Physical Optics Driven Method of Moments Based on Adaptive Grouping Technique

Miodrag S. Tasić, Branko M. Kolundžija[1]

Abstract – **Recently we introduced a new iterative method for analysing large perfectly conducting scatterers, called Physical Optics Driven Method of Moments (PDM). PDM performs grouping of original basis functions and creates macro basis functions using these groups. In this paper we present PDM results using variable number of groups per iteration.**

Keywords – **Method of moments, Basis functions, Physical optics, Perfectly conducting scatterers, Iterative methods**.

I. INTRODUCTION

Surface integral equations (SIEs) of electromagnetic field in frequency domain can be solved using method of moments (MoM) [1]. MoM transforms SIEs into a system of linear equations, which unknowns are weighting coefficients of adopted basis functions (BFs). MoM solution is expressed as a finite series (linear combination of BFs) so, essentially, it is approximate. However, by proper choice of BFs, the solution converges toward exact solution when number of BFs increases (complete set of BFs), i.e. it is numerically exact. The main drawback of MoM is poor scalability - the number of BFs per wavelength squared is fixed, hence total number of BFs (N) raising fast by increasing frequency. Furthermore, memory occupancy is $O(N^2)$, and CPU solution time is $O(N^3)$.

Different strategies for overcoming this problem are proposed: hybridization with asymptotic techniques [2,3], speeding up matrix vector product in iterative solution of MoM system of equations [4], compressing MoM matrix [5], and, among all, using specific BFs [6,7]. The idea behind specific BFs is to construct BFs which covers larger surfaces (than typical BFs), having in mind particular geometry and excitation of the problem. Recently we proposed method [8] which, in a way, belongs to specific BFs category. The method is iterative and it converges toward MoM solution by employing correctional currents created in physical optics (PO) manner. That is why the method was called PO driven MoM (PDM).

PDM is formulated as method for analyzing perfectly conducting closed scatterers. Particularly, PDM is well suited for electrically large problems. PDM starts with physical optics (PO) solution and then tries to improve it. To do so, PDM estimate correctional values for MoM unknowns (weighting factors for BFs), then groups BFs which might have the similar level of estimation quality, create macro basis functions (MBFs) using BFs groups, and finally determines weighting coefficients for MBFs in a way to minimize difference with respect to MoM. PDM needs less memory and less CPU time, paid with poorer accuracy than MoM. However, from engineering point of view, PDM can provide sufficient accuracy in CPU time unreachable to MoM.

In [8] we noticed that convergence rate of PDM decreases trough the iterations. In this paper we will present simple modification that can improve convergence of PDM to some extent.

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II. PO DRIVEN MOM (PDM)

A. Formulation of the Problem

A closed body made of perfect electric conductor (PEC), is placed in vacuum and excited by a time–harmonic incident electromagnetic field of frequency f and electric and magnetic field vectors $\mathbf{E}_{\text{inc}}$ and $\mathbf{H}_{\text{inc}}$. As a result, electric currents of density $\mathbf{J}_s$ are induced over the body surface, so that the incident field inside the body is annihilated. Once these currents are determined, all other quantities of interest can be easily evaluated. Currents $\mathbf{J}_s$ can be determined by solving electric field integral equation (EFIE) [9], which belongs to the linear operator equations in general form

$$L\mathbf{f} = \mathbf{g} \tag{1}$$

where $\mathbf{g}$ is known vector function (excitation), L is linear operator, and $\mathbf{f}$ is unknown vector function to be determined (response).

B. MoM

MoM is general method for solution of linear operator equation given by (1). Unknown function $\mathbf{f}$ is approximated by linear combination of N known vector BFs $\mathbf{f}_k$ multiplied by unknown coefficients a_k

$$\mathbf{f}_{\text{a}} = \sum_{k=1}^{N} a_k \mathbf{f}_k \ . \tag{2}$$

This approximation results in error of solution, $\mathbf{E} = \mathbf{f} - \mathbf{f}_{\text{a}}$, and error in satisfying equation (1), called residuum, $\mathbf{R} = L\mathbf{f}_{\text{a}} - \mathbf{g}$. What MoM does is choosing BFs that can represent unknown

function $\mathbf{f}_k$ and then to adjust coefficients a_k in order to minimize errors of solution in some sense. For that purpose residuum is forced to be "orthogonal" to the space of so called test functions w_j using inner product $\langle \mathbf{w}_j, \mathbf{R} \rangle = 0,\ j = 1,\ldots,N$.

Thus we obtain the system of linear equations, which solution gives us coefficients a_k, i.e. approximate solution (2),

$$\sum_{k=1}^{N} z_{jk} a_k = v_j, \quad j = 1,\ldots,N \tag{3}$$

where $z_{jk} = \langle \mathbf{w}_j, L\mathbf{f}_k \rangle$ and $v_j = \langle \mathbf{w}_j, \mathbf{g} \rangle$.

C. Theory of PDM

PDM takes over MoM geometrical modelling, BFs ($\mathbf{f}_k$), elements of MoM matrix (z_{jk}), and right hand side terms (v_j). The difference is that PDM does not solve the system of equations (3) – which is the most costly MoM operation – but determines a_k in iterative procedure.

Suppose we have PDM solution for $\mathbf{J}_s$ in iteration i-1, $\mathbf{f}_a^{(i-1)}$. Since the solution is approximate, total magnetic field just below the surface of the (closed) PEC body, $\mathbf{H}_{tot}^{(i-1)}$, will exist (whereas it should be zero for exact solution). Note that this total field is a sum of magnetic field from sources outside the body (i.e. known incident magnetic field $\mathbf{H}_{inc}$), and magnetic field due to surface currents (equivalent surface sources) $\mathbf{f}_a^{(i-1)}$, $\mathbf{H}\left(\mathbf{f}_a^{(i-1)}\right)$.

We introduce correctional surface currents

$$\Delta\mathbf{J}_s^{(i)} = 2\mathbf{n} \times \mathbf{H}_{tot}^{(i\text{-}1)}(\mathbf{r}), \mathbf{r} \in S^- \tag{4}$$

where $\mathbf{r}$ is a position vector, and $\mathbf{n}$ is unit vector normal to the surface S of the body and directed outwards. Eq. (4) states that correctional current at some point of the surface is calculated using magnetic field just below that point. Locally, these currents should cancel existing magnetic field below the surface (at spatial point). Globally, the cancelation will not happen because all other surface currents (other than that particular at spatial point) will change the field at spatial point. Nevertheless, currents given by (4) are useful lead in what direction the correction should go. In order to use them, we need to express them as linear combination of basis functions, in form given by (2)

$$\Delta\mathbf{J}_s^{(i)} \cong \sum_{k=1}^{N} a_k^{(i)} \mathbf{f}_k \ . \tag{5}$$

Coefficients $a_k^{(i)}$ are determined in a way to minimize residuum given by

$$R_{\Delta\mathbf{J}_s^{(i)}} = \int_S \left| \Delta\mathbf{J}_s^{(i)} - \sum_{k=1}^{N} a_k^{(i)} \mathbf{f}_k \right|^2 \mathrm{d}S \ . \tag{6}$$

Once we determine these coefficients, we can use them to build macro basis functions (MBFs)

$$\mathbf{F}_l^{(i)} = \sum_{k=1}^{N} b_{lk}^{(i)} a_k^{(i)} \mathbf{f}_k \ , \quad l = 1,\ldots,M^{(i)}. \tag{7}$$

Let us look closer at (7). In ith iteration we build $M^{(i)}$ MBFs. At first glance it seems that each MBF is linear combination of all BFs $\mathbf{f}_k$. But, by setting weighting coefficient $b_{lk}^{(i)}$ to zero, kth BF is omitted from lth MBF in ith iteration. Generally, $0 \le b_{lk}^{(i)} \le 1$, so we can include kth BF to one or more MBFs.

PDM approximate solution in nth iteration (n>0) is expressed as linear combination of all existing MBFs

$$\mathbf{f}_a^{(n)} = \sum_{i=0}^{n} \sum_{l=1}^{M^{(i)}} c_{il}^{(n)} \mathbf{F}_l^{(i)}, \ n > 0 \tag{8}$$

where $c_{il}^{(n)}$ are unknown coefficients that should be determined. By substituting expression for MBF from (7) in (8), and after some rearrangements, (8) can be written as

$$\mathbf{f}_a^{(n)} = \sum_{k=1}^{N} A_k^{(n)} \mathbf{f}_k \tag{9}$$

where

$$A_k^{(n)} = \sum_{i=0}^{n} \sum_{l=1}^{M} c_{il}^{(n)} b_{lk}^{(i)} a_k^{(i)} \ . \tag{10}$$

PDM solution given by (9) has the same form as MoM solution given by (2). However, coefficients $A_k^{(n)}$ generally won't satisfy system of equations given by (3), and instead will generate residuum for each equation

$$R_j^{(n)} = \sum_{k=1}^{N} z_{jk} A_k^{(n)} - v_j, \quad j = 1,\ldots,N \ . \tag{11}$$

The mean square residuum after nth iteration is calculated as $R^{(n)} = \frac{1}{N} \sum_{j=1}^{N} \left| R_j^{(n)} \right|^2$. After some rearrangements, it can be written as

$$R^{(n)} = \frac{1}{N} \sum_{j=1}^{N} \left| \sum_{i=0}^{n} \sum_{l=1}^{M} c_{il}^{(n)} Z_{jl}^{(i)} - v_j \right|^2 \tag{12}$$

where

$$Z_{jl}^{(i)} = \sum_{k=1}^{N} z_{jk} b_{lk}^{(i)} a_k^{(i)} . \quad (13)$$

By imposing condition that residuum (12) should be minimized (with proper choice of coefficients $c_{il}^{(n)}$) we obtain (PDM) system of equations

$$\sum_{i=0}^{n} \sum_{l=1}^{M^{(i)}} c_{il}^{(n)} \left(\sum_{j=1}^{N} Z_{jl}^{(i)} Z_{jm}^{(k)*} \right) = \sum_{j=1}^{N} v_j Z_{jm}^{(k)*} \quad (14)$$
$$k = 0,\ldots,n, \quad m = 1,\ldots,M^{(i)}$$

from which we determine coefficients $c_{il}^{(n)}$, i.e. approximate PDM solution in *n*th iteration, given by (7).

In system of equations (13), *i*=0 refers to initial solution (0th iteration). As initial solution we use physical optics (PO) currents, obtained as

$$\mathbf{J}_{\mathrm{s}}^{\mathrm{PO}}(\mathbf{r}) = \begin{cases} 0, \mathbf{r} \in \text{shadow region of } S \\ 2\mathbf{n} \times \mathbf{H}_{\mathrm{inc}}(\mathbf{r}), \mathbf{r} \in \text{lit region of } S \end{cases} . \quad (15)$$

We treat these PO currents similar to correctional currents (4) – we express them in form (2)

$$\mathbf{J}_{\mathrm{s}}^{\mathrm{PO}} \cong \sum_{k=1}^{N} a_k^{(0)} \mathbf{f}_k \quad (16)$$

and then use coefficients $a_k^{(0)}$ to create MBFs in 0th iteration. Though these MBFs can be used to create solution of the form (8), we use (16) as initial PDM solution.

The key point of using PDM is to avoid solution of MoM system of equations (3) – for large *N*, it becomes highly inefficient. Instead, in each iteration we solve PDM system of equations (14). The order of PDM system (i.e. the number of MBFs) is (for large *N*) much lower than *N*, enabling greater efficiency than MoM.

D. Realization of PDM

For the MoM part of PDM we use WIPL-D kernel [10]. EFIE is transformed into the system of linear equations using Galerkin testing (BFs are used as testing functions) and the system is solved using LU decomposition.

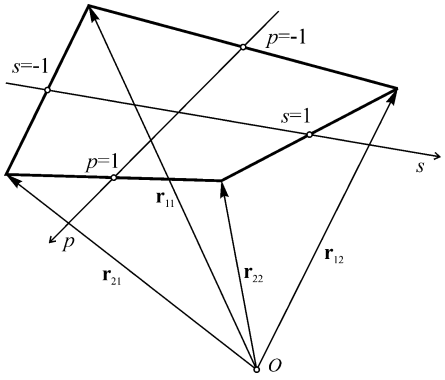


Fig. 1. Sketch of bilinear surface

Geometrical modeling is performed using curved quadrilaterals (plates), shown in Fig. 1, described by parametric equation

$$\mathbf{r}(p,s) = \frac{1}{4} \sum_{i=1}^{2} \sum_{j=1}^{2} \mathbf{r}_{ij} [1 + (-1)^i p][1 + (-1)^j s] . \quad (17)$$
$$-1 \le p, s \le 1$$

The currents distributed over a plate are decomposed into two components in local *pOs* coordinate system. In particular, the *s*-component is expanded as

$$\mathbf{J}_{\mathrm{s}}(p,s) = \sum_{i=0}^{n_p - 1} \sum_{j=0}^{n_s} a_{ij} \mathbf{P}_{ij}(p,s) \quad (18)$$

$$\mathbf{P}_{ij}(p,s) = \frac{\mathbf{a}_s}{2|\mathbf{a}_p \times \mathbf{a}_s|} p^i \begin{cases} (1-s)/2 & j=0 \\ (1+s)/2 & j=1 \\ s^j - s^{j-2} & j>1 \end{cases} \quad (19)$$

where n_p and n_s are orders of approximation along *p* and *s* coordinates, a_{ij} are unknown coefficients, $\mathbf{P}_{ij}(p,s)$, j = 0,1, are edge BFs, $\mathrm{P}_{ij}(p,s)$, $j > 1$, are patch BFs, $\mathbf{a}_p = \partial \mathbf{r}(p,s)/\partial p$, and $\mathbf{a}_s = \partial \mathbf{r}(p,s)/\partial s$. The *p*-component expansion is obtained by interchanging the coordinates *p* and *s* in expressions (18) and (18). Edge BFs are common for two plates and are called doublets, whereas patch BFs are defined on a single plate and are called singlets.

Correctional currents (4) are calculated in discreet points on the surface of the body – sufficient number is $(n_p+1)(n_s+1)$. Condition (6) is imposed for each patch separately. Hence doublets will be calculated twice and mean value will be adopted as corresponding coefficient $a_k^{(i)}$ in (5).

Magnetic field should be calculated in the body, just below these points. In each calculation point this field can be decomposed into the part due to currents in the point just above that point and the part due to all other currents. Since we can calculate the first part using boundary condition, and the second part is practically the same on the surface and just

below the surface, we can use points on the surface for magnetic field calculations also.

In creating MBFs according to (7) initially (in 0th iteration) we use indirect approach. First we create groups of physically connected plates (see [8] for details). For each group of plates we create one group of BFs, including all BFs defined over the plates of the group. Now we can create MBFs using groups of BFs. Coefficient $b_{lk}^{(0)}$ will be 1 if BF $\mathbf{f}_k$ belongs only to lth group of BFs, will be 1/2 if BF $\mathbf{f}_k$ belongs to the lth group and one other group of BFs, and will be 0 otherwise. In subsequent iterations such scheme (grouping) can be preserved or can be changed. Note that, in 0th iteration, MBFs created using groups of BFs which are completely in the shadow area (where PO currents are zero) are zero, so we omit them.

III. NUMERICAL RESULTS

Consider PEC airplane, about 40λ long, placed along *z*-axis in Cartesian coordinate system, with wings spanned in *xOz* plane, as shown in Fig. 2a. The airplane is excited by circularly polarized plane wave incoming from direction given by angles $\phi = -45°$ and $\theta = 45°$ (θ is measured from *xOy* plane to *z*-axis). The airplane is modeled by 7445 plates. The 2nd order approximation is used for almost all plates, so that total number of MoM basis functions is $N = 59201$. We use $M = 643$ groups of plates (386 are in lit zone), shown in Fig. 2b, for creation of initial MBFs.

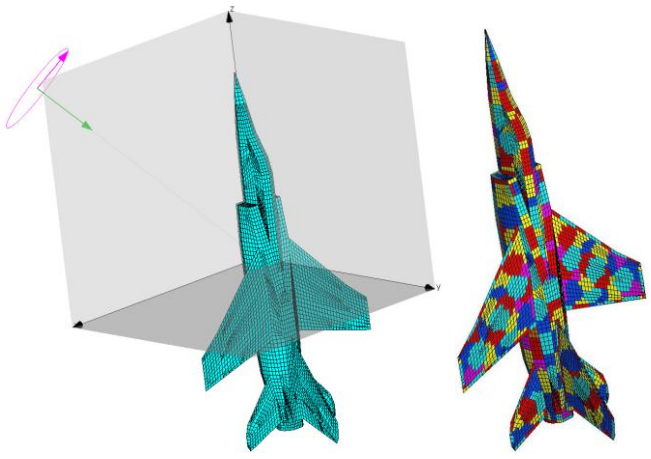


Fig. 2. a) PEC airplane, excited by circularly polarized wave, modeled with 7445 plates, and b) 643 groups of plates

First we performed PDM analysis keeping initial grouping scheme throughout the PDM analysis. It means that we have 643 fixed groups of BFs (additional 643 MBFs in each iteration). In each iteration we calculate residuum (11) for each BF, and then we square and normalize its module

$$R_{\mathrm{BF}k}^{(n)}(\%) = \frac{\left|R_k^{(n)}\right|^2}{\sum_{j=1}^{N}\left|R_j^{(n)}\right|^2} 100,\ k = 1,..,N\ . \tag{20}$$

Then we calculate residuum for each group of BFs

$$R_{\mathrm{G}l}^{(n)} = \sum_{k=1}^{N} b_{lk}^{(n)} R_{\mathrm{BF}k}^{(n)},\ l = 1,..,M\ , \tag{21}$$

and finally we calculate cumulative residuum

$$R_{\mathrm{CUM}}^{(n)}(j) = 100 - \sum_{l=1}^{j} R_{\mathrm{G}l}^{(n)},\quad j = 0,..,M\ . \tag{22}$$

When calculating cumulative residuum using (22) we suppose that $R_{\mathrm{G}l}^{(n)} \geq R_{\mathrm{G}l+1}^{(n)}$, i.e. we sorted residua of groups of BFs in non-growing order. Hence, cumulative residuum says which percent of total residuum will preserve after we "remove" *j* BFs groups with highest residua. Cumulative residuum for number of iterations *n* from 1 to 7 is shown in Fig. 3. We can see that e.g. after seven iterations about 100 groups of BFs are carrying about 60% of total residuum.

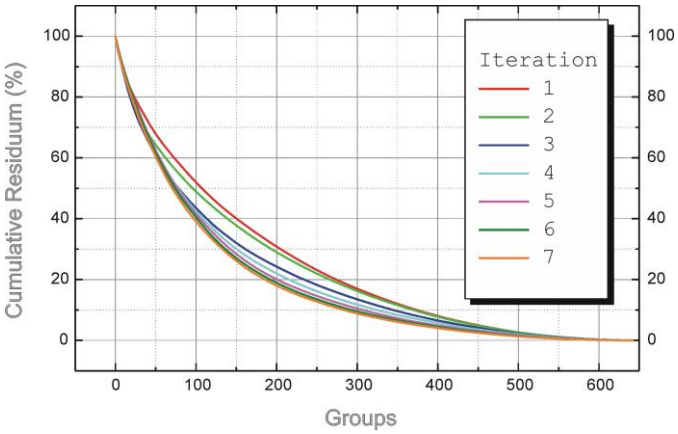


Fig. 3. Normalized cumulative residuum for different number of iterations

Can we improve convergence if we split these "bad" groups? In order to examine this, we will modify grouping scheme a little bit. After each iteration *n* (*n*>0) we will calculate cumulative residuum (22) for existing groups of BFs. Then we will find minimal j_c such that

$$R_{\mathrm{CUM}}^{(n)}(j_c) \leq R_{\mathrm{CUM}}^{\mathrm{cutoff}} \tag{21}$$

where $R_{\mathrm{CUM}}^{\mathrm{cutoff}}$ is cumulative residuum cut off value for groups, which we will adopt. Then we will split each group of BFs from 1 to j_c in two (if $j_c = 0$ there is no splitting).

Does it make difference how we will split the groups of BFs? To check this, we will calculate normalized cumulative residuum for each group and set cut off value for BFs in the

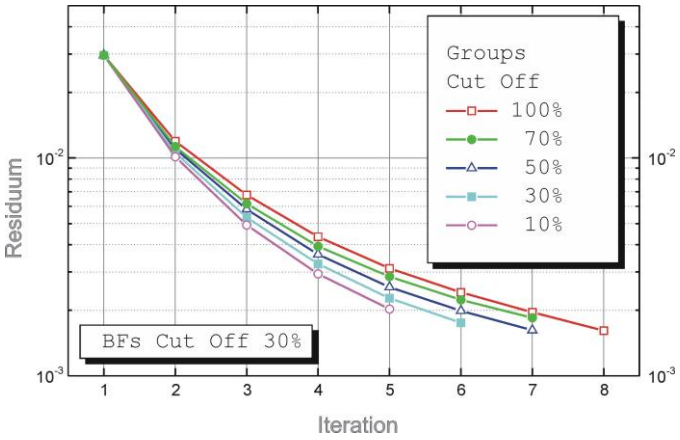
same way as for groups. Then we will split BFs from single group in two groups using cut of value as threshold.

Presented technique for splitting groups will be referred to as adaptive grouping technique (AGT), and the method as PDM based AGT.

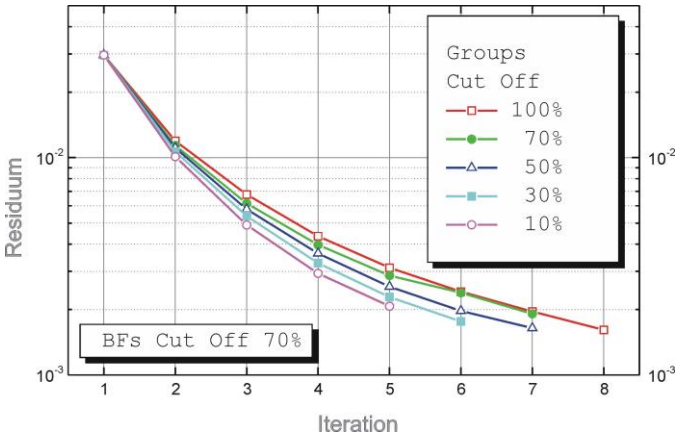
Integral measure of PDM convergence is Residuum

$$R_{\text{nom}}^{(n)} = \frac{R^{(n)}}{\frac{1}{N}\sum_{j=1}^{N}\left|v_j\right|^2}. \tag{23}$$

Residuum, as defined in (23), can take values between 0 (for MoM solution) and 1 (zero solution). We expect value about 0.01 for sufficiently accurate solution, and about 0.001 for excellent agreement with MoM result. The rate of Residuum decrease is the speed of PDM convergence.



a)

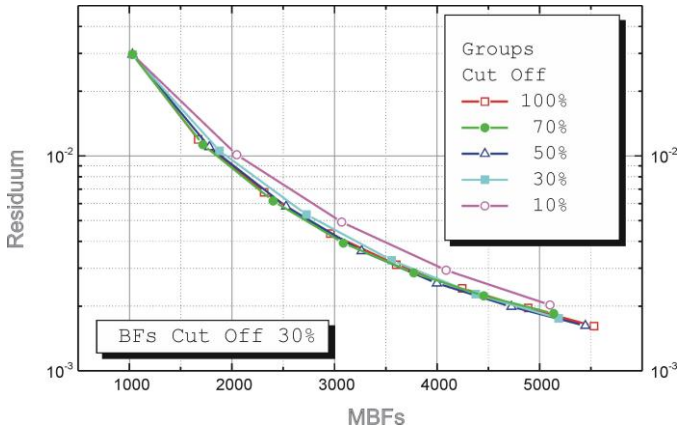


b)

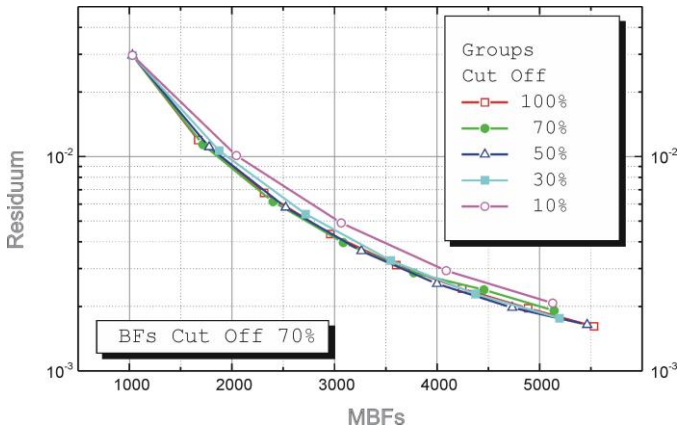
Fig. 4. PDM-AGT applied to airplane, Residuum vs Iterations, for cut off value for BFs of (a) 30%, and (b) 70%

Fig. 4 shows convergence trough PDM iterations, for different cumulative residuum cut off values for groups (100%, 70%, 50%, 30% and 10%). Value 100% means that there is no splitting at all, value 70% means that groups of BFs (with highest residuum) making 30% of total residuum will be split (each in two new groups), and so on. By decreasing cut off value for groups, number of groups that will be split increases and, hopefully, convergence too. The splitting of the groups was performed according to cumulative residuum cut off value for BFs. In Fig. 4a this value is 30%, whereas in Fig. 4b it is 70%. Obviously results are very similar, and the same refers to values in between, which are not presented here. So it seems that 50% is good practical choice for BFs cut off. Now, looking at any of the Figs. 4a-b, we see that Residuum of 0.002 is reached after 5 iterations with 10% groups cut off value, but in 7 iterations without splitting (100% groups cut off). Since CPU time per iteration is similar for both procedures (number of MBFs become significant only for large number of iterations), procedure with 10% groups cut off will reach the same Residuum in shorter CPU time.

Fig. 5 shows how residuum changes as a function of number of MBFs (i.e. efficiency of the method).



a)



b)

Fig. 5. PDM-AGT applied to airplane, Residuum vs MBFs for BFs cut off value of (a) 30%, and (b) 70%

Obviously, PDM-AGT for groups cut off 10% is the least efficient (given Residuum is reached with the largest number of MBFs), but all other values gives similar results. It means that we can obtain the same Residuum with similar number of MBFs.

Fig. 6. PEC helicopter with 7445 plates and has 420 groups

Second example is PEC helicopter, about 65λ long, placed along *x*-axis, as shown in Fig. 6. The helicopter is excited by circularly polarized plane wave incoming from direction given by angles $\phi = -135°$ and $\theta = -45°$. The helicopter is modelled by 13727 plates. The 2nd order approximation is used for almost all plates, so that total number of BFs is $N = 109636$. We use $M = 420$ groups of plates (324 are in lit zone).

Residuum trough the iterations of PDM-AGT is shown in Fig. 7. BFs cut off is 50%. We see that groups cut off 50% curve drops almost linearly, and that reaches Residuum of 0.01 in 5 iterations, "more than" 2 iterations earlier than 100% curve (no splitting).

Residuum versus number of MBFs is shown in Fig. 8 (iterations are marked for each curve) – slightly better for 100% curve, but nothing dramatic.

RCS (Radar Cross Section) obtained by PDM-AGT solution (groups cut off 50%) in 5th iteration is compared to MoM result in Fig. 9. very good agreement.

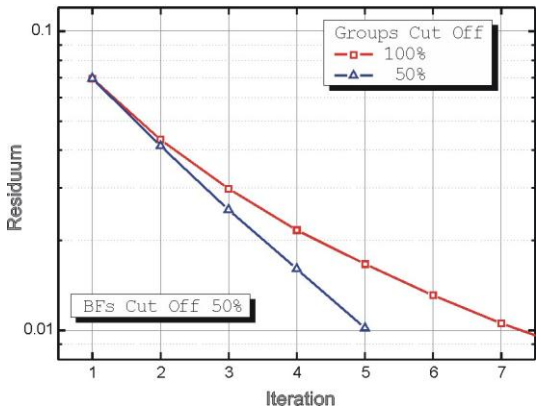


Fig. 7. PDM-AGT analysis of helicopter, Residuum vs Iterations

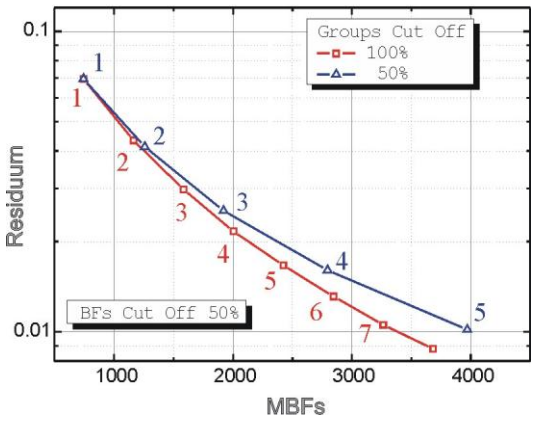


Fig. 8. PDM-AGT analysis of helicopter, Residuum vs MBFs

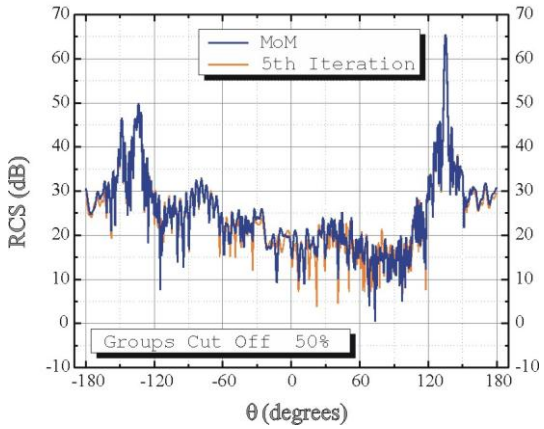


Fig. 9. RCS for helicopter MoM vs PDM-AGT (cut off 50%)

IV. CONCLUSION

In this paper we introduced simple technique for improving convergence rate of PDM method (PDM-AGT). In each iteration groups of BFs with highest residuum (error of solution) were divided (each in two new groups), thus allowing better correction in the following iterations. Since there is no certain rule how to choose initial number of groups of BFs for each particular model, this technique enables adaptive correction once the PDM is started. Numerical results shows that the technique indeed improves speed of convergence.

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