On Optimization Control Parameters in an Adaptive Error-Control Scheme in Satellite Networks

Ranko Vojinović, Aleksandar Obradović

Abstract – **This paper presents a method for optimization of control parameters of an adaptive GBN scheme in error-prone satellite channel. Method is based on the channel model with three state, where channel have the variable noise level.**

Keywords – **Optimization of control parameters, Adaptive GBN scheme, Error-prone satellite channel.**

I. INTRODUCTION

Communication satellite has several properties that are radically different from terrestrial point-to-point links. Even though signals to and from satellite travel at speed of light, the large round trip distance introduces a substantial delay. Further, satellite channels are caused by a variety of different phenomena. The large round-trip propagation causes increasing transfer error probability.

Various adaptive methods for error control in the dynamic conditions of channel operation are developed. In [1], a n-copy GBN (nGBN) scheme has been presented and used to improve the system throughput in case of poor channel behavior. N-copy GBN is a generalization of Standard GBN (SGBN). Instead of a single copy, n packet copies are sent.

The adaptive two state system, using the standard GBN mode when packet error rate (PER) is small, or the nGBN (n>1) mode if the case of higher PER values, has been proposed and analyzed in [2]. In [3], three-state adaptive system had been adopted and analyzed. It was assumed that this adaptive system (TS-GBN) acted in the SGBN, when PER was increased, and in the continuous GBN (CGBN) mode, when transmitter continuously sent the same packet until the positive acknowledgement (ACK) of its reception was received. In [4], the precise formulation of the model, and the exact analysis of throughput for the three-mode GBN (TM-GBN) mechanism are proposed.

Let us review the basic foundations of this concept, that has been presented in details in [3] and [4].

The transmitter sends the data packets to the receiver over the channel. The receiver provides ACK for each packet received, including the copies in the multiply mode. The packets have been emitted in one of three possible ways. In L mode, standard GBN procedure is followed. Upon the receipt of α consecutive negative acknowledgements (NAKs), the transmitter switches to H mode, where the packets are emitted by nGBN procedure. If the transmitter receives β consecutive ACKs in H mode, the transmitter returns to L mode. If the transmitter receives γ consecutive NAKs, while it is in H mode, the transmitter will switch to VH mode, in which it continuously emits the same packet. The transmitter returns to H mode upon the receipt of δ consecutive ACKs.

Therefore, estimation on the state of the channel is based on counting ACKs and NAKs. Number of this acknowledgments are parameters α, β, γ and δ, control parameters that practically control the system. Knowing its values is of essential importance for adaptive systems, because system state commutation is performed in moments defined by those parameters. In papers [3], [4] and [5], for system throughput analysis, those parameters are randomly selected. However, it is necessary to elaborate additional method of their calculation, in order to optimize process of system control. Proposal of the process of optimization of control parameters is the subject of this paper.

The paper is organized in the following way: Section II presents the system model; Section III defines optimization criteria, while system analysis and results are presented in Section IV.

II. MODEL ANALYSIS

Throughput analysis in [3] was based on simplified method, presented in [2]. In both of these papers it was assumed that the transmission condition change slowly over time, so the adaptive system could be modeled as a simple semi-Markov chain.

The throughput of the proposed scheme is:

$$S = S_1 P_L + S_2 P_H + S_3 P_{VH} \tag{1}$$

where S_1, S_2 and S_3 are the throughputs, given as –

standard GBN:

$$S_1 = \frac{1 - P_e}{1 + (N-1)P_e} \tag{2}$$

n-copy GBN:

$$S_2 = \frac{1 - P_e^n}{n + (N-1)P_e^n} \tag{3}$$

Ranko Vojinović and Aleksandar Obradović are with the Faculty of Information Technology, University Mediterranean, Vaka Đurovića bb, 81000 Podgorica, Montenegro

continuous GBN:

$$S_3 = \frac{1 - P_e}{1 + (N-1)(1-P_e)} \tag{4}$$

Where are:
P_e - packet error probability
N - standard value for GBN protocol
n - number of copies sent

P_L, P_H and P_{VH}, in Eq. (1), are the state probability, i.e. the probabilities for channel to be in states L, H and VH respectively.

Transitions from state L to state H, and from state H in state L, are defined by transition probabilities p_{12} and p_{21} respectively. Transitions from state H to state VH, and from state VH to state H, are defined by transition probabilities p_{23} and p_{32}. Cases of direct transition from state L to state VH and vice versa are disregarded, as rare.

Probabilities of transition are dependant on control parameters α, β, γ and δ. In [3] is shown that they are:

$$p_{12} = P_e^{\alpha} \tag{5}$$

$$p_{21} = (1 - P_e)^{\beta} \tag{6}$$

$$p_{23} = P_e^{\gamma} \tag{7}$$

$$p_{32} = (1 - P_e)^{\delta} \tag{8}$$

Also, it is shown in [3] that:

$$P_H = \frac{1}{\frac{p_{21}}{p_{12}} + \frac{p_{23}}{p_{32}} + 1} \tag{9}$$

$$P_L = \frac{p_{21}}{p_{12}} P_H \tag{10}$$

$$P_{VH} = \frac{p_{23}}{p_{32}} P_H \tag{11}$$

III. CRITERIA OF OPTIMIZATION OF CONTROL PARAMETERS

Denote with S and S_{opt} real and optimal throughput of the TS-ARQ system. The objective of the optimization is to define control parameters in order for S to be as close as possible to the value of S_{opt}. This objective is very complex, and it is difficult to solve it by well known optimization methods. First, there are many variables that need to be optimized. Apart quoted four control parameters, dynamic conditions are being defined by N and n. Second, proposed adaptive model is nonlinear, therefore in strict mathematical sense this problem can not be solved in closed form.

For illustration, we will present optimization method of this system using the mean square error method. According to this method, for the purpose of defining optimal control parameters in order to have S approximate S_{opt} in best possible way, it is required to minimize the value of the function:

$$F(\alpha,\beta,\gamma,\delta) = \int_0^1 \left(S_{opt}(P_e) - S(P_e)\right)^2 dP_e \tag{12}$$

with limitations:

$$\alpha_{\min} < \alpha < \alpha_{\max},\ \beta_{\min} < \beta < \beta_{\max},$$
$$\gamma_{\min} < \gamma < \gamma_{\max},\ \delta_{\min} < \delta < \delta_{\max}$$

while the optimization variables are considered to have integer values.

Function minimization with additional limitations is highly complex problem, and methods of its solving cannot be provided analytically. For these reasons full optimization of control parameters will not be done, but we will suggest the method that can optimize their ratios.

System switches its working modes, based on control parameters, depending on system state, more precisely, channel error probability, where the system does not know the error probability in every moment. However, it is correct to assume that the system will change from one to another mode near the probability values that can be approximately calculated. So, in order to optimize the control parameters, it is necessary to calculate switching points P_{C1} and P_{C2} of frame error probability, where adaptive TS-ARQ system switches its modes.

Switching regime modes in points with probability of frame error assume values P_{C1} and P_{C2} is shown in figure 1.

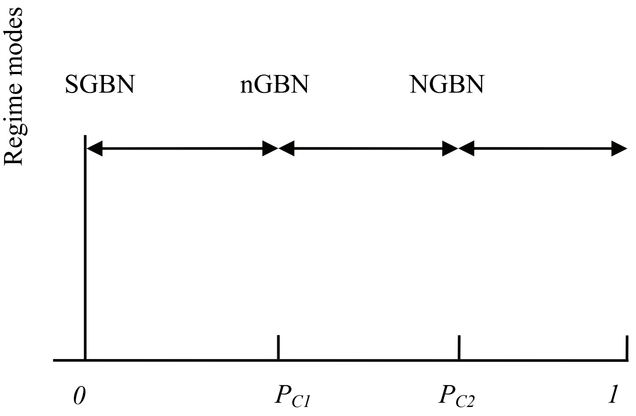

Fig. 1. Switching mode of adaptive TS-ARQ system

SGBN, nGBN and NGBN denote working modes of standard, n-copy and continuous GBN, respectively.

Values P_{C1} and P_{C2} are in ideally, such that valid:

- For $0 \le P_e < P_{C1}$, system acts in SGBN mode.
- For $P_{C1} < P_e < P_{C2}$, system acts in nGBN mode.
- For $P_{C2} < P_e \le 1$, system acts in NGBN mode.

At first, we will calculate probability for which TS-ARQ system switches from SGBN to nGBN mode. For a system modeled like this, it is logical to assume that for $P_e > P_{C1}$, n-copy mode will provide bigger throughput than SGBN mode. This means that $S_2 > S_1$, where S_2 and S_1 are throughput of the system in n-copy and standard GBN mode, respectively. Hence, we have:

$$\frac{1-P_e^n}{n+(N-1)P_e^n}-\frac{1-P_e}{1+(n-1)P_e}>0 \qquad (13)$$

After algebraic transformation and taking into account the physical properties of the channel (P_e and $(1-P_e)N$ are always greater than zero) we get:

$$\frac{P_e\left(1-P_e^{n-1}\right)}{1-P_e}-\frac{n-1}{N}>0 \qquad (14)$$

For $P_{C1}=P_e$, left side of inequality (14) becomes equal to zero, so the value for the P_{C1} is calculated from the expression:

$$\frac{P_{C1}\left(1-P_{C1}^{n-1}\right)}{1-P_{C1}}=\frac{n-1}{N} \qquad (15)$$

Calculation of the value P_{C2} is performed similarly. Analogous to prior elaboration, for $P_e > P_{C2}$, continuous GBN should provide greater throughput than n-copy GBN mode. This means that in this case $S_3 > S_2$, and $S_3 - S_2 > 0$, where S_3 is throughput of continuous GNB. According to this, probability of transition P_{C2} can be calculated from the equation $S_3 - S_2 = 0$:

$$\frac{1-P_e}{1+(N-1)(1-Pe)}-\frac{1-P_e^n}{n+(n-1)P_e^n}=0 \qquad (16)$$

It follows:

$$\begin{aligned}&\left(1-P_{C2}\right)\left[n+(n-1)P_{C2}^n\right]-\\&\left(1-P_{C2}^n\right)\left[\left(1-P_{C2}^n\right)(N-1)+1\right]=0\end{aligned} \qquad (17)$$

In the previous section it was emphasized that the objective of this method is to calculate the optimal ratio between control parameters. First we will calculate the optimal ratio between the parameters α and β.

In order to do this, let us assume that in the ideal case, in the point P_{C1}, where the system switches its mode from SBGN to nGBN (or vice versa), probabilities that the TS-ARQ system will be in P_L or P_H state are equal. Therefore we have:

$$P_L=P_H=0.5$$
$$P_{VH}=0$$

From the conditions above, using equations for P_L, P_H and P_{VH} (Eqs. (10), (9) and (11)), we calculate that for $P_e=P_{C1}$ probabilities of the transition from L state to H, and vice versa are equal, or that $p_{12}=p_{21}$ is valid.

By using this ratio and relations that connect those probabilities of transition with appropriate control parameters $p_{12}=P_e^{\alpha}$ and $p_{21}=(1-P_e)^{\beta}$, we get:

$$\frac{\alpha^*}{\beta^*}=\frac{\ln\left(1-P_{C1}\right)}{\ln P_{C1}} \qquad (18)$$

where α^* and β^* denote values of control parameters α and β for $P_e=P_{C1}$.

In analog way to the previous, we can calculate optimal ratio between parameters γ and δ. Namely, in the ideal case for $P_e=P_{C2}$ we have:

$$P_L=0$$
$$P_H=P_{VH}=0.5$$

From the equations above, and the relations that connect probabilities of transition p_{23} and p_{32} with parameters γ and δ, similar to previous consideration we calculate optimal ratio of parameters γ and δ:

$$\frac{\gamma^*}{\delta^*}=\frac{\ln\left(1-P_{C2}\right)}{\ln P_{C2}} \qquad (19)$$

Note that the Eqs. (18) and (19), strictly mathematically are not correct. Namely, we idealized the case that in point P_{C1} TS-ARQ system switches from SGBN to nGBN state (and vice versa) with probability of 1, and that in point P_{C2} system switches from nGBN mode to NGBN mode with probability of 1, which is not the real case. Namely, proposed system switches its state on the basis of the control parameters, while it does not know the error probability of the frame transfer in the moment of switching. Theoretically, in the moment when the error probability is near the P_{C1}, the system can take any state, as well as the state of continuous GBN. So, with this idealization we neglected relations of other control parameters, which due to the nature of the system, influence each other as well (e.g. relations between parameters α and δ, β and γ, etc.). However, with this optimization method, by practically splitting tri-state system into two weakly related two-state systems, optimal ratios between parameters α and β, as well as γ and δ, are quite well approximated, which will be illustrated by numerical examples in the next section.

IV. NUMERICAL RESULTS

As an example, consider a 50 kbps satellite channel with 500 ms round trip propagation delay. Let us imagine trying the GBN protocol to send 2500 bit frames via the satellite. At $t=0$, the sender starts sending the first frame. At $t=50ms$, the frame has been completely sent. Not until $t=550ms$ has the ACK arrived back to the sender, under the best circumstances. So, in an ideal case we assume that $N=10$.

We will determine ratio of α and β parameters for values $N=10$ and $n=2$. By substituting this values in the Eq. (15), we get the result $P_{C1}=0.1$. By substituting this value in the Eq. (18), the result is $\alpha^*/\beta^*=21.85$. In case that $N=10$ and $\underline{n=3}$, similarly we have that $P_{C1}=0.17$ and $\alpha^*/\beta^*=9.43$.

It is already said that with this method we approximate only the parameter ratio. So, for $\alpha=\alpha_{min}=2$, we calculate adequate values for β. In case $N=10$ and $n=2$, $\beta=44$, while in second case where $N=10$ and $n=3$, the parameter $\beta=18$. It is understood that for the ratios of the parameters, due to their nature, we took nearest integer ratios.

Fig. 2. illustrate throughput of TS-ARQ system for values of optimized parameters. It is taken that $N=10$, $n=2$, $\alpha=2$, $\gamma=10$, $\delta=2$, while β values are changing and they are *20, 30* and *40.*

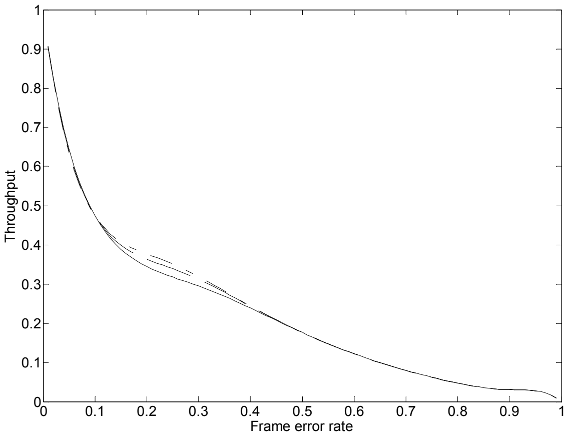

Fig. 2. Throughput of TS-ARQ system depending on the probability of transmission error; influence of parameter β: $N=10$, $n=2$, $\alpha=2$, $\gamma=10$, $\delta=2$, solid line for $\beta=20$, dashed line for $\beta=30$ and dash-dot for $\beta=40$.

It is clear from the Fig. 2. that in the range of error probability up to $P_e=0.3$, system throughput is increased with the increase of β parameter, and that maximum value is for $\beta=40$, the value calculated with proposed analysis. For all other P_e values, all three charts practically overlap.

Let us determine optimal ratio of parameters γ and δ. We will take, as in previous example, that $N=10$ and $n=2$. By solving the Eq. (17) by using divided interval method, with the accuracy of *0.0001,* we got that $P_{C2}=0.695$. By substituting this value into Eq. (19), we have $\gamma^*/\delta^*=3.27$. For $\delta^*=2$, we got approximate integer value for $\gamma^*=6$.

Fig. 3. illustrate throughput of TS-ARQ system for the values of optimized parameters. It is taken that $N=10$, $n=2$, $\alpha=2$, $\beta=2$, $\delta=2$, while γ values are changing and they are 6, 10 and 14.

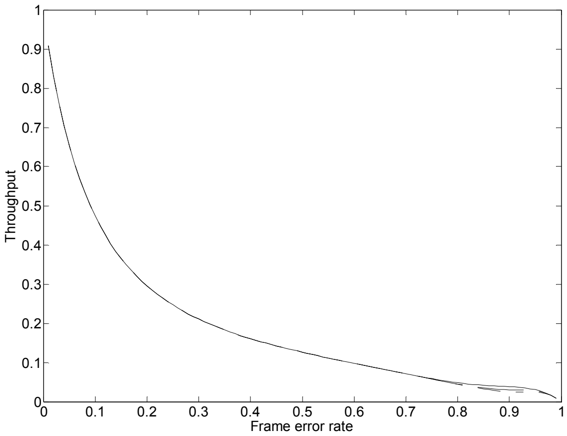

Fig. 3. Throughput of TS-ARQ system for values of optimized parameters: $N=10$, $n=2$, $\alpha=2$, $\beta=2$, $\delta=2$; parameter influence γ: full line for $\gamma=6$, dashed line for γ=10 and dash-dot for $\gamma=14$.

It is clear from the Fig. 3. that in the range of error probability up to $P_e=0.65$, where is expected for system to be in NGBN mode, system throughput is increased with the decrease of γ parameter and that maximum value is for γ=6, the value calculated with proposed analysis. For all other P_e values, all three charts practically match.

V. CONCLUSION

Proposed model, based on counting ACKs and NAKs, on the bases of which switching of the system mode into standard, n-copy and continual GBN is done, gives good results regarding throughput in regards to comparative results in case of high channel noise. From the standpoint of the proposed strategy, appropriate selection of control parameters, the optimization of satellite data transmission efficiency can be done.

Analysis of acquired results showed, with all stated limitations and approximations, correctness of the proposed method of optimization.

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