Theoretical Investigation on an Array of Dual Tuned Staggered Dipole Apertures by the Method of Moment and Comparison with Experimental Results

Arup Ray, Manisha Kahar, Partha Pratim Sarkar

Abstract – **A frequency selective surface (FSS) comprising of a two dimensional array of dipole apertures, resonant at two frequencies, within a metallic screen is proposed. A computationally efficient method for analyzing this FSS is used. The formulation is carried out in the spectral domain where the convolution form of the integral equation for the induced current reduces to an algebraic one and the Spectral-Galerkin technique is used to solve the resulting equation. Entire- domain basis function that satisfies the edge condition is introduced to expand the unknown induced current on the complementary structure i.e. an array of printed dipoles. Using Babinet's principle for complementary screen, the transmitted electric field for the structure of an array of aperture dipoles has been calculated. The theoretical practical data indicate that this structure can be used as a highly selective tuned bandpass filter for GPS and Wi-Max applications which is very resistant to variations of RF incidence angle of 90° (degree) rotations about any vertical axis, perpendicular to the FSS plane and passing through its centre**.

Keywords – **Frequency selective surface, Spectral-Galerkin technique, highly selective bandpass filter, Method of Moments, slotted dipoles, polarization independent**

I. INTRODUCTION

Frequency selective surfaces [1] (FSS), which find widespread applications as filters for microwave and optical signals, have been the subject of extensive studies in recent years. These surfaces comprise of periodically arranged metallic patch elements or aperture elements within a metallic screen and exhibit total reflection (patches) or transmission (aperture) in the neighbourhood of the element resonance. Typically an FSS consists of one or more grids supported by one layer or multiple layers of dielectrics. The grids are thin conducting periodic elements etched on thin substrates. As these grids can be designed to resonate at specified frequencies, the FSS will reflect waves at these frequencies and pass waves at other frequencies.

The IEEE 802.16 working group has established a standard known as Worldwide Interoperability for Microwave Access (WiMAX) which can reach a theoretical up to 30-mile radius coverage. Moreover, in the case of WiMAX, the highest theoretically achievable transmission rates are possible at 70 Mbps. As defined through IEEE Standard 802.16, a wireless MAN provides network access to buildings through exterior antennas communicating with central radio base stations (BSs).The potential applications of WiMAX are to provide backhaul support for mobile WiFi hotspots, providing a wireless alternative to cable and digital subscriber line (DSL) for "last mile" broadband access, providing data, telecommunications (VoIP) with IPTV services (triple play) and providing a source of Internet connectivity as part of a business continuity plan.

The WiMAX system with the IEEE 802.16e standard for broadband wireless access in the frequency bands between 2 and 6 GHz has also been proposed [2-3]. It has been projected that the avail-ability of communication devices capable of WiMAX operation will become more pronounced for users all over the globe. We propose in this paper a FSS system suitable to be applied in WiMAX communication devices, with the E911 function for locating wireless users. The E911 function is a technology that involves automatic user identification and locating wireless users. The Federal Communications Commission (FCC) mandated that all mobile phones in the United States must be equipped with E911 after December 2001. In order to provide the E911 function, GPS signal at 1575 MHz has been recommended, especially for operating in heavy multipath environments [4-5].

For this promising application, we have designed and analyzed an energy saving highly selective FSS that is transparent to GPS and Wi-Max radio wave frequencies. This FSS can be used in the metallic coating of the house windows [6] to provide the needed transparency. The periodic pattern used for the FSS is of the aperture type and the elements are slotted dipoles.

Experimental investigations on the frequency selective property of an array of tuned printed dipoles [7-9] have been carried out earlier. These FSSs suffered from two serious problems, firstly they were not very selective to the resonant frequency and secondly their frequency selective property was susceptible to changes of source polarization. In this paper our goal is to design a highly selective, polarization independent FSS, which will act like a band-pass filter to GPS and Wi-Max frequencies rejecting all other frequencies in the near vicinity and thus may be used in a multi-band reflector system.

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II. DESIGN OF THE FSS

Dipoles resonant at 1.5 GHz and 3.5 GHz were spaced as shown in Figure 1 and were repeated to form a two dimensional array of dual tuned staggered aperture dipoles. These dipole apertures were fabricated on a metallic screen on one side of a thin dielectric slab of thickness 1.6mm and the copper coating on the other side of the slab was completely removed. Area of the glass epoxy dielectric slab was 260mm x 260mm. Its dielectric constant was 4.7. At the above mentioned resonant frequencies the corresponding free space wavelengths are 200.00 mm and 85.72 mm. Hence the lengths of the rectangular dipoles should have been made equal to 100.00 mm and 42.86 mm (half wave length [10]). If f_r be the resonant frequency of the slotted dipoles etched on a thin copper sheet supported by thin dielectric substrate of dielectric constant ε_d then its effective resonant frequency

$$f_{eff} = \frac{f_r}{\sqrt{\frac{\varepsilon_a + \varepsilon_d}{2}}}$$

where ε_a = dielectric constant of air. This formula is valid for normal incidence of the electromagnetic wave.

As the dielectric constant of the substrate , ε_d , was 4.7 so due to dielectric loading effect [10] for the above mentioned length of dipoles the resonant frequencies become 0.88 GHz and 2.07 GHz respectively.

So instead tuned dipole lengths of 63.5 mm and 26.5 mm were taken which were close to half wave length corresponding to frequencies of 2.53 GHz and 5.91 GHz considering the loading effect. Thus although we are actually designing the FSS to resonate at 2.53GHz and 5.91GHz due to inherent loading effect it is expected to resonate at 1.5GHz and 3.5GHz. Breadth of each dipole was taken as 2mm (maximum allowed thickness for a dipole is one tenth of its length). Two types of dipoles were placed one after another in crossed rows and columns as shown in Figure 1. The spacing between any two dipole apertures was so chosen that the rule governing a conventional array antenna is maintained. Grating lobes appear when spacing between two adjacent patches becomes electrically large. A general rule is that the spacing between adjacent similar FSS element should be less than one wavelength for the broadside-incident case (0-degree incident angle) [11]. Here the optimized spacing between two adjacent shorter dipole apertures in rows and columns was 43.00 mm in one direction and 59.5mm in the orthogonal perpendicular direction. Likewise the spacing between adjacent taller dipole apertures in rows and columns was 86.00 mm in one direction and 108.5 mm in the orthogonal perpendicular direction. All the dimensions are shown in Fig 1 and a fabricated prototype of the FSS is shown in Fig 2.

III. FORMULATION OF THEORITICAL INVESTIGATION

In this section the integral equation is developed [10] for the scattering from a complementary frequency selective surface of the type shown in Figure 1 (i.e an array of printed dipole apertures) which is assumed to be perfectly conducting in nature. Let $\vec{J}$ be the current density induced on the FSS due to a given incident field and k_0 is the free space wave number. The scattered electric field in the Fourier transformed domain $\tilde{\tilde{E}}_x^s(\alpha,\beta)$ at Z = 0 can be derived from [12].

$$\begin{bmatrix} \tilde{E}_x^S(\alpha,\beta) \\ \tilde{E}_y^S(\alpha,\beta) \end{bmatrix} = \frac{1}{j\omega\varepsilon_0} \begin{bmatrix} k_0^2-\alpha^2 & -\alpha\beta \\ -\alpha\beta & k_0^2-\beta^2 \end{bmatrix} \tilde{\tilde{G}}(\alpha,\beta) \begin{bmatrix} \tilde{J}_x(\alpha,\beta) \\ \tilde{J}_y(\alpha,\beta) \end{bmatrix} \quad (1)$$

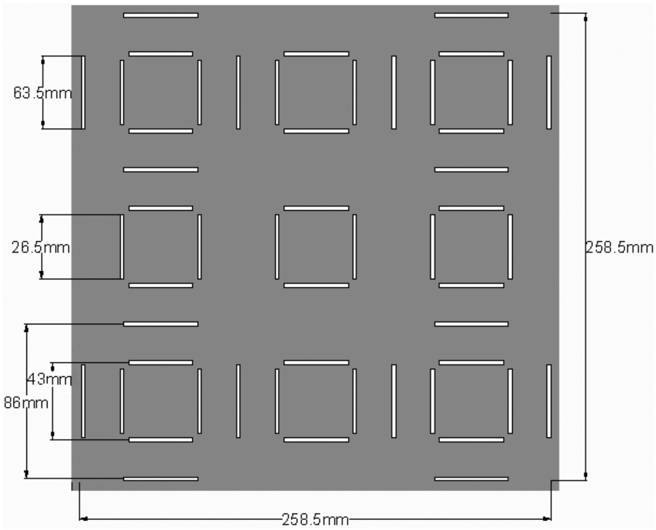

Fig. 1. Frequency Selective Surface(FSS) under investigation

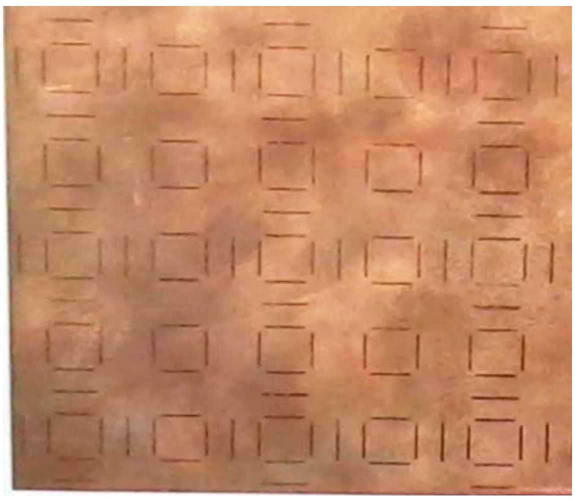

Fig. 2. Prototype of Frequency Selective Surface

Where,

$$\tilde{\tilde{G}}(\alpha,\beta)=\frac{-j}{2\sqrt{k_0^2-\alpha^2-\beta^2}}\vec{I} \quad \text{for } k_0^2>\alpha^2+\beta^2$$

$$=\frac{1}{2\sqrt{\alpha^2+\beta^2-k_0^2}}\vec{I} \quad \text{for } k_0^2<\alpha^2+\beta^2$$

On the conducting patch, the tangential electric field vanishes,
So,

$$E_{Tan}^{S}+E_{Tan}^{inc}=0 \ \text{[on the conducting patch]}$$

i.e.

$$E_{Tan}^{S}=-E_{Tan}^{inc}$$

Since the structure is periodic $\tilde{\tilde{J}}(\alpha,\beta)$ has a discrete spectrum in the transformed domain i.e. $\tilde{\tilde{J}}(\alpha,\beta)$ is non-zero for discrete values of α,β viz. α_{mm},β_{mm} which correspond to the Floquet's modes. So while taking the inverse Fourier transform of (1), the integral form can be converted into a summation form.

Putting $E_{Tan}^{S}=-E_{Tan}^{inc}$ = $E^{S}=-E^{inc}$ and taking inverse Fourier transform of (1) we can get,

$$\begin{bmatrix}\tilde{E}_x^S(x,y)\\ \tilde{E}_y^S(x,y)\end{bmatrix}=-\begin{bmatrix}E_x^{inc}\\ E_y^{inc}\end{bmatrix}=\frac{1}{j\omega\varepsilon_0}\times\frac{1}{(2\pi)^2ab}\sum_{m=-\alpha}^{+\alpha}\sum_{n=-\alpha}^{+\alpha}\begin{bmatrix}k_0^2-\alpha_m^2 & -\alpha_m\beta_n\\ -\alpha_m\beta_n & k_0^2-\beta_n^2\end{bmatrix}$$

$$\times\tilde{\tilde{G}}(\alpha_m,\beta_n)\begin{bmatrix}\tilde{J}_x(\alpha_m,\beta_n)\\ \tilde{J}_y(\alpha_m,\beta_n)\end{bmatrix}e^{j\alpha_m x}e^{j\beta_n y} \qquad (2)$$

where, a is the periodicity in x direction, and b is the periodicity in y direction.

$$\alpha_m=\frac{2m\Pi}{a}+k_x^{inc}$$

$$\beta_n=\frac{2n\Pi}{b}+k_y^{inc}$$

$$k_x^{inc}=k_0\sin\theta\cos\varphi$$

$$k_y^{inc}=k_0\sin\theta\sin\varphi$$

[Where θ, is the angle of incident wave with the Z-axis and φ is the angle of the projection of the incident wave on the XY-plane with the X- axis. (Shown in Fig. 3.)]

Next our goal is to solve the equation (2) using Galerkin's procedure and to calculate $\tilde{\tilde{J}}$, surface current density in the transformed domain. For this we first express the unknown $\vec{J}$ in terms of an appropriate basis function.

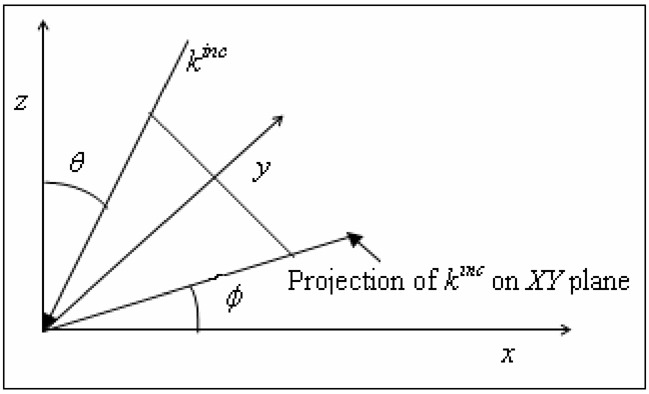

Fig. 3. The direction of electromagnetic wave incident on the FSS

IV. BASIS FUNCTION SELECTION , SOLUTION AND EXPERIMENTAL MEASUREMENT

For easy and faster calculation Raj Mittra et. al. [13] suggested the basis functions for different FSS structures. Here we have considered an array of printed dipoles as the frequency selective structure. For this FSS structure Basis function suggested by Mittra [13] is given below.

$$\vec{J}=\sum_i(\hat{x}C_{x_i}f_{x_i}+\hat{y}C_{y_i}f_{y_i}) \qquad (3)$$

Where

$$f_{x_i}=0 \quad \text{i.e.} \quad J_x=0 \qquad (4)$$

because the x-dimension is 1/10th the y-dimension, the former dimension can be ignored, and

$$J_{y_i}=f_{y_i}=Sin\frac{i\Pi}{L}(y+\frac{L}{2})\times P_y(0,L) \quad i=1,\ldots$$

L = Length of the dipole

$$P_y(0,L)=1 \quad \text{for } |y|=\frac{L}{2}$$

$$=0 \quad \text{otherwise} \qquad (5)$$

Let us write the matrix equation (2) in the symbolic dyadic form.

$$L(u)=g \quad \text{where } g \text{ is the known matrix} \quad \begin{bmatrix}\vec{E}_x^{inc}\\ \vec{E}_y^{inc}\end{bmatrix}$$

$$u \quad \text{is the matrix to be determined} \quad \begin{bmatrix}\tilde{\tilde{J}}_x(\alpha_m,\beta_n)\\ \tilde{\tilde{J}}_y(\alpha_m,\beta_n)\end{bmatrix}$$

and L is the total operator involved in matrix equation (2)
Now we can write,

$$\left(\hat{x}\hat{x}L_{xx}+\hat{x}\hat{y}L_{xy}+\hat{y}\hat{x}L_{yx}+\hat{y}\hat{y}L_{yy}\right).\left(\hat{x}u_x+\hat{y}u_y\right)=\hat{x}g_x+\hat{y}g_y \qquad (6)$$

So, considering y-dimensions only the above equation (6) is rewritten as

$$L_{yx}u_y+L_{yy}u_y=g_y$$

$$L_{yx}\sum_{j=1}^{N}C_{x_j}f_{x_j}+L_{yy}\sum_{j=1}^{N}C_{y_j}f_{y_j}=g_y \qquad (7)$$

u is the basis function which is to be chosen. In generalized form

$$u=\sum_{i=1}^{N}(\hat{x}C_{x_i}f_{x_i}+\hat{y}C_{y_i}f_{y_i}) \qquad (8)$$

taking the inner product of f_{y_i} and equation (6) we get,

$$\langle f_{y_i},L_{yx}\sum_{j=1}^{N}f_{x_j}C_{x_j}\rangle+\langle f_{y_i},L_{yy}\sum_{j=1}^{N}f_{y_j}C_{y_j}\rangle=\langle f_{y_i},g_y\rangle \qquad (9)$$

$$\text{or,}\ \sum_{j=1}^{N}\langle f_{y_i},L_{yx}f_{x_j}\rangle C_{x_j}+\sum_{j=1}^{N}\langle f_{y_i},L_{yy}f_{y_j}\rangle C_{y_j}=\langle f_{y_i},g_y\rangle \qquad (10)$$

In short form, equations (10) is written in the following manner

$$\sum_{j=1}^{N}a^{yx}{}_{ij}C_{x_j}+\sum_{j=1}^{N}b^{yy}{}_{ij}C_{y_j}=\lambda^y{}_i \qquad (11)$$

where, $a^{yx}{}_{ij}=\langle f_{y_i},L_{yx}f_{x_j}\rangle=0$, from (4);

$b^{yy}{}_{ij}=\langle f_{y_i},L_{yy}f_{y_j}\rangle$ and $\lambda^y{}_i=\langle f_{y_i},g_y\rangle$

In short form after simplification, equations (11) is written in the following manner

$$\begin{bmatrix} b^{yy}_{11} & b^{yy}_{12}\ldots\ldots\ldots\ldots b^{yy}_{1N} \\ b^{yy}_{21} & b^{yy}_{22}\ldots\ldots\ldots\ldots b^{yy}_{2N} \\ . & . \qquad . \\ . & . \qquad . \\ . & . \qquad . \\ b^{yy}_{N1} & b^{yy}_{N2}\ldots\ldots\ldots\ldots b^{yy}_{NN} \end{bmatrix}\begin{bmatrix} C_{y_1} \\ C_{y_2} \\ . \\ . \\ . \\ C_{y_N} \end{bmatrix}=\begin{bmatrix} \lambda^y_1 \\ \lambda^y_2 \\ . \\ . \\ . \\ \lambda^y_N \end{bmatrix} \qquad (12)$$

(12)

Here,

$$b^{yy}{}_{ij}=\langle f_{y_i},L_{yy}f_{y_j}\rangle$$

$$=\int_{-\frac{L}{2}}^{+\frac{L}{2}}\sin\frac{i\Pi}{L}\left(y+\frac{L}{2}\right)L_{yy}\sin\frac{j\Pi}{L}\left(y+\frac{L}{2}\right)dy \qquad (13)$$

and

$$\lambda^y_i=\langle f_{y_i},g_y\rangle=\int_{-\frac{L}{2}}^{+\frac{L}{2}}\sin\frac{i\Pi}{L}\left(y+\frac{L}{2}\right).1dy \qquad (14)$$

[Here g_y is the incident electric field which has been assumed to be 1.]

So, using the values of b^{yy}_{ij} and λ^y_i from equations (13) and (14); the matrix equation (12) may be solved to calculate $C_{y_1},C_{y_2},...,C_{y_N}$. So, now, u is known in equation (8), that is,

$$u=J_{yi}=\sum_{i=1}^{N}C_{y_i}f_{y_i}=C_{y_1}\sin\frac{\Pi}{L}\left(y+\frac{L}{2}\right)+C_{y_2}\sin\frac{2\Pi}{L}\left(y+\frac{L}{2}\right)+ ...+C_{y_N}\sin\frac{N\Pi}{L}\left(y+\frac{L}{2}\right)$$

Considering this basis function we have calculated using MATLAB *Jxi* (which is 0 here) and *Jyi* and their Fourier transforms $\tilde{J}_x(\alpha,\beta)$ and $\tilde{J}_y(\alpha,\beta)$ assuming $E^{inc}_y=e^{-jk_0y}$ and $E^{inc}_x=0$. Next the values of $\tilde{J}_x(\alpha,\beta)$ and $\tilde{J}_y(\alpha,\beta)$ have been put in equation (2) to get the scattered electric field $E^s_x(x,y)$ and $E^s_y(x,y)$. Now from this scattered electric field, the transmission coefficients of the structure of mode *m, n* due to incident mode *k, l* may be readily calculated [12]-[13] as.

$$T_{TM}=\frac{j\omega\varepsilon_0E^t_z}{\left(k_0^2+\gamma_{mn}^2\right)}\sqrt{\frac{\gamma_{mn}\left(k_0^2+\gamma_{mn}^2\right)}{\gamma_{kl}\left(k_0^2+\gamma_{kl}^2\right)}}$$

where

$$E^t_z=j\frac{(\alpha_mE^s_x+\beta_nE^s_y)}{\gamma_{mn}}$$

$$\gamma_{mn}=-j\left(k_0^2-\alpha_m^2-\beta_n^2\right)^{1/2} \qquad for\ k_0^2>\alpha_m^2+\beta_n^2$$

$$or,\quad -\left(\alpha_m^2+\beta_n^2-k_0^2\right)^{1/2} \qquad for\ k_0^2<\alpha_m^2+\beta_n^2$$

We know that

$$\text{transmission coefficient} = \frac{\textit{Transmitted Electric Field}}{\textit{Incident Electric Field}}$$

Here we have considered incident electric field = 1.

So, transmission coefficient is equal to the transmitted electric field here.

From the transmission coefficients transmitted electric field has been calculated for the array of printed dipoles. Then following Babinet's principle for complementary screen [10, 14], the transmission coefficients for the array of slotted dipoles (i.e our designed FSS structure shown in Figure 1) have been calculated and normalized transmitted electric fields in dB vs frequency have been calculated separately for the frequencies of 1.5 GHz and 3.5 GHz. Then averaging these two results the composite plot of normalised transmitted electric fields in dB vs frequency has been drawn as shown in Fig. 4.

Transmission test for the FSS structure has been performed using standard microwave test bench, shown in Fig. 5 .The transmitting horn antenna is connected to a variable R.F oscillator (Marconi Microwave Source, Model No: 6058B) and the receiving horn antenna to a Agilent Power meter (Model No. Agilent E4418B) with power sensor (Model No. Agilent E4412A). The proposed FSS is kept on a wooden frame stand at far field distance of the transmitting horn antenna. The transmitting horn antenna connected to the R.F oscillator radiates power which is blocked at all frequencies except at 1.5 GHz and 3.5 GHz. This is indicated from the readings of the power meter in dBm. These values were normalised and a graph of measured normalised transmitted electric fields vs frequency for the FSS structure, shown in Fig.2, was obtained as shown by dashed lines in Fig. 4. It is interesting to note that computed results are in good agreement with measured data.

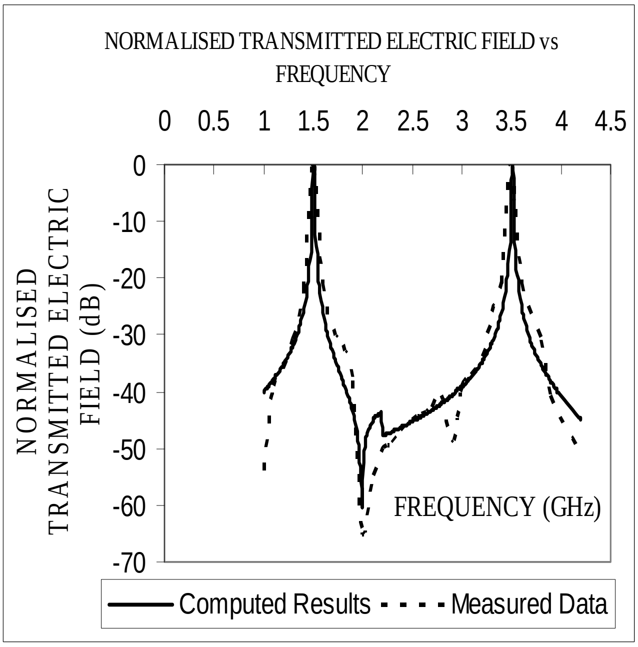

Fig. 4. Normalized Transmitted Electric Field vs Frequency

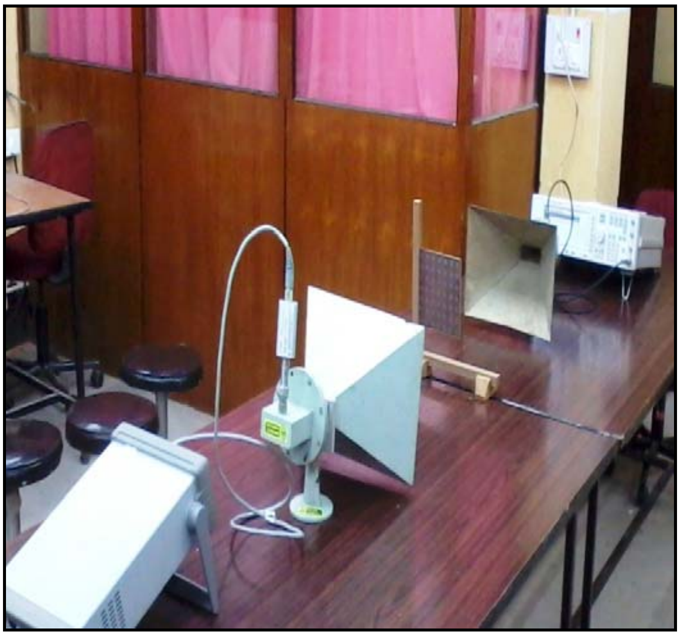

Fig 5. Experimental setup

V. Conclusion

Maximum transmission through the FSS occurs at the frequencies of l.5 GHz and 3.5 GHz. The -10 dB transmission band widths from the FSS structure are 0.03 GHz and 0.04 GHz for resonant frequencies of 1.5 GHz and 3.5 GHz respectively .It is interesting to note that measured results are in good agreement with theoretical data. This is not only interesting but also encouraging as we have been able to achieve a tuned, highly selective FSS resonating at desired frequencies to allow GPS and Wi-Max communications .More over this FSS structure although constituted of aperture type dipoles, which are inherently very sensitive to changes of source polarization angles, [10] this designed FSS is however very resistant to variations of RF incidence angle of 90° (degree) rotations about any vertical axis, perpendicular to the FSS plane and passing through its centre.

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