider the variation of the electron density only along the *z* axis (see Fig. 2) assuming an initially homogeneous distribution of the electron density between the plates.

Taking into account the symmetry of the problem we can assume that the electron density depends only on the variables (z,t), thus we can apply the separation of variables technique for 1D. Regarding the boundary conditions, we can expect

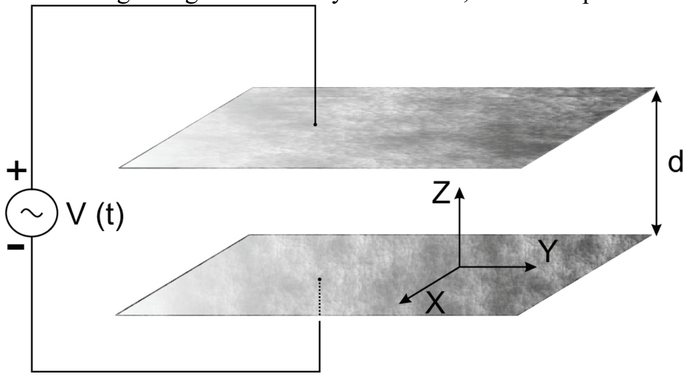

Fig. 2. Parallel-plate waveguide.

that the electrons will quickly recombine with the atoms of the metal if one reaches any plate, so that the electron density can be considered zero on the metals. This can be expressed by the following boundary conditions

$$n\big|_{z=0} = n\big|_{z=d} = 0 \quad \forall (x,y,t) \, . \qquad (9)$$

Therefore, we find the solution to be

$$n(z,t) \propto \sin\left(\sqrt{\frac{\gamma}{D}} z\right) e^{(\nu_i - \gamma)t} \, , \qquad (10)$$

where the parameter γ must read $\gamma = \left(\frac{m\pi}{d}\right)^2 D$, in order to verify the boundary conditions, and m is a natural number. As the ionization rate depends on the electric field and the pressure, and the diffusion coefficient does on the latter one, too, there will exist, at a given pressure, a breakdown electric field, E_B, corresponding to $m = 1$, which establishes the breakdown condition as

$$\nu_i(p, E_B) - \gamma(p) = 0 \, , \qquad (11)$$

above which Corona discharge will occur.

III. Results

A. Paschen curve

For a given distance between the plates we can calculate the electric field that verifies Eq. (11) for each pressure by a root search algorithm. The bisection algorithm has been used in the simulations shown along this work since, although its convergence is slow, it is always ensured. We have studied two different waveguide dimensions and frequencies. The first study corresponds to the values *d*=4.74 cm, *f*=0.994 GHz; and the second one to *d*=1.26cm, *f*=9.4 GHz. We show the threshold E_B vs. the pressure obtained with the different aforementioned models and added the experimental results obtained by MacDonald [11] for these dimensions (see Figs. 3 and 4). The plots representing the breakdown electric field vs. pressure are called the Paschen curve of the microwave device.

The effect of the discontinuity of the ionization rate in Tang's model (squared markers) shown in Fig. 1, can be observed in Figs. 3 and 4. The change of trend in the curve obtained by Tang's model from some pressure on comes from the search of the root in Eq. (11). Around the pressure for which the predicted breakdown electric field has associated a value of E_e=54*p*, the ionization rate oscillates between the values for which the ratio of effective electric field over pressure is greater or lower than 54. Thus, at pressures that have associated roots corresponding to values of E_e/p<54 the tendency of the curve is different than at those ones that have associated roots corresponding to values of E_e/p>54. This fact also shows that small differences in the value of the ionization rate can noticeably change the predicted breakdown electric field. Strictly speaking for $E_e/p=54$ no breakdown threshold can be defined due to the discontinuous ionization rate.

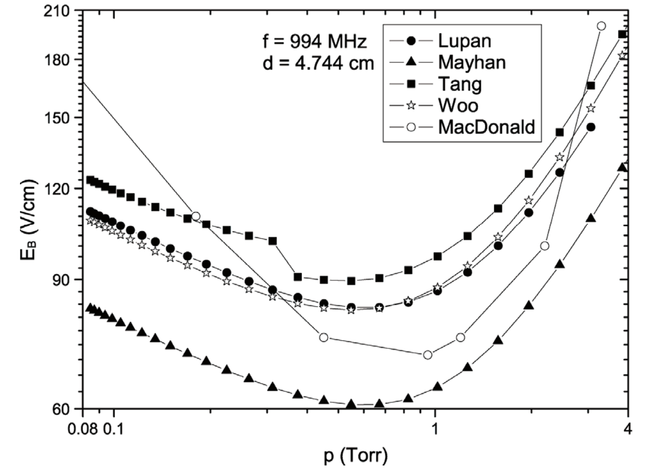

Fig. 3. Paschen curve for air in a parallel-plate waveguide together with MacDonald measurements.

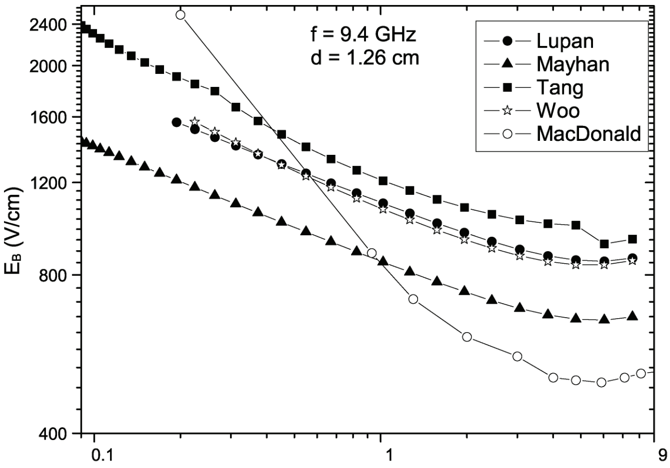

Fig. 4. Paschen curve for air in a parallel-plate waveguide together with MacDonald's measurements.

Comparing Fig. 1 with Figs. 3 and 4 we can see that, as expected, the higher the ionization rate, the lower the predicted breakdown threshold. From MacDonald's measurements we can see that for different values of RF frequency and gap distance, they show either a higher or a lower breakdown threshold than the one predicted by Mayhan's model around the *critical pressure*. Let us recall that Mayhan's ionization rate is always greater than the one of any other model for the complete range of validity; therefore it is expected to predict the minimum breakdown electric field at all pressures.

The predicted breakdown electric fields obtained by Lupan and Woo models miss some points in Fig. 4 below a certain pressure. This occurs because under the represented range of pressures the predicted values of E_e/p increase beyond one hundred, and this lies outside the range of validity of both models.

B. Critical pressure

In the design of microwave components for space applications it is usual to look for the *critical breakdown power threshold*. This one can be linked to the breakdown electric field at the *critical pressure* by means of the following expression [12]:

$$\frac{dP_{rms}}{dy}=\frac{d}{240\pi}\left|E_0\right|^2 \tag{12}$$

to calculate the predicted *critical breakdown power*. We remark that Eq. (12) is understood as power per unit length since the parallel-plate configuration is assumed to have an infinite width. In Tables 1 and 2 the predicted values obtained by the models for the two waveguide geometries considered before are shown.

Table 1 shows the predicted critical power discharge for the geometry considered in Fig. 3 together with the corresponding model. For these values of gap distance and frequency we can see that the ratio between the maximum and the minimum predicted value is more than 3dB assuming the same plate width.

TABLE 1

CRITICAL BREAKDOWN POWER ASSOCIATED TO FIG. 3

Models	$\frac{dP_B^{crit}}{dy}$ (kW/m)
Lupan	4.3
Mayhan	2.3
Tang	5.1
Woo	4.2

Table 2 shows the same for the geometry considered in Fig. 4. The differences are also noticeable in this case, though this time the ratio between maximum and minimum is slightly smaller than 3dB.

TABLE 2
CRITICAL BREAKDOWN POWER ASSOCIATED TO FIG. 4

Models	$\frac{dP_B^{crit}}{dy}$ (kW/m)
Lupan	454.1
Mayhan	271.2
Tang	527.9
Woo	440.0

C. Influence of parameters on the breakdown

It is interesting to fix the pressure and sweep other parameters to see the influence of the frequency and the gap distance in the predicted breakdown electric field for the different models. Some correlation between the values of the parameters and the closer agreement in the predicted breakdown threshold obtained by different models might be found.

Indeed, in Fig. 5 the critical breakdown electric field is represented vs. the gap distance for a given frequency. The plot shows how, as the gap distance decreases, the models tend to converge to the same values except Mayhan's model. We cannot reduce the distance as much as we want to check whether Mayhan's model agrees with the others as d decreases because, below a certain distance, the predicted breakdown electric field increases too much to stay within the range of validity of Lupan and Woo models. In any case, other simulations for smaller distances have shown that the predicted values by Mayhan's model diverges from the tendency of the other models for values of d below the ones shown in Fig. 5.

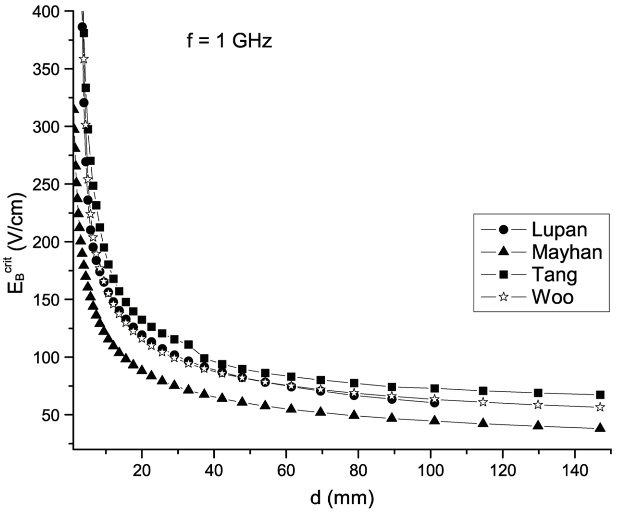

Fig. 5. Critical breakdown electric field vs. gap distance.

From the description of the models in the literature it is not clear why the models of Lupan, Tang and Woo predict nearly the same breakdown values as the distance decreases. One possible explanation might be that their ionization rate models had been obtained from experiments done with waveguides of similar dimensions, all of them corresponding to values of $d < 10$ mm.

Another interesting study is made fixing the pressure and the distance between the plates to calculate the breakdown electric field sweeping the RF frequency. In this study a curious correlation between the working pressure and the agreement of the models with each other can be seen when comparing the results at two different given pressures.

Some simulations have been undertaken sweeping the frequency and keeping constant the distance between the plates at a given pressure. For instance, in Fig. 6 we present the results of the aforementioned study at 1mBar for a gap distance of 1cm. In this case we can see that all the models, even Mayhan's one, tend to the same predicted values of the breakdown threshold as the frequency tends to low values.

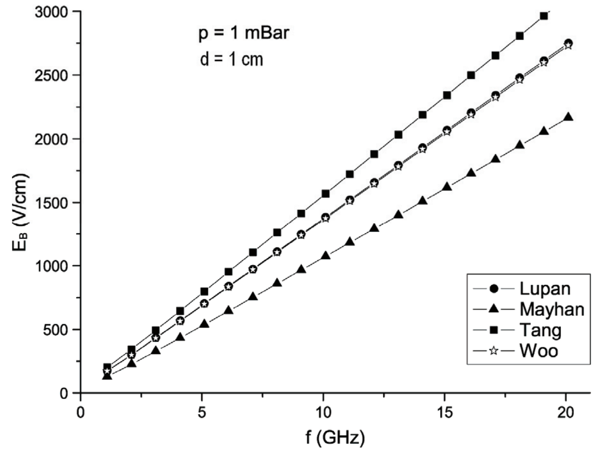

Fig. 6. Breakdown electric field vs. frequency at 1 mBar.

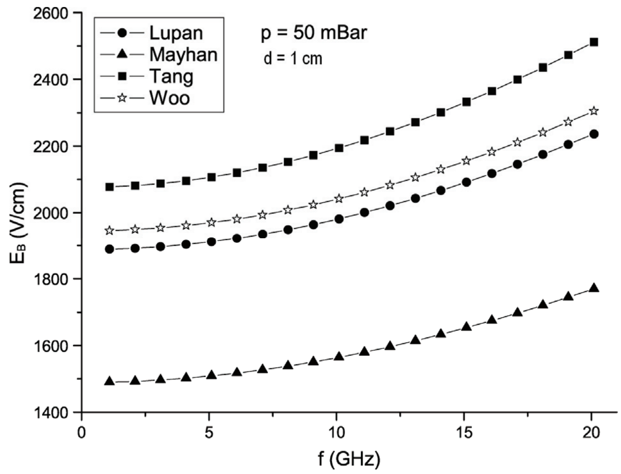

Fig. 7. Breakdown electric field vs. frequency at 50 mBar.

However, in Fig. 7, where the pressure is 50mBar and the distance is the same as in the previous case, this effect cannot be observed. These results are not surprising if we think about the frame under which all the ionization rate models, based on semi-empirical approaches, have been developed. On the one hand, all attempts to develop a model for the RF ionization rate have been based on corrections of the ionization rate models developed for DC conditions. This is the reason why the models tend to the same predictions for low frequencies.

On the other hand, most of the DC models are based on measurements made under low pressure conditions, what explains the discrepancies of the models shown in Fig. 7 when the frequency tends to low values while the pressure is higher than the one considered in Fig. 6.

IV. CONCLUSION

The critical breakdown power threshold of a microwave device for space applications is fundamental to ensure the good performance of satellites and space vehicles. This factor provides an important constraint in the design of electron devices on board of satellites. Four different available models of the ionization rate for low-pressure microwave discharges are studied in this paper. Although all of them use the same model of the effective electric field, noticeable discrepancies in the predicted critical breakdown power obtained by the different models could be observed.

For most of the models the ionization rate is only valid for a relatively small range of values of the ratio of effective electric field over pressure. Apparently, there are many technical difficulties to measure the ionization rate under conditions of very low pressure and very high effective electric fields. This fact limits the values of the RF frequency and gap distance for which the search of the Corona breakdown power can be done without leaving the validity range of the models. We have shown that the predicted breakdown power for a given frequency increases as the gap distance decreases. This means that at certain pressures the breakdown effective electric field predicted for some parallel-plate waveguides will lie beyond the upper or the lower limit of validity range. Thus none of the models available can be used to calculate the predicted breakdown threshold for all geometries and RF frequencies at every pressure. It would be interesting to propose experimental setups to measure the ionization rate dealing with lower pressures and higher effective electric fields than considered so far. This will increase the validity range of the models and thus a larger range of waveguide dimensions, frequencies and pressures could be used to simulate the *critical pressure*.

A study of some parameters to see under which conditions the models tend to predict the same breakdown threshold has been done. It has been shown that the breakdown values predicted by the different models at low pressures tend to be equal for low RF frequencies. On the contrary, for high pressures and high frequencies the models differ significantly. This can be explained by the fact that the currently available models studied are based on experimental data mainly obtained from DC experiments at low pressures. It would be interesting to develop ionization rate models based on microwave frequency experiments at higher pressures.

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