Synthesis and Use of Wave Digital Networks of Admittance Inverters

Biljana P. Stošić, Nebojša S. Dončov

Abstract — **In this paper, a generalization of the wave digital models of admittance inverters is done, aiming to increase a number of types of microstrip structures that can be modeled bz using wave digital filter theory. Wave digital networks of admittance inverters (J,+90^{o}) and (J,–90^{o}), are described in details and developed here based on scattering parameter formalism and two-port networks of parallel or series adaptors.**

Keywords — **Wave digital approach, admittance inverter, microstrip resonator filters, network synthesis.**

I. Introduction

The basic theory of wave digital filters (WDF) was developed by Alfred Fettweis [1-3] in 1971. It was used for digitizing lumped electrical circuits composed of inductors, capacitors, resistors, and other elements of classical network theory. In the past, wave digital approach has been widespread used in modeling and analysis of different physical systems. Wave digital filters offer computational efficiency, stability under finite-arithmetic conditions and facilitate interfacing with wave variables, making them a worthwhile subject of study. The wave digital structures are specially tailored with respect to a hardware implementation.

A detailed review of application of WDF structures for electromagnetic (EM) field simulation is given in [4-5]. It is shown in [6] that simulations of different microstrip structures based on their wave digital network representations can be carried out in ADS (Advanced Design Software). In [7], a framework for the automated generation of the wave digital structures is presented, and the reference circuit is assumed to comprise arbitrary connection types.

Recently, a novel technique combining one-dimensional wave digital approach with an equivalent discontinuity model, obtained from a full-wave EM tool, has been presented in [8-10] to accurately and efficiently describe microstrip structures with specific topological features, which characterize its connectivity. The combined wave digital/full-wave EM approach, has been applied to generate appropriate wave digital network models of microstrip structures with symmetrical step discontinuities [8], and gap discontinuities [9-10] (an end-coupled half-wavelength resonator filter [9], and a parallel-coupled bandpass filter [10]). Proposed approach follows the procedure to replace the presence of considered discontinuity by appropriate equivalent model, derived from scattering matrix, and to accurately perform S-parameter analysis of complete observed structure by using wave digital network models.

Biljana P. Stošić and Nebojša S. Dončov are with the University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, P.O.Box 73, 18000 Niš, Serbia, E-mails: biljana.stosic@elfak.ni.ac.rs, nebojsa.doncov@elfak.ni.ac.rs

In order to apply the wave digital approach for modeling and analyzing of microstrip structures with gaps (e.g. end-coupled and parallel-coupled bandpass resonator filters), one needs to obtain their J-inverter models consisting of J-inverters and half-wavelength resonators. The equivalent admittance inverter models of microstrip filters with gaps will be described shortly in this paper. The resulting wave digital networks of these filters can be formed based on these equivalent representations.

In wave digital structures adaptors are used as the connection elements. Alfred Fettweis has introduced elementary two-port parallel and series adaptors representing parallel and series connections of two-ports.

In [11], the wave digital models of admittance inverter (J,+90^{o}) was defined and explained in details. Two models of inverter were formed by use of two-port parallel or series adaptors. They were formed directly in the Simulink toolbox of MATLAB environment.

This paper is devoted to the synthesis and use of wave digital network models of admittance inverters with characteristic impedance $1/J$ that transform they load admittance by $+90^{\circ}$ or -90°. Obtained models are based on two-port adaptors for an easy integration into equivalent wave digital networks of complex microstrip structures.

The paper is organized in the following nine parts. In Section II, the equivalent circuits of microstrip bandpass resonator filters (end-coupled and parallel-coupled structures) are presented in order to give an explanation where the admittance inverters have to be used. A short description of equivalence development from a layout to a wave digital model is given.

The $ABCD$ and S matrices of the admittance inverter are described in Sections III and IV, respectively. The scattering matrices are used to form wave digital networks of inverters (J,+90^{o}) and (J,–90^{o}) as described in Sections VI and VII, respectively. The equations and models drawn in MATLAB/Simulink are given there. Models of inverters are formed based on two-port networks of parallel or series adaptors, given in Section V. Standard digital elements, such as delay element, adder, and multiplier, are used in developing the inverter network models.

Proof of the inverter feature is given in Section VIII, and a short conclusion is addressed in Section IX.

II. Microstrip Structures with Gaps

A. An End-coupled Microstrip Bandpass Filter

An end-coupled microstrip bandpass filter consists of several microstrip resonators capacitively coupled to each other [12]. The capacitive gaps serve as admittance (J-)

inverters.

Development of an equivalence of an end-coupled resonator bandpass filter, which layout is shown in Fig. 1*a*, is described in [9]. Its equivalent circuit is shown in Fig. 1*b*, having a form of the *J*-inverter filter model, consisting of *J*-inverters and $\lambda/2$ resonators. The equivalent wave digital network of the *J*-inverter filter model is formed, Fig. 1*c*, and then analyzed. It consists of blocks corresponding to *J*-inverters (assigned as ADP-Inv_J*k*, k=1,2,...,*N*+1) and blocks corresponding to resonators (assigned as Resonator_*m*, $m=1,2,\ldots,N$).

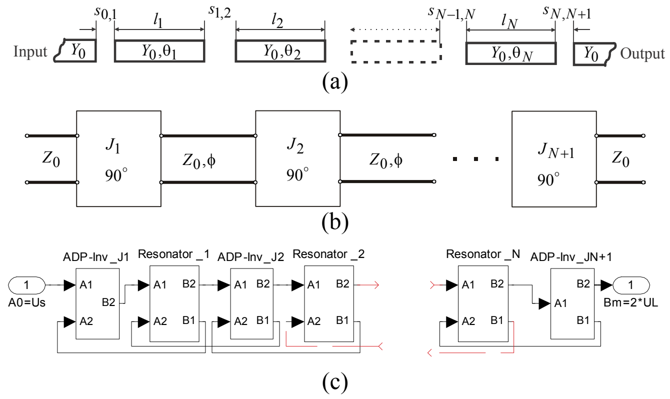


Fig. 1. Development of an equivalence of an end-coupled resonator bandpass filter. (a) The capacitive-gap (end-)coupled resonator bandpass filter (top view), (b) *J*-inverter filter model, (c) Wave digital network from MATLAB/Simulink toolbox

B. A Parallel-coupled Microstrip Bandpass Filter

The parallel-coupled bandpass filter consists of series of half-wavelength line resonators. They are positioned in such a way that adjacent resonators are parallel to each other along one-half of their physical lengths.

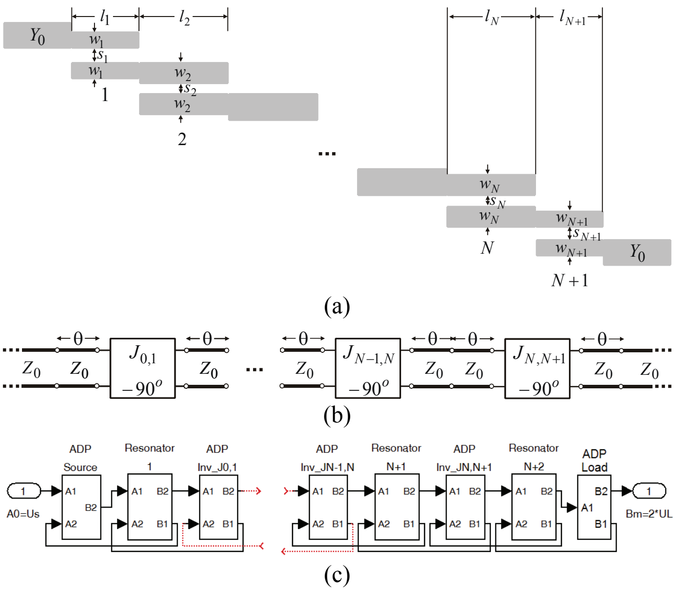


Fig. 2. Development of an equivalent wave digital network model for a parallel-coupled line filter: (a) Layout of $N+1$ section coupled-line bandpass filter (top view), (b) Equivalent circuit using equivalent circuits for each coupled line section, (c) Wave digital network drawn in Simulink toolbox

Fig. 2*a* shows a general structure containing $N+1$ coupled-line section which are of quarter-wavelength at the center frequency. Its equivalent circuit containing *J*-inverters and $\lambda/2$ resonators is shown in Fig. 2*b*. The resulting wave digital network of filter can be formed based on this equivalent representation. It will contain wave digital network models of inverters (assigned as ADP-Inv_J*k*,*k*+1, $k=0,1,\ldots,N$) and cascaded unit elements [13] which are used to model resonators (assigned as Resonator_*m*, $m=1,2,\ldots,N+2$). Analysis of resulting network is then done using earlier developed one-dimensional wave digital approach [13].

III. The *ABCD* Matrix of Admittance Inverters

Admittance inverter is an idealized device operating electrically like a quarter-wave lossless transmission line of characteristic impedance $1/J$, thus transforming the load admittance Y_L by $\pm 90^\circ$ and modifying its magnitude, resulting in an input admittance $Y_{in} = J^2/Y_L$, Fig. 3*a*. The constant J is real and it is called the characteristic admittance of inverter.

In general, the $ABCD$ parameters of the admittance inverter ($J, \pm 90^\circ$) can be easily obtained by considering it as a transmission line of either positive or negative quarter-wave length and with characteristic impedance of $1/J$ (see Fig. 3*b*).

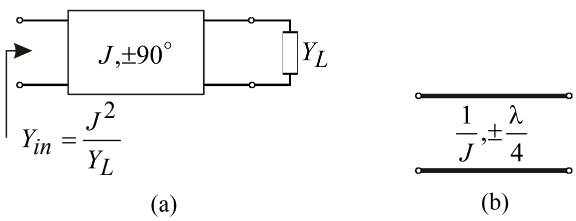


Fig. 3. (a) A schematic symbol of admittance inverter, (b) its equivalent transmission line model

The transmission line of length l and characteristic impedance Z_0 has the $ABCD$ matrix of the form

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\beta \cdot l) & j \cdot Z_0 \cdot \sin(\beta \cdot l) \\ j \cdot Y_0 \cdot \sin(\beta \cdot l) & \cos(\beta \cdot l) \end{bmatrix}. \quad (1)$$

For chosen characteristic admittance $Y_0 = J$, and for the case of a positive quarter-wave length of transmission line, $l = \lambda/4$ i.e. electrical length $\theta = \beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$, the $ABCD$ matrix given by Eq. (1) for *J*-inverter ($J, +90^\circ$) becomes

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & j/J \\ j \cdot J & 0 \end{bmatrix}. \quad (2)$$

Similarly, the $ABCD$ parameters of the admittance inverter ($J, -90^o$) can be easily obtained by considering it as a transmission line of negative quarter-wave length $l = -\lambda/4$, i.e. electrical length $\theta = \beta \cdot l = -\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = -\frac{\pi}{2}$, and with characteristic admittance of $Y_0 = J$ (see Fig. 3*b*). $ABCD$ matrix given by Eq. (1) becomes

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -j/J \\ -j \cdot J & 0 \end{bmatrix}. \quad (3)$$

IV. The Scattering Parameters of Two-port Network

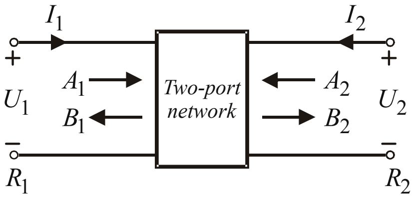


Fig. 4. Two-port network

Synthesis of wave digital network of admittance inverter can be done by starting from its scattering parameters. For a two-port network shown in Fig. 4, *ABCD* matrix formulation is

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}, \quad (4)$$

and scattering matrix formulation is

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad (5)$$

where A_1 and B_1 are the incident and reflected wave at port 1 with port resistance R_1, and A_2 and B_2 are the incident and reflected wave at port 2 having port resistance R_2.

Classic $ABCD$ matrix to S matrix transformation formulas for two-port case give

$$S_{11} = \frac{A + B \cdot G_2 - C \cdot R_1 - D \cdot R_1 \cdot G_2}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (6a)$$

$$S_{12} = \frac{2 \cdot R_1 \cdot G_2 \cdot (A \cdot D - B \cdot C)}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (6b)$$

$$S_{21} = \frac{2}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (6c)$$

$$S_{22} = -\frac{A - B \cdot G_2 + C \cdot R_1 - D \cdot R_1 \cdot G_2}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (6d)$$

where $G_2 = 1/R_2$, Fig. 4.

A. The Scattering Parameters of J-inverter ($J, +90^o$)

S-parameters of the *J*-inverter ($J, +90^o$) can be found starting with its $ABCD$ matrix given in Eq. (2) and the previously given classic formulas (6), as follows

$$S_{11} = \frac{1 - J^2 \cdot R_1 \cdot R_2}{1 + J^2 \cdot R_1 \cdot R_2}, \quad (7a)$$

$$S_{12} = \frac{-j \cdot 2 \cdot J \cdot R_1}{1 + J^2 \cdot R_1 \cdot R_2}, \quad (7b)$$

$$S_{21} = \frac{-j \cdot 2 \cdot J \cdot R_2}{1 + J^2 \cdot R_1 \cdot R_2}, \quad (7c)$$

$$S_{22} = \frac{1 - J^2 \cdot R_1 \cdot R_2}{1 + J^2 \cdot R_1 \cdot R_2}. \quad (7d)$$

B. The Scattering Parameters of J-inverter ($J, -90^o$)

S-parameters of the *J*-inverter can be found starting with its $ABCD$ matrix given in Eq. (3) and given classic formulas (6), as follows

$$S_{11} = \frac{1 - J^2 \cdot R_1 \cdot R_2}{1 + J^2 \cdot R_1 \cdot R_2}, \quad (8a)$$

$$S_{12} = \frac{j \cdot 2 \cdot J \cdot R_1}{1 + J^2 \cdot R_1 \cdot R_2}, \quad (8b)$$

$$S_{21} = \frac{j \cdot 2 \cdot J \cdot R_2}{1 + J^2 \cdot R_1 \cdot R_2}, \quad (8c)$$

$$S_{22} = \frac{1 - J^2 \cdot R_1 \cdot R_2}{1 + J^2 \cdot R_1 \cdot R_2}. \quad (8d)$$

The scattering parameters of admittance inverters, given in Eqs. (7) and (8), are valid for arbitrary inverter constant J and port resistances R_1 and R_2.

Wave digital models of admittance inverters can be formed based on two-port parallel or series adaptor networks as shown in the following three paper sections.

V. Two-port Adaptors

Adaptors are memoryless digital elements whose task is to perform transformations between pairs of wave variables that are referred to different port resistances. They have low sensitivity to coefficient quantization. In the symbolic representations of the two-port parallel and series adaptors, given in Figs. 5 and 6, the adaptor coefficient β is shown explicitly. This coefficient is usually written on the side corresponding to port 2.

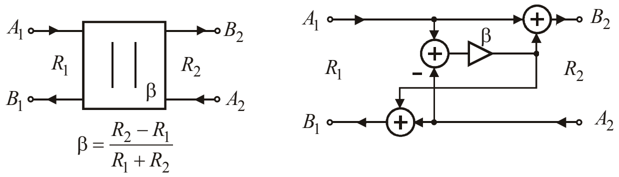
A. Parallel Adaptor

The equations for two-port parallel adaptor, whose schematic symbol is shown in Fig. 5*a*, are

$$B_1 = A_2 + \beta \cdot (A_1 - A_2)\,, \tag{9}$$

$$B_2 = A_1 + \beta \cdot (A_1 - A_2)\,. \tag{10}$$

Its wave digital network is depicted in Fig. 5*b*.



(a) (b)

Fig. 5. Two-port parallel adaptor: (a) symbol, and (b) wave digital network

B. Series Adaptor

In Fig. 6, schematic symbol for two-port series adaptor is given. As can be seen, a series adaptor can be easily derived from parallel adaptor by adding two multipliers.

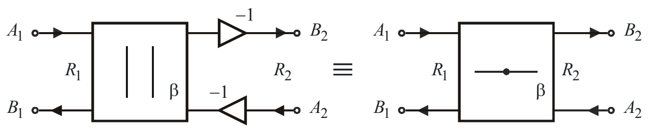


Fig. 6. Symbol for two-port series adaptor

Generally, the equations for two-port series adaptor are

$$B_1 = -A_2 + \beta \cdot (A_1 + A_2)\,, \tag{11}$$

$$B_2 = -A_1 - \beta \cdot (A_1 + A_2)\,. \tag{12}$$

VI. The Inverter $(J, +90^{o})$ Models

A. The Inverter Model based on Two-port Parallel Adaptor

In order to represent J -inverter with the network based on two-port parallel adaptor, adaptor coefficient is chosen to be

$$\beta = \frac{J^2 \cdot R_1 \cdot R_2 - 1}{J^2 \cdot R_1 \cdot R_2 + 1} = \frac{R_2 - \dfrac{1}{J^2 \cdot R_1}}{R_2 + \dfrac{1}{J^2 \cdot R_1}}\,. \tag{13}$$

For this case, the *S*-parameters of the J -inverter $(J, +90^{o})$ given in (7) can be written in the form

$$\mathbf{S} = \begin{bmatrix} -\beta & -\dfrac{j}{\gamma} \cdot (1-\beta) \\ -j \cdot \gamma \cdot (1+\beta) & -\beta \end{bmatrix} \tag{14}$$

where the arbitrary chosen constant is $\gamma = \frac{1}{J \cdot R_1}$. For these obtained scattering parameters, according to formulation given in Eq. (5), the set of equations for J -inverter $(J, +90^{o})$ is

$$B_1 = S_{11} \cdot A_1 + S_{12} \cdot A_2 = -\beta \cdot A_1 - j \cdot \frac{1}{\gamma} \cdot (1-\beta) \cdot A_2\,, \tag{15}$$

$$B_2 = S_{21} \cdot A_1 + S_{22} \cdot A_2 = -j \cdot \gamma \cdot (1+\beta) \cdot A_1 - \beta \cdot A_2\,. \tag{16}$$

To synthesized the wave digital network of inverter, Eqs. (15)-(16) are written in the form of Eqs. (9)-(10) as following

$$B_1 = -j \cdot \frac{1}{\gamma} \cdot \left[A_2 + \beta \cdot (-j \cdot \gamma \cdot A_1 - A_2)\right], \tag{17}$$

$$B_2 = -j \cdot \gamma \cdot A_1 + \beta \cdot (-j \cdot \gamma \cdot A_1 - A_2)\,. \tag{18}$$

According to the last equations, wave digital network of J -inverter $(J, +90^{o})$ based on two-port parallel adaptor network is depicted in Fig. 7.

In addition to wave digital network of two-port parallel adaptor given in Fig. 5*b*, this inverter model has two multipliers more.

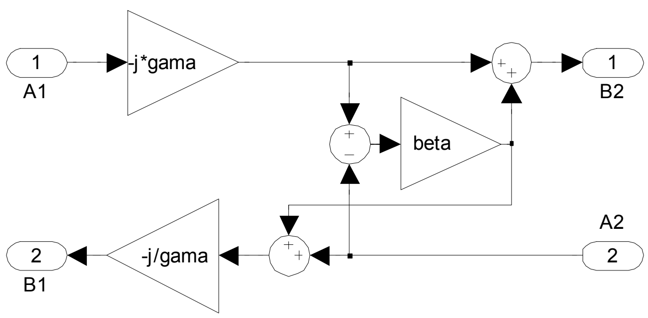


Fig. 7. The wave digital network of J -inverter $(J, +90^{o})$ based on two-port parallel adaptor network drawn in MATLAB Simulink toolbox

B. The Inverter Model based on Two-port Series Adaptor

For the inverter model based on two-port series adaptor network, the coefficient β is chosen to be as given in (13). The S-parameters of the J-inverter are given in (14). For obtained scattering parameters given in (14), set of equations for wave variables of J-inverter becomes as shown in (15)-(16).

In order to form the wave digital network model of inverter, Eqs. (15)-(16) have to be written in the form of Eqs. (11)-(12) as shown

$$B_1 = j \cdot \frac{1}{\gamma} \cdot \left[-A_2 + \beta \cdot \left(j \cdot \gamma \cdot A_1 + A_2 \right) \right], \tag{19}$$

$$B_2 = -j \cdot \gamma \cdot A_1 - \beta \cdot \left(j \cdot \gamma \cdot A_1 + A_2 \right). \tag{20}$$

Starting with Eqs. (19) and (20), the wave digital model for admittance inverter based on two-port series adaptor network is obtained and its representation in MATLAB Simulink toolbox is shown in Fig. 8. It contains three multipliers with constant coefficients, three adders and two sign-inversions.

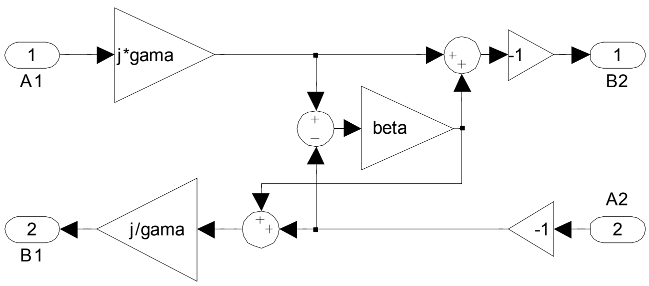


Fig. 8. The wave digital network of J-inverter ($J, +90^{\mathrm{o}}$) based on two-port series adaptor network drawn in MATLAB Simulink toolbox

VII. THE INVERTER ($J, -90^{\mathrm{o}}$) MODELS

A. The Inverter Model based on Two-port Parallel Adaptor

In order to represent J-inverter ($J, -90^{\mathrm{o}}$) with the network based on two-port parallel adaptor, adaptor coefficient β is chosen to be as given in (13). In this case, the S-parameters of J-inverter given in (8) can be written in the form

$$\mathbf{S} = \begin{bmatrix} -\beta & \frac{j}{\gamma} \cdot (1-\beta) \\ j \cdot \gamma \cdot (1+\beta) & -\beta \end{bmatrix} \tag{21}$$

where the arbitrary chosen constant is $\gamma = \frac{1}{J \cdot R_1}$. For these obtained scattering parameters, the set of equations for J-inverter ($J, -90^{\mathrm{o}}$) is

$$B_1 = -\beta \cdot A_1 + j \cdot \frac{1}{\gamma} \cdot (1-\beta) \cdot A_2, \tag{22}$$

$$B_2 = j \cdot \gamma \cdot (1+\beta) \cdot A_1 - \beta \cdot A_2. \tag{23}$$

To get the wave digital network of inverter, the equations (22)-(23) are written in the form of the equations (9)-(10) as following

$$B_1 = j \cdot \frac{1}{\gamma} \cdot \left[A_2 + \beta \cdot \left(j \cdot \gamma \cdot A_1 - A_2 \right) \right], \tag{24}$$

$$B_2 = j \cdot \gamma \cdot A_1 + \beta \cdot \left(j \cdot \gamma \cdot A_1 - A_2 \right). \tag{25}$$

According to the last equations, a possible realization of J-inverter ($J, -90^{\mathrm{o}}$) based on two-port parallel adaptor network using only three constant coefficient multipliers and three adders is depicted in Fig. 9.

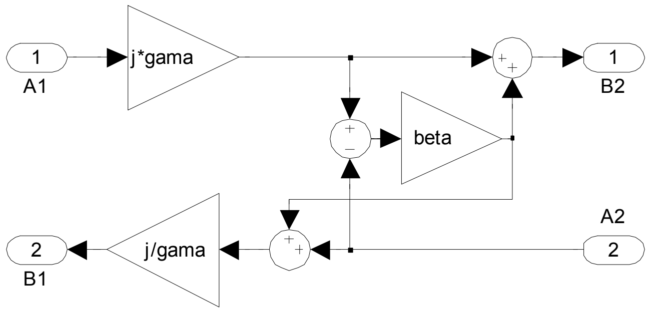


Fig. 9. The wave digital network of J-inverter ($J, -90^{\mathrm{o}}$) based on two-port parallel adaptor network drawn in MATLAB Simulink toolbox

B. The Inverter Model based on Two-port Series Adaptor

For the two-port series adaptor, the coefficient β is chosen to be as given in (13). The S-parameters of the J-inverter are given in (21). For obtained scattering parameters given in (21), set of equations for wave variables of J-inverter ($J, -90^{\mathrm{o}}$) becomes as shown in (22)-(23).

In order to form the wave digital network model of inverter, Eqs. (22)-(23) have to be written in the form of Eqs. (11)-(12) as shown

$$B_1 = -j \cdot \frac{1}{\gamma} \cdot \left[-A_2 + \beta \cdot \left(-j \cdot \gamma \cdot A_1 + A_2 \right) \right], \tag{26}$$

$$B_2 = j \cdot \gamma \cdot A_1 - \beta \cdot \left(-j \cdot \gamma \cdot A_1 + A_2 \right). \tag{27}$$

A possible realization of J-inverter ($J, -90^{\mathrm{o}}$) based on two-port series adaptor network using only three constant coefficient multipliers, three adders and two sign-inversions is given in Fig. 10.

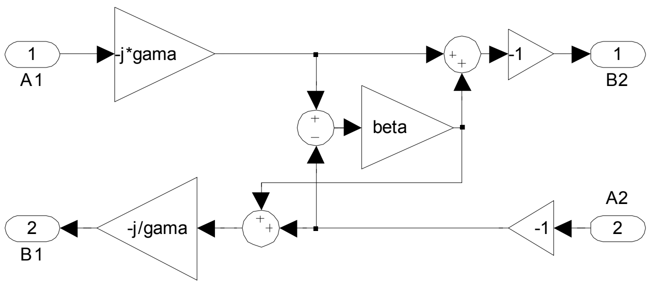


Fig. 10. The wave digital network of J -inverter ($J,-90^{\circ}$) based on two-port series adaptor drawn in MATLAB Simulink toolbox

VIII. Proof of the Inverter Feature

The wave digital network of J -inverter ($J,+90^{\circ}$) based on two-port parallel adaptor will be used in order to prove the main inverter feature (inverter is a two-port network providing an input admittance that is inverse of the load admittance). At the far end (port 2), it is terminated by a wave digital element corresponding to inductor which inductance is chosen to be equal to its port resistance $L=R_2$ (it consists of multiplier and delay element). It will be shown that this network corresponds to an one-port capacitor for which is chosen $C \neq \frac{1}{R_1}$, and R_1 is its port resistance.

Let's consider first a capacitor shown in Fig. 11.

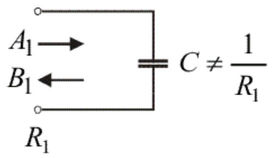


Fig. 11. One-port capacitor

The reflection coefficient is

$$\Gamma(z)=\frac{B_1}{A_1}=\frac{Z-R_1}{Z+R_1}=\frac{\frac{1}{SC}-R_1}{\frac{1}{SC}+R_1}. \quad (28)$$

The discretization is performed by the well-known bilinear frequency transformation. The bilinear transform is defined by the substitution $S=\frac{1-z^{-1}}{1+z^{-1}}$, where $z=e^{j\Omega T}$ is a complex variable, Ω is a normalized frequency, and T is a sampling period. By replacing the normalized complex frequency S in the relation (28), the reflection coefficient becomes

$$\Gamma(z)=\frac{\alpha_1+z^{-1}}{1+\alpha_1\cdot z^{-1}} \quad (29)$$

where the coefficient α_1 is chosen to be

$$\alpha_1=\frac{1/C-R_1}{1/C+R_1}. \quad (30)$$

In case of $C=J^2\cdot L$ (obtained for $Y_{in}=J^2/Y_L$ and $Y_L=\frac{1}{j\omega L}$) and $L=R_2$, this coefficient becomes

$$\alpha_1=\frac{1-J^2R_1R_2}{1+J^2R_1R_2}. \quad (31)$$

Second, let's consider wave digital network shown in Fig. 12. The coefficient β is given by Eq. (13), and $\gamma=\frac{1}{J\cdot R_1}$ is arbitrary chosen constant.

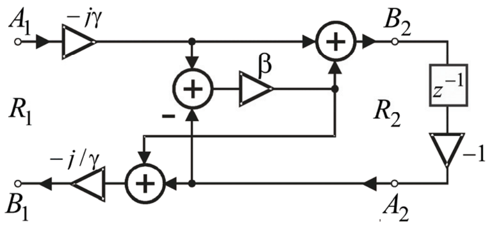


Fig. 12. A wave digital network representing an admittance inverter closed by inductor

The set of equations for J -inverter ($J,+90^{\circ}$) is given by Eqs. (15) and (16). By substituting relation

$$A_2=-z^{-1}\cdot B_2 \quad (32)$$

in those equations, a new equation system is obtained

$$B_1=-\beta\cdot A_1+\frac{j}{\gamma}\cdot(1-\beta)\cdot z^{-1}\cdot B_2, \quad (33)$$

$$B_2=-j\cdot\gamma\cdot(1+\beta)\cdot A_1+\beta\cdot z^{-1}\cdot B_2. \quad (34)$$

If B_2 is computed from Eq. (34), and then eliminated from Eq. (33), the reflection coefficient is found to be

$$\Gamma(z)=\frac{B_1}{A_1}=\frac{-\beta+z^{-1}}{1-\beta\cdot z^{-1}}. \quad (35)$$

It is evident that the reflection coefficients given by Eqs. (29) and (35) are the same because of $-\beta=\alpha_1$. In this way, it is shown that this wave digital network operates exactly like admittance inverter, it modifies load admittance i.e. inverts inductor into capacitor, and vice versa.

IX. CONCLUSION

The admittance inverters are very suitable to transform different filter layouts into equivalent circuits that can be further analyzed by using recently proposed combined wave digital/full-wave electromagnetic approach.

This paper shows how wave digital networks representing digital models of admittance inverters are synthesized based on scattering parameter formalism. A wave digital models of admittance inverters (J,$+90^{o}$) and (J,-90^{o}) are developed based on two-port parallel/series adaptor networks.

Types of microstrip structures that can be analyzed through the idea based on the equivalent circuits with admittance inverters are those of the end-coupled and parallel-coupled half-wavelength resonator filters. Response in formed network model can be easily found by use of block-diagram network drawn in Simulink toolbox and some basic MATLAB functions allowing for accurate and fast modeling and analyzing of microstrip circuits.

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