

# Localization of Stochastic Electromagnetic Sources by using Correlation Matrix Trained MLP Neural Network

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**Abstract –** In this paper, MLP neural network-based approach is proposed for an efficient direction of arrival (DOA) estimation of multiple narrow-band electromagnetic sources of stochastic nature in far-field. Neural network is trained to perform the mapping from the space of signals described by correlation matrix, obtained by signal sampling in far-field scan area, to the space of DOA in angular positions. Accuracy and efficiency of the proposed approach is validated on two examples determining the location of one and three stochastic sources in far-field, respectively, placed at fixed angle distance.

**Keywords** –Stochastic electromagnetic sources, DOA estimation, MLP neural network, correlation matrix.

## I. INTRODUCTION

The source localization by using passive sensor arrays has numerous applications in various fields, such as communications, radar, geophysics, acoustics, biomedical engineering and etc. Direction-of-arrival (DOA) estimation of impinging electromagnetic signal radiated from sources is usually required to determine the source position. DOA estimation is especially important in wireless communications, as it can help to optimize the radiation pattern of an antenna array by using an adaptive beam-forming algorithm and thus minimize the interference. There are a number of algorithms developed to tackle DOA estimation problem. They are mostly based on processing of spatial covariance matrix of received signals by antenna elements. The subspace-based methods called MUSIC (MULTiple SIgnal Classification) is the most known for its super-resolution capabilities [1]. However, the MUSIC conducts an intensive spectral search procedure in order to provide an accurate DOA estimation and therefore it is not very suitable for real-time applications due to high computational complexity. Artificial neural networks (ANNs) [2,3] offer a fast but accurate alternative as they can provide an efficient DOA solution performing basic mathematical operations and calculating elementary functions.

Due to their abilities, ANNs are very suitable for determining of angular positions of source signals [4,5]. Two-dimensional (2D) DOA estimation was performed in [6] to determine angular coordinates (azimuth and elevation) of narrow-band electromagnetic (EM) source by using a novel approach that combines MLP (Multi-Layer Perceptron) and RBF (Radial Basis Function) ANNs. In order to include the environmental conditions knowledge (such as mutual coupling between antenna array elements or inaccuracies in the manufacturing process) into the ANN structure, an empirical model was proposed in [7], capable to outperform 2D MUSIC algorithm in terms of accuracy when small number of snapshots are used to estimate the source angular position.

All previously mentioned ANN models are developed for DOA estimation of deterministic narrow-band EM sources. In this paper, we have applied MLP ANNs approach for localization of multiple stochastic narrow-band EM sources in far-field. The MLP neural model is developed with the purpose to perform the mapping from the space of signals described by correlation matrix, obtained by signal sampling via antenna array elements in far-field scan area, to the space of DOA in angular positions. The results provided by the successfully trained neural model for the considered case of three stochastic sources placed at fixed angle distance in azimuth plane demonstrate model efficiency and accuracy. The proposed approach avoids intensive and time-consuming numerical calculations with a potential to be employed in more general real-time application cases of DOA estimation of a number of stochastic sources located at arbitrary mutual angular distance.

## II. STOCHASTIC SOURCE RADIATION MODEL

We represent a radiation of stochastic source in far-field by linear uniform antenna array with  $N$  elements at the mutual distance  $d = \lambda/2$  (Fig.1). In this radiation model representing stochastic source, antenna elements feed currents (defined by vector  $\mathbf{I}=[I_1, I_2, \dots, I_N]$ ) are in general mutually correlated. The level of their correlation can be defined by correlation matrix,  $\mathbf{C}^I(\omega)$ , describing stochastic nature of antenna array radiation [8]:

$$\mathbf{C}^I(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[ \mathbf{I}(\omega) \mathbf{I}(\omega)^H \right] \quad (1)$$

Using Green function to map the domain of radiation source currents into the domain of electric field in far-field, vector  $\mathbf{M}$  can be defined as:

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$$\mathbf{M}(\theta, \varphi) = jz_0 \frac{F(\theta, \varphi)}{2\pi r_c} [e^{jkr_1} \ e^{jkr_2} \dots \ e^{jkr_N}] \quad (2)$$

where  $z_0$  is free-space impedance,  $F(\theta, \varphi)$  is the radiation pattern of antenna array elements,  $r_c$  is the distance of far-field point to the centre of array,  $k$  is the phase constant ( $k=2\pi/\lambda$ ) and  $r_1, r_2, \dots, r_N$  are the distances of far-field point from the first to the  $N$ -th element of antenna array.

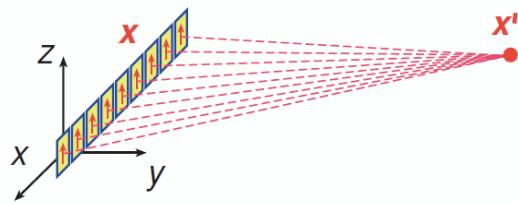


Fig 1. Representation of the stochastic source radiation in the far-field by linear uniform antenna array with  $N$  elements at the mutual distance  $d = \lambda/2$

The electric field intensity in sampling point in far-field (sampling point is at the angles  $\theta$  and  $\varphi$  in azimuth and elevation planes, respectively, defined by the first element of antenna array) can be calculated as:

$$E(\theta, \varphi) = \mathbf{M}(\theta, \varphi)\mathbf{I} \quad (3)$$

If  $M$  points are simultaneously observed in far-field, then more general notation can be used to describe the antenna array elements distance from particular points in far-field. In this notation  $r_{i,m}$  represents the distance between  $i$ -th element ( $1 \leq i \leq N$ ) in the antenna array and  $m$ -th point in the far-field ( $1 \leq m \leq M$ ) (see Fig. 2).

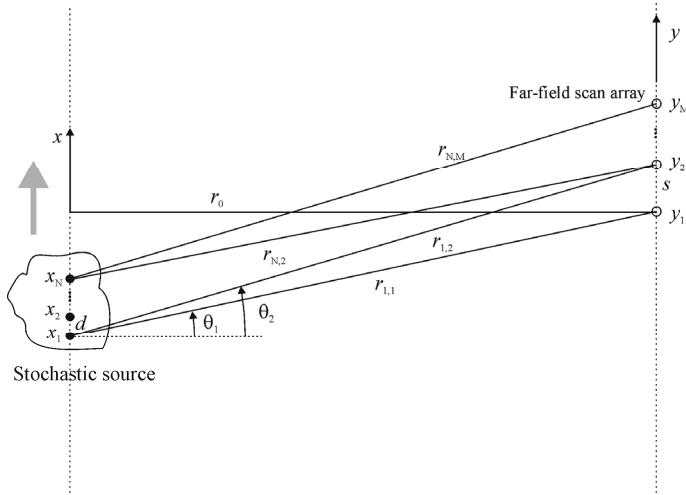


Fig. 2. The position of stochastic source in azimuth plane with respect to the location of EM field sampling points in the far-field scan area

The distance between the linear trajectories of the stochastic sources and the linear array of sampling points marked with  $r_0$ . Angle  $\theta_l$  represents the angle position of the first sampling point relative to the first antenna element in the antenna array. Also, this angle represents the position angle of stochastic sources in the azimuthal plane relative to sampling points  $\theta=\theta_l$ . Azimuthal angle of the  $m$ -th sampling point related to the first antenna element in the antenna array can be calculated as

$$\theta_m = \arctan \left[ \frac{(m-1) \cdot s}{r_0} + \tan \theta_l \right] \quad (4)$$

The distance between the first sampling point and the first element in the antenna array can be calculated as

$$r_{1,1} = \frac{r_0}{\cos \theta_l} \quad (5)$$

This distance represents the distance between the stochastic source and the set of sampling points  $r = r_{1,1}$ . Distance between first element in the antenna array and  $m$ -th point in the far-field ( $1 \leq m \leq M$ ) can be calculated as

$$r_{1,m} = \frac{r_0}{\cos \theta_m} \quad (6)$$

Distance between  $i$ -th element ( $1 \leq i \leq N$ ) in the antenna array and  $m$ -th point in the far-field ( $1 \leq m \leq M$ ) can be calculated as

$$r_{i,m} = \sqrt{\left(r_m^{(1)} \cdot \cos \theta_m\right)^2 + \left|r_m^{(1)} \cdot \sin \theta_m\right| + (i-1) \cdot (x_2 - x_1)} \quad (7)$$

alternatively

$$r_{i,m} = \sqrt{r_0^2 + \left|r_0 \cdot \tan \theta_m\right| + (i-1) \cdot (x_2 - x_1)} \quad (8)$$

Based on the mapping defined by the equation (3) the spectral density of electric energy in sampling point in far-field can be determined

$$W_E(\theta, \varphi) = \frac{\varepsilon}{2} \mathbf{M}(\theta, \varphi) \mathbf{c}^T \mathbf{M}(\theta, \varphi)^H \quad (9)$$

$M$  points ( $Y_1, Y_2, \dots, Y_M$ ) in far-field, in which electric field levels are sampled, are at the azimuth and elevation plane angles  $(\theta_1, \varphi_1), (\theta_2, \varphi_2), \dots, (\theta_M, \varphi_M)$ , determined by the first element of antenna array. The correlation matrix of signals received in these sampling points can be obtained from the correlation matrix of antenna elements feed currents as:

$$\mathbf{C}_E[i, j] = \mathbf{M}(\theta_i, \varphi_i) \mathbf{c}^T \mathbf{M}(\theta_j, \varphi_j)^H \quad (10)$$

$$i = 1, \dots, M \quad j = 1, \dots, M$$

For the case of several stochastic sources in azimuth plane, the radiation of each source can be represented by one antenna array with  $N$  elements as previously shown. The level of EM field in far-field sampling point, as well as the elements of correlation matrix are determined by the superposition of radiation from all sources. In accordance with this, when the number of stochastic sources is  $S$ , vector  $\mathbf{M}$  has a form :

$$\mathbf{M}(\theta, \varphi) = jz_0 \frac{F(\theta, \varphi)}{2\pi r_c} \cdot [e^{jkr_1^{(1)}} \dots e^{jkr_N^{(1)}} e^{jkr_1^{(2)}} \dots e^{jkr_N^{(2)}} \dots e^{jkr_1^{(S)}} \dots e^{jkr_N^{(S)}}] \quad (11)$$

where  $r_i^{(j)}$  is the distance between  $i$ -th element in antenna array, representing  $j$ -th stochastic source, and the sampling point in far-field, while the feed currents vector is:

$$\mathbf{I} = [I_1^{(1)} \dots I_N^{(1)} I_1^{(2)} \dots I_N^{(2)} \dots I_1^{(S)} \dots I_N^{(S)}] \quad (12)$$

where  $I_i^{(j)}$  is the feed current of  $i$ -th element in antenna array representing  $j$ -th stochastic source.

Combining Eqs.(3), (9) and (10) with Eqs. (11) and (12) that correspond to the case of several stochastic sources, it is possible to determine the intensity of EM field in the sampling point in far-field, as well as the elements of correlation matrix  $\mathbf{C}_E$ . It should be also pointed out that in the case of unknown level of correlation between antenna elements feed currents, its correlation matrix can be obtained by measuring the intensity of electric field in the sampling points in near-field.

### III. NEURAL NETWORK MODEL

The neural model based on MLP ANN is developed with the purpose to perform the mapping from the space of signals described by correlation matrix  $\mathbf{C}_E$  to the space of DOA in azimuth

$$\boldsymbol{\theta} = f(\mathbf{C}_E) \quad (13)$$

where  $\boldsymbol{\theta}$  is azimuthal angles vector of stochastic sources,  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_S]$  and  $S$  is number of stochastic sources (Fig. 3). In the observed case elevation coordinates of radiation sources is neglected. The architecture of developed neural model is shown in Fig.4. Its MLP network can be described by the following function:

$$\mathbf{y}_l = F(\mathbf{w}_l \mathbf{y}_{l-1} + \mathbf{b}_l) \quad l = 1, 2 \dots \quad (14)$$

where  $\mathbf{y}_{l-1}$  vector represents the output of  $(l-1)$ -th hidden layer,  $\mathbf{w}_l$  is a connection weight matrix among  $(l-1)$ -th and  $l$ -th hidden layer neurons and  $\mathbf{b}_l$  is a vector containing biases of  $l$ -th hidden layer neurons.  $F$  is the activation function of neurons in hidden layers and in this case it is a hyperbolic tangent sigmoid transfer function:

$$F(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}} \quad (15)$$

In order to perform mapping it is sufficient to take only the first column of correlation matrix and therefore  $\mathbf{y}_0 = [Re\{\mathbf{C}_E[1,1]\} \ Im\{\mathbf{C}_E[1,1]\}, \dots, Re\{\mathbf{C}_E[1,M]\} \ Im\{\mathbf{C}_E[1,M]\}]$ . Also,  $\boldsymbol{\theta}$  is given as  $\boldsymbol{\theta} = \mathbf{w}_3 \mathbf{y}_2$  where  $\mathbf{w}_3$  is a connection weight matrix between neurons of last hidden layer and neurons in output layer. The optimization of weight matrices and biases values during the training allows ANN to approximate the mapping with the desired accuracy.

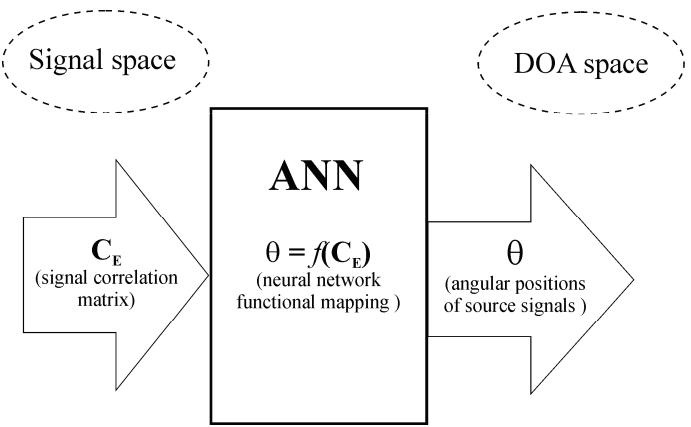


Fig. 4. ANN in the role of mapping from the space of signals described by correlation matrix  $\mathbf{C}_E$  to the space of DOA in azimuth

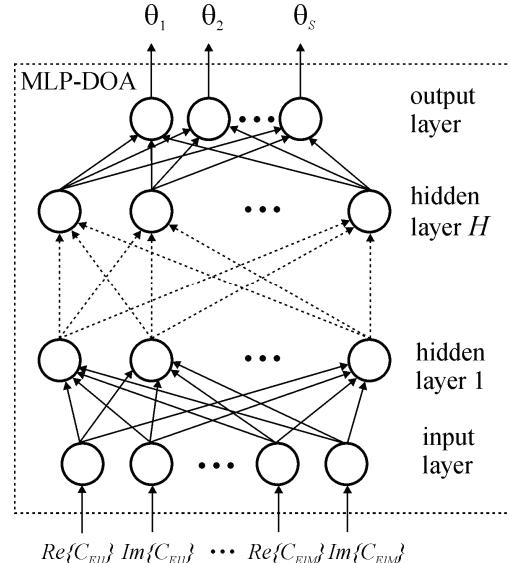


Fig. 4. Architecture of MLP neural model for DOA estimation of stochastic EM source signal in azimuth plane

In accordance with architecture of MLP neural model for DOA estimation of stochastic EM source signal in azimuth plane, presented in Fig. 4, general notation of this MLP model is  $MLPH-N_1 \dots N_{i-1} \dots N_H$ ; where  $H$  represents the total number of hidden layers of MLP network and  $N_i$  represents the total number of neurons in  $i$ -th hidden layer.

### IV. MODELING RESULTS

The training of ANNs is conducted for the case of four equidistant sampling points, at the mutual distance  $d = \lambda/2$ , located 1 km from the stochastic source. Two cases are considered: the first case is when  $S=1$  when DOA angle for one stochastic source in azimuth plane has to be determined and the second case is when  $S=3$ . The DOA angles of the

three stochastic sources at fixed angle distance of  $10^\circ$  in azimuth plane have to be estimated.

### 2.1 Case 1 ( $S=1$ )

By using Eq. (2) and (10) for  $N=4$  and  $M=4$ , at the working frequency of  $f = 7.5$  GHz, 401 and 320 uniformly distributed samples are generated for training and testing, respectively, in the range  $[-80^\circ, 80^\circ]$ . The values of antenna array parameter which used in sampling process are described in Table I. It is assumed that stochastic source can be described accurately by the antenna array consisting of four vertical dipoles feed by uncorrelated currents ( $\mathbf{c}'$  is the unit diagonal matrix). Levenberg-Marquardt method with prescribed accuracy of  $10^{-5}$  is used as a training algorithm. In order to obtain MLP model with the highest possible accuracy, training of several MLP networks with different number of neurons in hidden layers is conducted. The testing results for six MLP models with the lowest average case error (ACE [%]) of the same training set are shown in Table II together with the values of worst-case error (WCE [%]). Based on these criteria, MLP2-12-12 is chosen as representative neural model. The neural model simulation of the testing samples set shows a very good agreement between the output values of neural model and referent azimuth values (Fig.5).

TABLE I

THE VALUES OF ANTENNA ARRAY PARAMETER WHICH USED IN SAMPLING PROCESS

Frequency	$f = 7.5$ GHz
Number of sources	$S = 1$
Sampling points distance from source trajectory	$r_0 = 1000$ m
Number of antenna array elements	$N=4$
Mutual distance of the antenna array elements	$s = \lambda/2$ (0.02 m)
Number of sampling points	$M = 4$
Mutual distance of the sampling points	$s = \lambda/2$ (0.02 m)

TABLE II

TESTING RESULTS FOR SIX MLP NEURAL MODELS WITH THE BEST AVERAGE ERRORS STATISTICS (CASE 1)

MLP model *	WCE [%]	ACE [%]
MLP2-12-12	1.26	0.17
MLP2-12-9	1.35	0.18
MLP2-18-16	2.03	0.18
MLP2-16-11	1.71	0.20
MLP2-20-20	1.85	0.21
MLP2-18-7	1.12	0.22

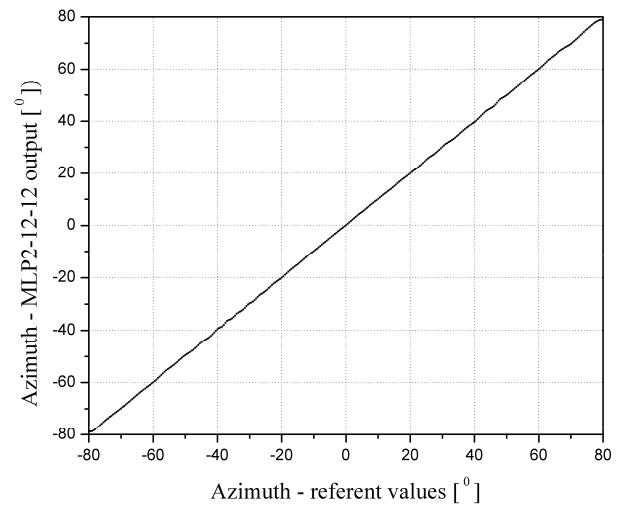


Fig. 5. Comparison of MLP2-12-12 model output with the referent azimuth values

### 2.1 Case 2 ( $S=3$ )

By using Eq.(10) and (11) for  $N=1$  and  $M=4$ , at the working frequency of  $f = 7.5$  GHz, 71 and 47 uniformly distributed samples are generated for training and testing, respectively, in the range  $[-80^\circ, 60^\circ]$  for first source (source 1),  $[-70^\circ, 70^\circ]$  for second source (source 2) and  $[-60^\circ, 80^\circ]$  for third source (source 3). The values of antenna array parameter which used in sampling process are described in Table III.

TABLE III  
THE VALUES OF ANTENNA ARRAY PARAMETER WHICH USED IN SAMPLING PROCESS

Frequency	$f = 7.5$ GHz
Number of sources	$S = 3$
Sampling points distance from source trajectory	$r_0 = 1000$ m
Number of antenna array elements	$N=1$
Mutual distance of the antenna array elements	$s = \lambda/2$ (0.02 m)
Number of sampling points	$M = 4$
Mutual distance of the sampling points	$s = \lambda/2$ (0.02 m)

In this case it is assumed that each stochastic source is described by one vertical dipole. The feed currents of two dipoles are mutually uncorrelated so that  $\mathbf{c}'$  is the unit diagonal matrix. As in the previous case, Levenberg-Marquardt method with prescribed accuracy of  $10^{-4}$  is used as a training algorithm. The testing results for six MLP models with the lowest average case error are shown in Table II, and MLP2-16-4 is chosen as representative neural model. The neural model simulation of testing samples set shows a very good agreement between the output values of neural model and referent azimuth values for all sources (Fig. 6, Fig. 7 and

Fig. 8). In addition, a good agreement with results obtained by MUSIC algorithm can be observed.

TABLE IV  
TESTING RESULTS FOR SIX MLP NEURAL MODELS WITH THE BEST AVERAGE ERRORS STATISTICS

MLP model	WCE [%]	ACE [%]
MLP2-16-4	1.59	0.29
MLP2-12-5	1.26	0.30
MLP2-15-11	1.15	0.31
MLP2-16-10	1.14	0.36
MLP2-12-9	1.49	0.37
MLP2-18-14	1.19	0.37

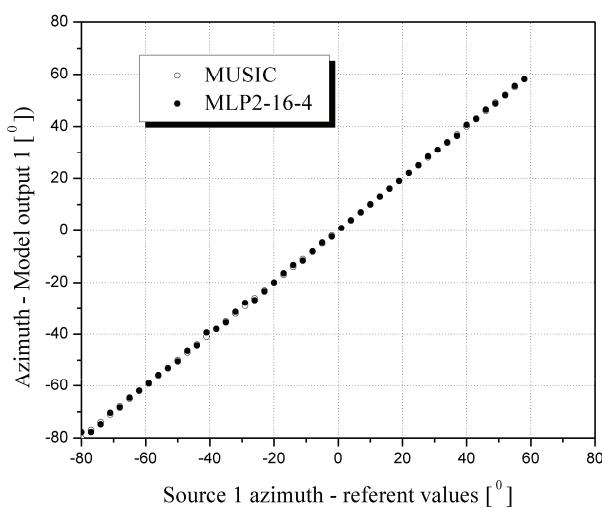


Fig. 6. Comparison of MLP2-16-4 model output 1 (azimuth of source 1) with MUSIC and referent azimuth values

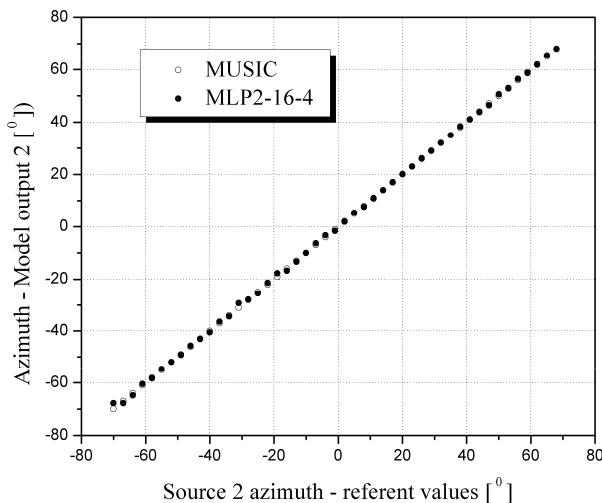


Fig. 7. Comparison of MLP2-16-4 model output 2 (azimuth of source 2) with MUSIC and referent azimuth values

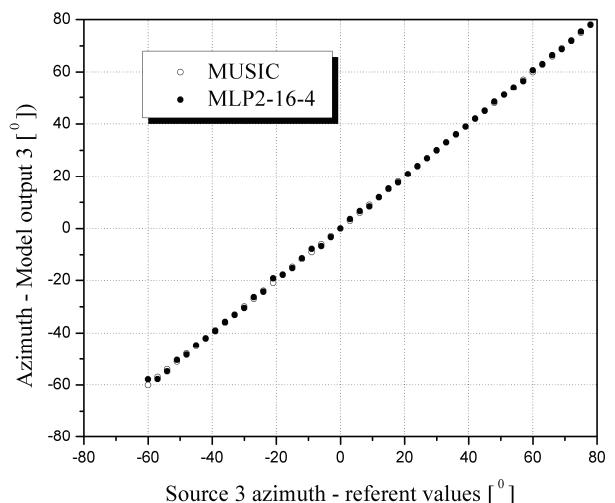


Fig. 8. Comparison of MLP2-16-4 model output 3 (azimuth of source 3) with MUSIC and referent azimuth values

## V. CONCLUSION

The neural network-based approach for DOA estimation of electromagnetic radiation of stochastic sources is presented in the paper. Only the first column of correlation matrix obtained by simple signal sampling in far-field scan area by linear uniform antenna array is used as an input of developed neural model. Neural model ability to accurately and efficiently determine the location of the stochastic source is illustrated on one example. As proposed neural model avoids intensive and time-consuming numerical calculations it is more suitable than conventional approaches for real-time applications. At the moment, neural model is capable to determine the location in azimuth plane for up to three stochastic sources placed at fixed mutual angle distance. Future research will be focused to the more general DOA estimation of multiple stochastic sources at arbitrary angular distance.

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