An Example of Suppression of Spurious Stop-Bands of EBG Band-Stop Filter

Dusan A. Nesic[1] and Branko M. Kolundzija[2]

Abstract— **Based on the well known theory of infinite periodic structures, analytical theory of EBG (electromagnetic band gap) cells suppressing 6 higher (spurious) stop-bands is developed. Using such cells in a cascade the straight-forward procedure for design of the corresponding EBG band-stop filter is proposed, with possibility to control the width and the depth of the stop-band. The analytical theory is confirmed by the EM simulation of the filter realized in the microstrip technology.**

Keywords – Band-stop filter, Suppression of spurious stop-bands, Microstrip

I. Introduction

In last decades many papers were published dealing with microstrip EBG (electromagnetic band-gap) structures. The EBG structures are obtained in different ways: by drilling holes in the substrate [1], by etching sinusoidal patterns in the substrate [2], or by varying the microstrip line pattern to cause the sinusoidal variation of its characteristic impedance [3]. Most of these papers rely on EM (electromagnetic) simulations and measurement using basic ideas from well known theory of infinite periodic structures [4]. Instead of experimenting with various patterns using EM simulation, analytical theory of EBG cells suppressing 3 higher stopbands is developed in ref. [5], thus enabling straightforward design of EBG filters with the same property. Using the similar method as in ref. [5], this paper presents analytical theory of EBG cells suppressing 6 higher stop-bands and their straightforward design.

II. Theoretical Basis

Consider one-dimensional (1D) microstrip periodic structure in form of infinite cascade of identical two port linear reciprocal passive cells. Following the general theory of periodic structures given in [4], the cell can be characterized in two ways: 1) with its ABCD matrix, and 2) with its physical length l and equivalent propagation coefficient $\gamma = \alpha + j\beta$ (α is equivalent attenuation coefficient, and β is equivalent phase coefficient). In that sense, the voltage and the current at the beginning of the n^{th} cell, V_n and I_n, can be expressed in terms of the voltage and the current at its end, V_{n+1} and I_{n+1}

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \tag{1a}$$

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = e^{\gamma l} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \tag{1b}$$

After replacing (1b) into (1a) and simple rearrangement we obtain the homogenous system of equations with condition that for nontrivial solution its determinant equals zero, i.e.

$$\begin{bmatrix} A - e^{\gamma l} & B \\ C & D - e^{\gamma l} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = 0 \tag{2a}$$

$$AD + e^{2\gamma l} - (A + D)e^{\gamma l} - BC = 0 \tag{2b}$$

Replacing $AD = 1 + BC$ (valid for reciprocal networks) into (2b) and subdividing left and right side by $2e^{\gamma l}$ the so-called dispersive relation is obtained in the form $\cosh(\gamma l) = (A+D)/2$. In the case of cells without losses A and D are always real, so that $\cosh(\gamma l)$ also must be real. Having in mind that $\cosh(\gamma l) = \cosh(\alpha l)\sinh(\beta l) + j\sinh(\alpha l)\sinh(\beta l)$, $\cosh(\gamma l)$ can be real only if $\sinh(\alpha l)\sinh(\beta l)$ equals zero. It is possible in two cases: 1) $\alpha = 0$, $\beta \neq 0$ (the wave propagates along the cascade without attenuation), and 2) $\alpha \neq 0$, $\beta = 0$, π/l (the wave attenuates), and dispersive relation reduces to

$$\left|\cos(\beta l)\right| = \left|\frac{A + D}{2}\right| \leq 1 \tag{3a}$$

$$\left|\cosh(\alpha l)\right| = \left|\frac{A + D}{2}\right| > 1 \tag{3b}$$

Frequency ranges in which relation (3a) is satisfied correspond to pass-bands, while frequency ranges in which relation (3b) is valid correspond to stop-bands. If infinite cascade is truncated the attenuation in stop-bands is limited by number of cells, while propagation in pass-bands is disturbed due to reflections in planes of truncation.

Consider a cell as cascade of n sections, i.e., transmission lines without losses, of equal angular length θ, and of different characteristic impedances, Z_i, $i = 1,...,n$. ABCD matrix of the cell having i sections can be represented as product of the ABCD matrix of the cell having i-1 sections and the ABCD matrix of single section as

$$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \begin{bmatrix} A_{i-1} & B_{i-1} \\ C_{i-1} & D_{i-1} \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & jZ_i \sin\theta \\ \dfrac{j}{Z_i}\sin\theta & \cos\theta \end{bmatrix} \tag{4}$$

[1]Dusan A. Nesic is with the IHTM-CMTM, Njegoseva 12, 11000 Belgrade, Serbia, E-mail: nesicad@nanosys.ihtm.bg.ac.rs
[2]Branko M. Kolundzija is with the School of Faculty of Electrical Engineering, University of Belgrade, Bul. Kralja Aleksandra 175, 11000 Belgrade, Serbia, E-mail: kol@etf.bg.ac.rs

After multiplying matrices on the left side of (4), recurrent formulas for ABCD parameters are obtained in the form

$$A_i = A_{i-1}\cos\theta + j\frac{B_{i-1}}{Z_i}\sin\theta \qquad A_1 = \cos\theta \tag{5a}$$

$$B_i = jA_{i-1}Z_i\sin\theta + B_{i-1}\cos\theta \qquad B_1 = jZ_1\sin\theta \tag{5b}$$

$$C_i = j\frac{D_{i-1}}{Z_i}\sin\theta + C_{i-1}\cos\theta \qquad C_1 = \frac{j}{Z_1}\sin\theta \tag{5c}$$

$$D_i = D_{i-1}\cos\theta + jC_{i-1}Z_i\sin\theta \qquad D_1 = \cos\theta \tag{5d}$$

It can be shown that for odd n the dispersive relation can be written in the form of odd polynomial of $\cos\theta$ in the form

$$\frac{(A_n + D_n)}{2} = \sum_{k=0}^{\left[\frac{n}{2}\right]} p_k^{(n)} (\cos\theta)^{n-2k} \tag{6}$$

where $[n/2]$ represent integer part of $n/2$, and coefficients $p_k^{(n)}$ depends only on characteristic impedances Z_i, $i = 1,\ldots, n$. This polynomial will be referred as dispersive polynomial. Obviously the polynomial is periodic function in θ with period π. Let us investigate its behaviour in range $0 \le \theta \le \pi$, for which $-1 \le \cos\theta \le 1$. It is easy to show that at boundaries of the range the polynomial has values ±1. In general case, inside the range the polynomial may have up to n zeros and n-1 extremes. If magnitudes of all extremes are greater than one, there are n-1 stopbands in the range.

In what follows all cells will be made of n = 9 sections of equal length. Array of characteristic impedances in one cell made of 9 sections are presented in fig.1.

Z_1	Z_2	Z_3	Z_4	$Z_5=Z_0$	Z_6	Z_7	Z_8	Z_9

Fig. 1. An array of the characteristic impedances in one cell made of 9 sections

For the beginning let us consider cascade of 4 identical cells, where the characteristic impedances of the cell's section are obtained by random choice in range from 25 to 100 Ω. Fig. 2 shows s_{12} versus frequency for 5 different combinations of characteristic impedances. The blue curve represents result for cell containing sections of 81, 25, 29, 78, 79, 50, 74, 37, and 28 Ω. The yellow curve represents result for cell containing sections of 31, 43, 26, 43, 64, 59, 75, 81, and 66 Ω. The red curve represents result for cell containing sections of 51, 63, 44, 78, 58, 82, 67, 73, and 60 Ω. The green curve represents result for cell containing sections of 25, 42, 65, 32, 52, 65, 79, 69, and 83 Ω. Finally, the grey curve represents result for cell containing sections of 46, 66, 45, 83, 97, 37, 98, 71, and 35 Ω. Frequency range of 27 GHz (0 GHz $\le f \le$ 27 GHz) corresponds to angular length of π ($0 \le \theta \le \pi$). It is seen that all 5 randomly chosen cells have more or less pronounced stop-bands in the vicinity of all 8 extremes.

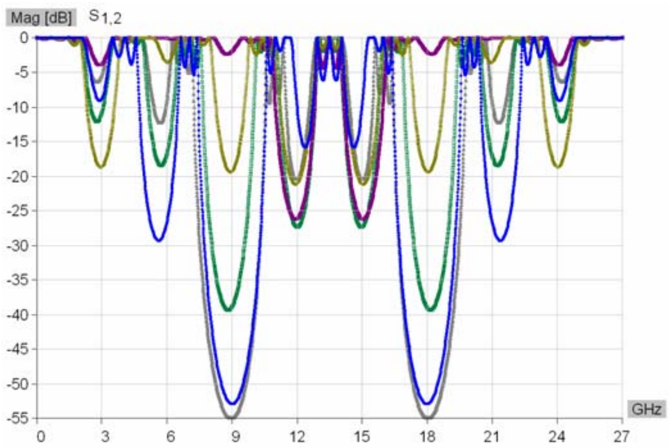


Fig. 2. S_{12} of cascade of four identical cells made of nine sections obtained for five random choices of section impedances.

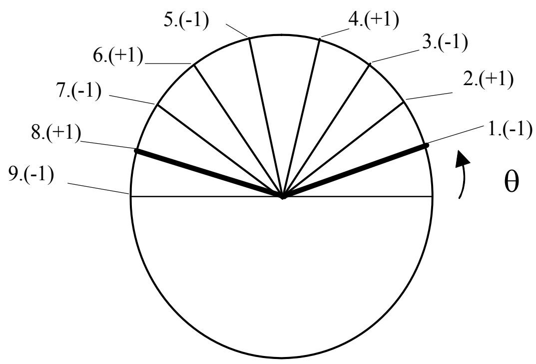


Fig. 3. Position of band-gaps depends on angle (bold lines mark desirable bandgaps) (-1) or (+1) correspond to boundaries of the dispersive polynomial

In case of band-stop filter it is desirable to control "depth" of the first stop-band and to suppress higher (spurious) stop-bands. The question is if it is possible to eliminate some of stop-bands by special choice of section's impedances. Having in mind that $\cos(\pi-\theta) = -\cos\theta$, the stop-bands are symmetrical with respect to $\theta = \pi/2$, the center of the range, so that elimination of the 1st, 2nd, 3rd, and 4th stop-band results in elimination of the 8th, 7th, 6th and 5th stop-band, respectively, and vice versa, fig.3. Since, we want to retain the 1st stop-band, such structure gives theoretical possibility to eliminate six (spurious) stop-bands, from the 2nd to the 7th, and to achieve this it is enough to find a way to eliminate stop-bands from the 2nd to the 4th. The elimination of a stop-band is provided if magnitude of the corresponding extremum is decreased to value not greater than one.

To decrease value of the 2nd, 3rd, and 4th extremum we need first to find their position. The positions of all extremes are obtained from the first derivative of the dispersive polynomial with respect to θ equated to zero, i.e.

$$\frac{d(A_9 + D_9)}{2d\theta} = 0 \tag{7}$$

After simple manipulations equation is obtained in the form

$$\sin\theta\,[\,9p_0^{(9)}(\cos\theta)^8 + 7p_1^{(9)}(\cos\theta)^6 + 5p_2^{(9)}(\cos\theta)^4 + 3p_3^{(9)}(\cos\theta)^2 + p_4^{(9)}\,] = 0 \quad (8)$$

In case of solution $\sin\theta = 0$ we have $\theta = k\pi$, which corresponds to the boundaries of range $0 \le \theta \le \pi$. Note that for these boundaries we previously found that dispersive polynomial has values ±1, i.e., we have passbands in the vicinity of these boundaries, as shown in fig.3. Other solutions are obtained if expression in braces is equated to zero. After replacing $(\cos\theta)^2 = t$, equation of 4th order in terms of t is obtained

$$9p_0^{(9)}t^4 + 7p_1^{(9)}t^3 + 5p_2^{(9)}t^2 + 3p_3^{(9)}t + p_4^{(9)} = 0 \quad (9)$$

The equation is solved analytically. Solving this equation we obtain the positions of extremes (θ_1, θ_2, θ_3, θ_4, θ_5, θ_6, θ_7 и θ_8), around which in general case stop-bands are placed. Since the first of them is the desirable stop-band, and the last four always go in couple with the first four, let us focus on the 2nd, 3rd and 4th extremum (θ_1, θ_2, and θ_3).

To investigate if there were such values of section impedances that decrease the extremum magnitude to value not greater than one, we applied the optimization procedure. At the beginning the impedance values of all sections are varied between given Z_{min} and Z_{max}. For optimization we used the multi-minima optimization method [6], so that in each run a bunch of minimums was obtained. For each minimum the magnitude of all three extremes were successfully decreased almost exactly to one (with error of about 10^{-3} which corresponds to error in single precision calculus). However, we found that most of these cells have different values of Bloch impedance in pass-bands on the two sides of stop-band, and different values from the desirable reference impedance Z_0 (e.g., $Z_0 = 50\ \Omega$), which is not convenient for making filters as finite cascades. By further experiments we found that Bloch impedance in pass-bands close to Z_0 can be achieved if central section is chosen to have characteristic impedance Z_0, and if product of characteristic impedances of sections symmetrically placed with respect to the central section is equal to Z_0^2, i.e.,

$$Z_k Z_{9-k+1} = Z_0^2 \quad (10)$$

After applying relation (10) only four characteristic impedances left to be optimized (Z_1, Z_2, Z_3, and Z_4). In all such optimizations it was found that minimum or maximum value is obtained for 3rd section. Moreover, for Z_3 fixed, the multi-minima method always finished in single minimum or maximum. Finally, note that if Z_3 is replaced by Z_0^2/Z_3, the same cell is obtained, but with sections counted in opposite direction. Hence, the final optimizations are performed by varying three characteristic impedances (Z_1, Z_2, and Z_4) for different values of characteristic impedance Z_3, where Z_3 is varied from small fraction of Z_0 to Z_0. As a result of these optimizations we obtained Z_1, Z_2, and Z_4 in terms of Z_3. These results could be accurately fitted by 2nd order polynomials

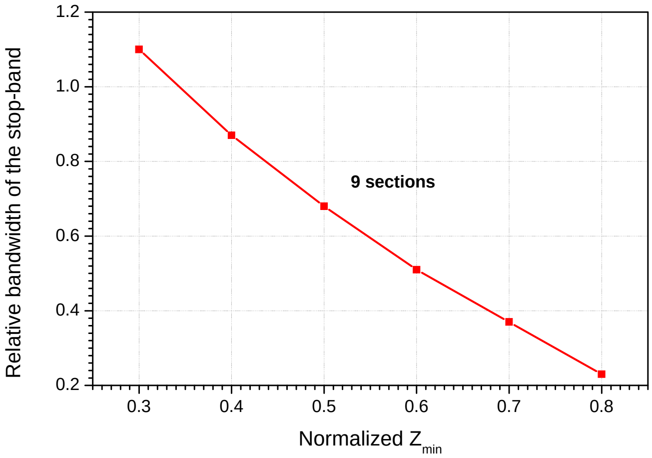
$$Z_1 = 22.82542 + 0.78206\,Z_3 - 0.0049\,Z_3^2 \quad (11a)$$

$$Z_2 = 2.59423 + 1.0158\,Z_3 - 0.00131\,Z_3^2 \quad (11b)$$

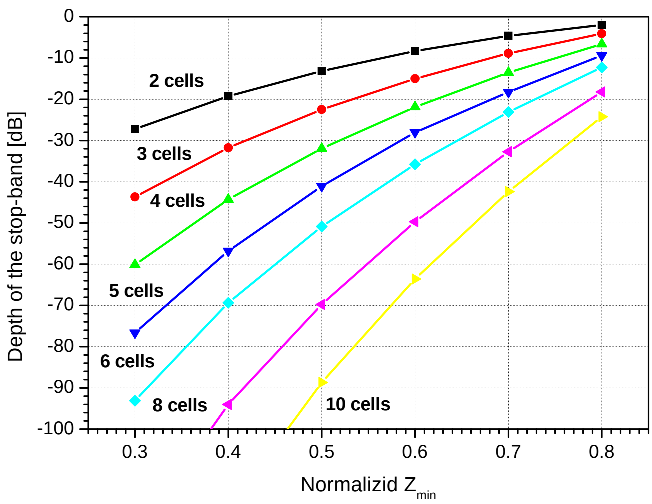
$$Z_4 = 8.19616 + 1.06278\,Z_3 - 0.00469\,Z_3^2 \quad (11c)$$

Finally, we can establish procedure that provides design of the cell, which suppresses six spurious stop-bands. As a first step we adopt reference impedance Z_0 (usually, 50 Ω). Then we adopt minimum characteristic impedance of the cell, Z_3. After that characteristic impedance Z_1, Z_2, and Z_4 are calculated using formulas (11a), (11b), and (11c). Finally, all other characteristic impedances are calculated using (10).

It is shown that by decreasing Z_3 width and depth of the bandstop increases. It is also shown that by cascading such cells the width of the bandstop practically does not change, while the depth of bandstop significantly increases. Variations of the width and the depth of the bandstop versus normalized Z_{min}, i.e. Z_3 / Z_0 , are presented in fig.4.



a) Relative bandwidth of the stop-band



b) Depth of the stop-band

Fig. 4. Curves for cells with 9 sections versus normalized Z_{min} (i.e. Z_3 / Z_0)

Hence, we propose the following procedure for design of bandstop filter with suppression of six spurious stopbands. First, decrease Z_3 starting from Z_0 to adjust width of stop-band. Then, increase the number of cells to adjust the depth of stop-band. In particular, after truncation of infinite cascade significant ripples occur in the pass-band, in the vicinity of the stop-band. The ripples can be decreased by increasing Z_3 of outer cells and decreasing Z_3 in inner cells.

III. IMPLEMENTATION

Using above theory the band-stop filter which suppresses six higher stop-bands is designed. In the first step, the model using ideal transmission lines is created. The central frequency of band-stop is 3 GHz, at which the attenuation of 30 dB is achieved. The width of the band-stop at attenuation level of 10 dB is about 1.85 GHz. The filter consists of 4 cells having equal electrical lengths. For two inner cells impedance Z_3 is 23.60 Ω, while for two outer cells impedance Z_3 is 28.20 Ω, decreasing maximum attenuation in pass-band bellow 0.5 dB.

In the second step the ideal model is converted to three-dimensional (3D) EM model in microstrip technology using teflon substrate ($\varepsilon_r = 2.1$, $tg\delta = 0.0004$ and $h = 0.508$ mm). In the EM modelling real thickness of metallization of 17 microns and skin effect losses are taken into account using $\sigma = 18$ MS/m instead of $\sigma = 58.8$ MS/m. All simulations are performed in WIPL-D software packages [7]. Fig 5 shows the metallization pattern, which is used in EM model. Fig. 6 shows comparison between results obtained by simulation of ideal (transmission line) model and 3D EM model. There is very good agreement between ideal and EM model

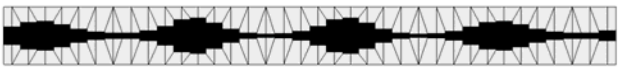
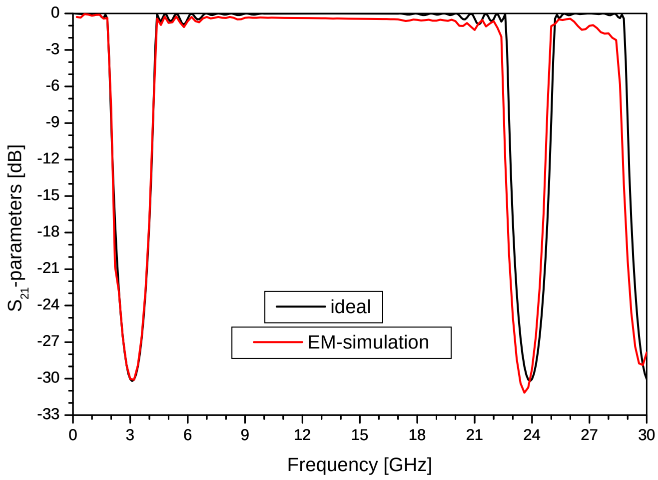
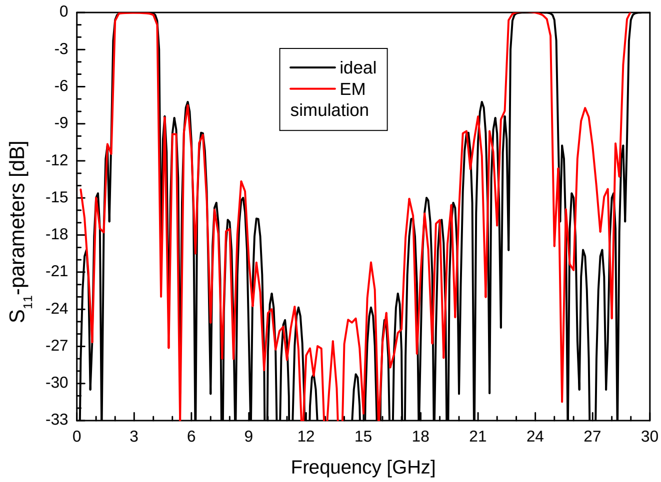


Fig. 5. Geometrical model of band-stop filter made of four cells, each cell consisting of 9 sections



a) S_{21}



b) S_{11}

Fig. 6. S_{21} and S_{11} parameters of band-stop filter shown in Fig. 5. Comparison of results obtained by ideal model (circuit simulation) and technological model (EM simulation)

IV. CONCLUSION

Design procedure of EBG filters suppressing 6 higher (spurious) stop-bands is proposed. The filter is in form of cascade of cells of the same type. Each cell is in form of cascade of sections (transmission lines of equal electrical length), whose characteristic impedances are strictly determined by the characteristic impedance of one section (lowest/ highest value), which is decreased/increased to adjust the width of the main stop-band. The depth of the stop-band is controlled by number of cells. Circuit and EM model agreed very well.

ACKNOWLEDGMENT

This work is supported by the Ministry of Science and Technological Development, Republic of Serbia (TR-11021). The paper is extended paper "EBG band-stop filter with suppression of 6 spurious stop-bands" on conference TELSIKS 2011, Niš, Serbia.

REFERENCES

[1] V. Radisic, Y. Qian, R. Coccioli, and T. Itoh, "Novel 2-D Photonic Bandgap Structure for Microstrip Lines," *IEEE Microwave and Guided Letters*, vol. 8, no. 2, pp. 69-71, Feb. 1998.

[2] M.A.G. Laso, T. Lopetegi, M.J. Erro, D. Benito, M.J. Garde, and M. Sorolla, "Multiple-Frequency-Tuned Photonic Bandgap Microstrip Structures," *IEEE Microwave and Guided Wave Letters*, vol. 10, no. 6, pp. 220-222, June 2000.

[3] D. Nesic and A. Nesic, "Bandstop Microstrip PBG Filter With Sinusoidal Variation of the Characteristic Impedance and without Etching in the Ground Plane," *Microwave and Optical Technology Letters*, vol. 29, no. 6, pp. 418-420, June 2001.

[4] D.M. Pozar, *Microwave Engineering*, 3rd ed., John Wiley & Sons, 2005.

[5] D.A. Nesic and B.M. Kolundzija, "EBG band-stop filter with suppression of 3 spurious stop-bands," *Proc. of 6th IWAT*, 2010.

[6] B.M. Kolundzija and D.I. Olcan, "Multiminima Heuristic Methods for Antenna Optimization," *IEEE Transactions on Antennas and Propagation*, vol. 54, no. 5, pp. 1405 - 1415, May 2006.

[7] *www.wipl-d.com*