Thinning of Elliptical and Concentric Elliptical Antenna Arrays Using Particle Swarm Optimization

Rajesh Bera, Jibendu Sekhar Roy

Abstract – **Design of thinned elliptical antenna arrays of uniformly excited isotropic antennas using particle swarm optimization (PSO), is reported. Thinned arrays can generate directive beam with minimum side lobe level (SLL) without degrading the performance of fully populated antenna arrays significantly. Thinning of elliptical antenna arrays using PSO are optimized with different values of eccentricity and variation of side lobe level with eccentricity of thinned antenna array is reported. Using optimization method, it is found that by changing eccentricity of elliptic arrangement of an antenna array, reduced side lobe levels can be obtained. Thinning of concentric elliptical array, using PSO, is also reported in this paper.**

Keywords – **Thinning, Elliptical antenna arrays, Concentric elliptical arrays, Particle swarm optimization, Side lobe level.**

I. INTRODUCTION

Antenna array is used to increase the overall gain of the antenna system and to control the radiation pattern (like, beam tilting, beam shaping etc.) of the array. Appearance of side lobe in array pattern is a physical phenomenon. These side lobes are consuming power and cause of interference in communication towards the undesired directions. Many array synthesis methods are available to reduce side lobe levels keeping desired gain of the array constant [1-2]. A linear array produces good directivity with very narrow beam width but radiation is not uniform in all azimuth directions. For many applications circular array is attractive because of capability of producing directive beams and nulls on the azimuth plane, on the plane of array [1-3]. An elliptical antenna array consists of a number of antenna elements arranged on an ellipse, as shown in Fig 1(a). The geometry of concentric elliptical array is shown in Fig 1(b). In Fig. 1(a), 'a' and 'b' are semi-major and semi-minor axes respectively and eccentricity of the ellipse is given by $e = \sqrt{(1-a^2/b^2)}$.

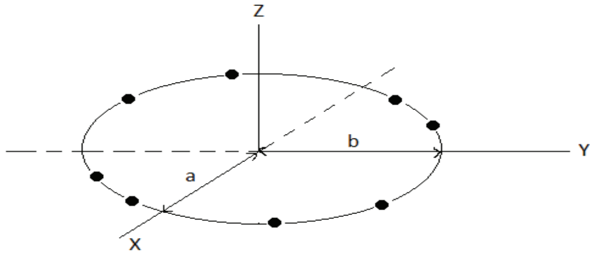

Fig. 1(a). Geometry of elliptic array

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In Fig 1(b), the antenna elements along each ellipse form an elliptical array, and the antenna elements along a radial direction constitute a linear array. In a fully populated array, all the antenna elements are 'on' that is all the elements are excited. Thinning of an antenna array means to switch off some of the antenna elements, to produce narrowest beam with lowest side lobe level, without degrading the performance of the array, seriously. In the 'off' state the antenna element is either open circuited or terminated by a matched load [4]. There are different methods of reducing side lobe level of an antenna array, some of which are statistically tapered thinned arrays [5], thinning based on empirical or analytical formula [6], and thinning using optimization techniques.

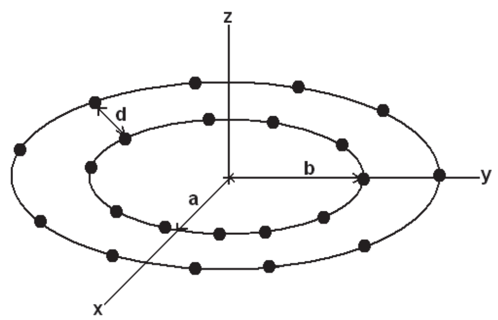

Fig. 1(b). Geometry of concentric elliptic array

During last few years, thinning using optimization technique became very much attractive for thinned array antenna design. Elliptic arrangement has one extra parameter 'eccentricity' compared to circular arrangement and in optimization problem, because of this parameter; it is possible to reduce side lobe level more than that of a circular array [7-8].

In this paper, particle swarm optimization (PSO) [9, 10] is used to minimize side lobe in a thinned elliptical array and concentric elliptical array without degrading the performance of a fully populated array significantly. Also, in this paper, optimization method is used to investigate the thinning of elliptical antenna arrays for various eccentricities. Researchers have applied soft computation techniques (like, genetic algorithm and particle swarm optimization, ant colony optimization) to optimize side lobe level of thinned array antennas [3, 4, 11-21]. But most of the cases, optimization methods are used for linear, planar, circular and concentric circular arrays. Reports on the application of optimization techniques to the thinning of elliptical antenna arrays and concentric elliptical arrays are relatively less.

II. Particle Swarm Optimization

PSO is a population based stochastic optimization method which relies on the social behaviour of the birds [9, 10]. Each single solution is a ‘bird’ in the search space and it is called ‘particle’. Particles have fitness values, evaluated from the fitness function to be optimized and particles have velocities which direct the flying of the particles. PSO does not use genetic operators like, mutation and crossover, but update themselves with the internal velocities. In this paper, programming for PSO algorithm is done using MATLAB. A flowchart for PSO optimization is shown in Fig. 2. First, initialize each particle with a random velocity and random position. Calculate the cost for each particle. If current cost is lower than the best value so far, remember this position (pBest). Choose the particle with the lowest cost of all particles. The position of this particle is gBest. Calculate, for each particle, the new velocity and position, according to the above equations. Repeat steps 2-4 until maximum iteration or minimum error criteria is not attained.

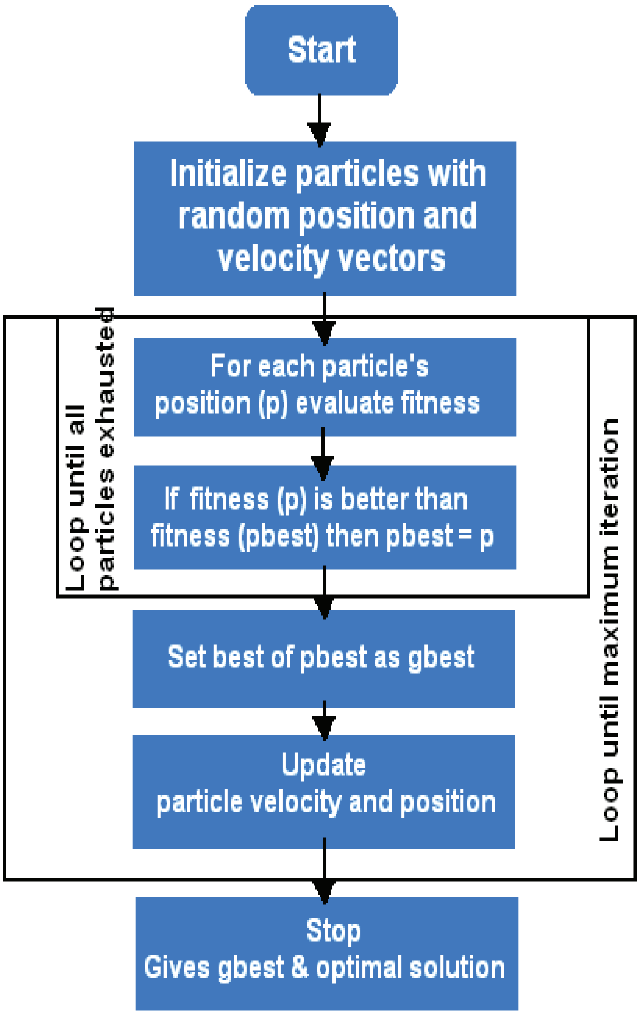

Fig. 2. Flowchart for PSO optimization

III. Elliptical Array Thinning Using PSO

Here three cases are considered where antenna elements in elliptical arrays are 10, 12 and 15. First the side lobe levels of fully populated arrays are computed. Here, the side lobe level (SLL) is defined as SLL (dB)=20$\log_{10}$ (amplitude of side lobe/amplitude of main beam). Due to different combinations of "on" and "off" states of the elements, SLL changes. The maximum amplitude of SLL for particular combinations of "on" and "off" states is defined as SLL_{max}. Then PSO optimization is applied to obtain the minimum side lobe level $min(SLL_{max})$ by changing combinations of "on" and "off" states of the elements in each case. Then eccentricity of the antenna array, in each case, is varied and PSO is applied to obtain the side lobe level of the array. The aim of applying PSO is to obtain minimum side lobe level of elliptical and concentric elliptical array by changing "on" and "off" states of the antenna elements for a particular inter-element spacing. The expression of array factor (AF) is the cost function for PSO optimization. In an antenna array the overall pattern of the array is determined by the array factor multiplied by the radiation pattern of the antenna. In general, the array factor of an N-element elliptic antenna array is expressed as [7, 8]

$$AF(\theta,\varphi) = \sum_{n=1}^{N} A_n (\exp(jk(a_n + R_n a_r))) \quad (1)$$

A_n and α_n are the relative amplitude and relative phase of the of the n-th element of the of the array, R_n is the position vector of the n-th element, a_r is the unit vector of the observation point in spherical coordinates, k is the wave number.

Here,

$$R_n = a\cos\varphi_n a_x + b\sin\varphi_n a_y \quad (2)$$

$$a_r = \sin\theta\cos\varphi\, a_x + \sin\theta\sin\varphi\, a_y + \cos\theta\, a_z \quad (3)$$

where, $\varphi_n=2\pi(n-1)/N$ is the angle in the x-y plane between the x axis and the n-th element.

Substituting Eq. 2 and Eq. 3 in Eq. 1, array factor can be expressed as

$$AF(\theta,\varphi) = \sum_{n=1}^{N} A_n (\exp(jk\sin\theta(a\cos\varphi_n \cos\varphi + b\sin\varphi_n \sin\varphi))) \quad (4)$$

If N is the number of antenna elements lie on ellipses and M is the number of concentric ellipses, then the total array factor of the concentric elliptical array arrangement of isotropic elements is expressed as [7,8]

$$AF(\theta,\varphi) = \sum_{m=1}^{M}\sum_{n=1}^{N} B_{mn} (\exp(jk\sin\theta(a_m\cos\varphi_n \cos\varphi + b_m \sin\varphi_n \sin\varphi))) \quad (5)$$

B_{nm} is the amplitude of excitation current, a_m and b_m are semi-major axis and semi-minor axis of *m*-th elliptical array, respectively. If "a" is the smallest semi-major axis and "d" is the spacing between ellipses [Fig. 1.(b)], then

$$a_m = a + (m-1)d \tag{6}$$

$$b_m = a_m\sqrt{1-e^2} \tag{7}$$

The fitness function for PSO optimization is given by

$$\text{Fitness}=(SLL_d-SLL_a)/SLL_d \tag{8}$$

Where, SLL_d and SLL_a are desired value of SLL and actual value of SLL, achieved in PSO optimization respectively. SLL values are obtained from the expressions of array factors, given by Eq. 1 and Eq. 5. Division by SLL_d in Eq. 8 is to normalize the array factor.

IV. Optimized Results

In PSO optimization, eccentricity is kept constant at 0.6 and inter-element arc spacing is d=0.7λ in all the three cases. Optimized result for 10 element thinned elliptic array is compared with that of a fully populated array and is shown in Fig. 3. Minimum SLL for fully populated array is -8 dB and for optimized thinned array, minimum SLL is -13.95dB. Corresponding array arrangement is shown in Fig. 4, where the number of "on" elements is 6, that is, the array is 60% filled up.

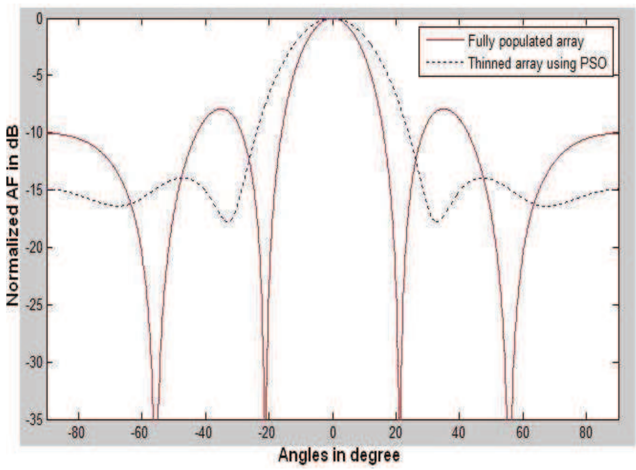

Fig. 3. Array factor (AF) for 10 elements elliptic array (eccentricity=0.6)

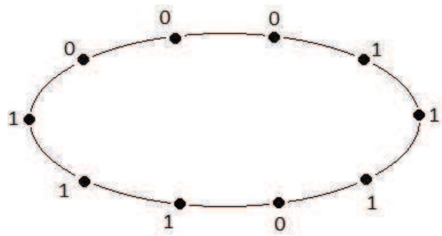

Fig. 4. PSO optimized thinned elliptic array (60% filling)

Optimized result for 12 element thinned elliptic array is compared with that of a fully populated array and is shown in Fig. 5. Minimum SLL for fully populated array is -8.5 dB and for optimized thinned array, minimum SLL is -23.86 dB. The corresponding array arrangement is shown in Fig. 6, where the number of "on" elements is 6, that is, the array is 50% filled up.

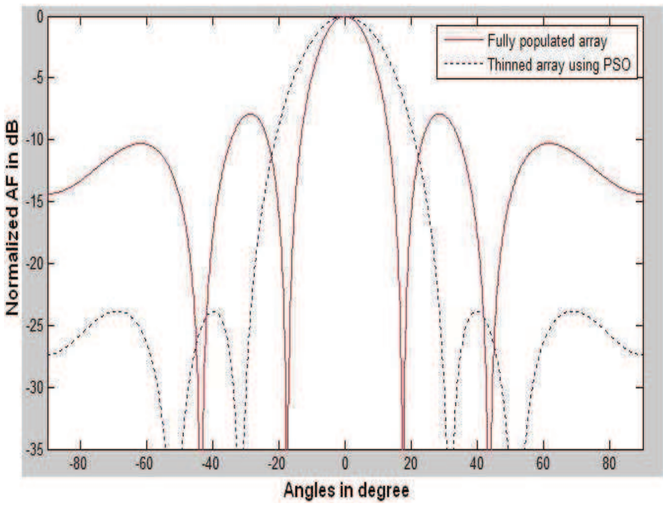

Fig. 5. Array factor (AF) for 12 element elliptic array (eccentricity=0.6)

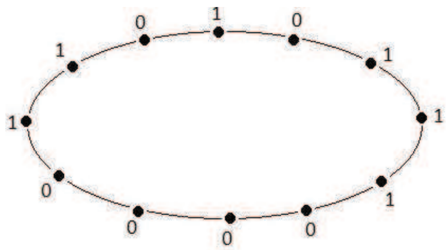

Fig. 6. PSO optimized thinned elliptic array (50% filling)

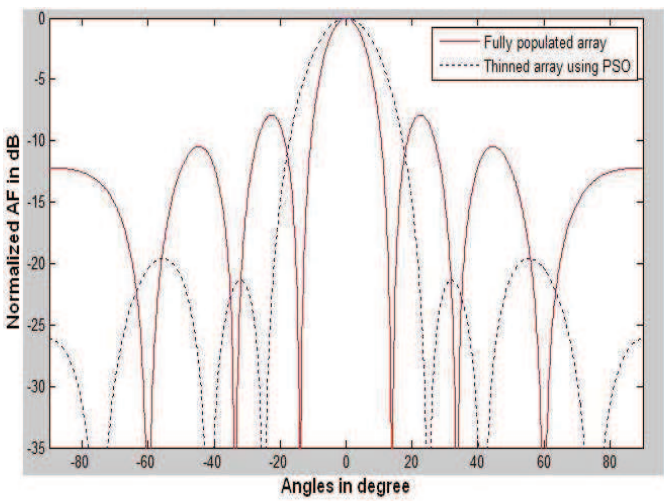

Fig. 7. Array factor (AF) for 15 elements elliptic array (eccentricity=0.6)

Optimized result for 15 elements thinned elliptic array is compared with that of a fully populated array and is shown in Fig. 7. Minimum SLL for fully populated array is -8 dB and for optimized thinned array, minimum SLL is -19.58 dB. The corresponding array arrangement is shown in Fig. 8, where the number of 'on' elements is 7, that is, the array is 46.66% filled up. In Fig. 4, Fig. 6 and Fig. 8, the spacing between the antenna elements is d=0.7λ. In Fig. 3, Fig. 5 and in Fig. 7, the beamwidths of fully populated arrays are less than those of thinned arrays. This is expected, because in thinned array some of the antenna elements are 'off' and hence gain of a thinned array is lower (higher beamwidth) than that of a fully populated array.

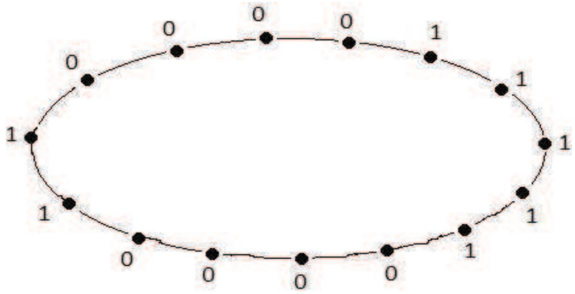

Fig. 8. PSO optimized thinned elliptic array (46.66% filling)

Variation of minimum SLL for different values of eccentricity and for different values of array elements is shown in Fig. 9. For each value of eccentricity and for a particular number of antenna elements, optimized normalized *min*(SLL_{max}) is obtained and plotted in Fig. 9.

In Fig. 9, in all the cases, after a certain value of eccentricity, the graph shows nearly constant value of SLL. This is because of the fact that at higher eccentricity, elliptical array arrangement becomes almost linear array arrangement, resulting in no improvement of SLL.

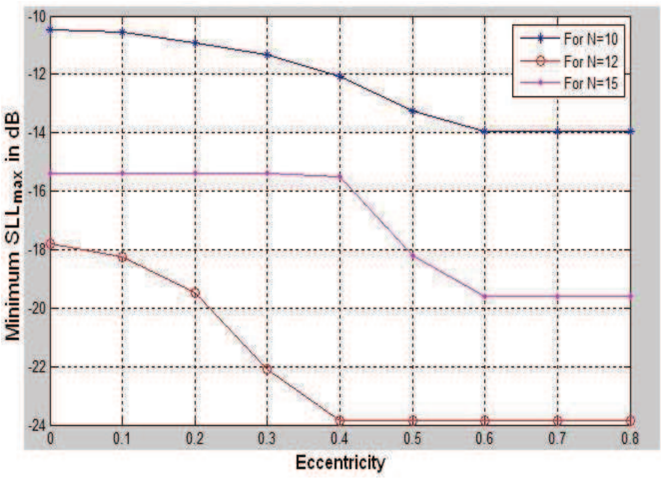

Fig. 9. Variation of minimum SLLmax for different values of eccentricity with d=0.7λ

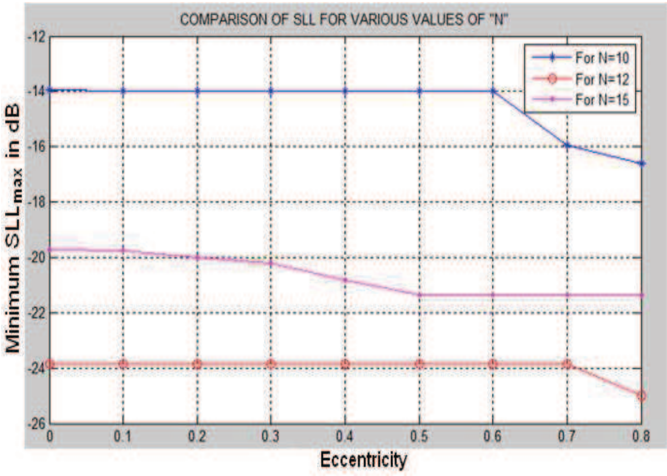

Fig. 10. Variation of minimum SLLmax for different values of eccentricity with d=0.5λ

Similarly, minimum SLL for elliptical array for different eccentricity with inter-element spacing of d=0.5λ are optimized using PSO and the variation of *min*(SLL_{max}) with eccentricity is plotted in Fig. 10.

For concentric elliptical array, element spacing on the inner ellipse is 0.6λ in all the cases and also eccentricity is kept fixed at e=0.8 in both the concentric ellipses. Then PSO is used to minimize the fitness function of Eq. 8, where, SLL is computed using the array factor of Eq. 5.

Radiation pattern of an optimized two thinned concentric elliptical arrays of 15 elements is compared with that of a fully populated concentric elliptical antenna array and plotted in Fig. 11. In the optimized array number of "on" element is 12 and the optimized concentric elliptical array geometry is shown in Fig. 12, where spacing between two ellipses is d=0.3λ. In Fig. 12 "0" and "1" indicate "off" and "on" elements in the array. For fully populated array, minimum SLL is -7.5 dB and for optimized thinned array, minimum SLL is -17.8 dB. Filled ratio is 40%.

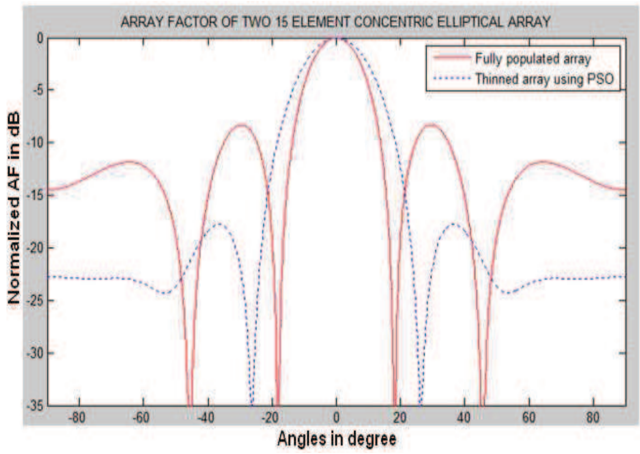

Fig.11. Radiation patterns of fully populated and optimized thinned concentric elliptical arrays (d=0.3λ)

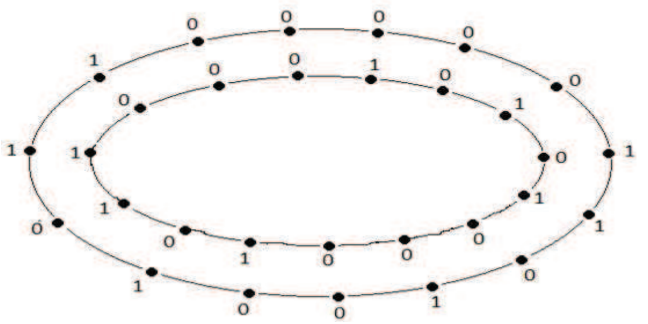

Fig. 12. Geometry of optimized thinned array

Radiation pattern of same array of 15 elements is compared with that of a fully populated concentric elliptical antenna array and plotted in Fig. 13, where spacing between two ellipses is d=0.7λ. In the optimized array number of "on" element is 17 and the optimized concentric elliptical array geometry is shown in Fig. 14. For fully populated array, minimum SLL is -9 dB and for optimized thinned array, minimum SLL is -18 dB. Filled ratio is 56.67%.

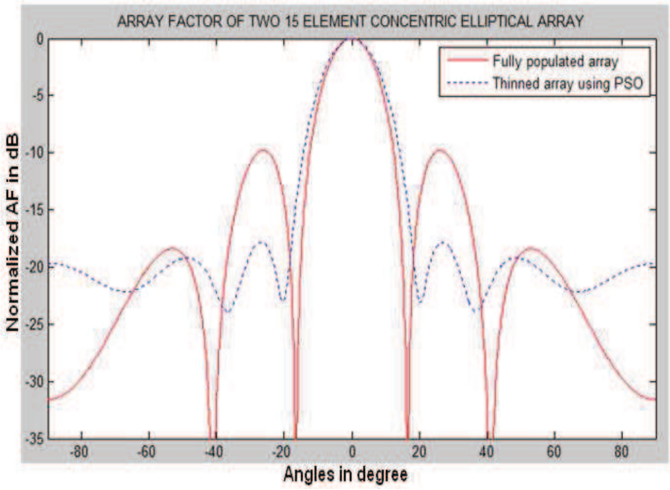

Fig. 13. Radiation patterns of fully populated and optimized thinned concentric elliptical arrays (d=0.7λ)

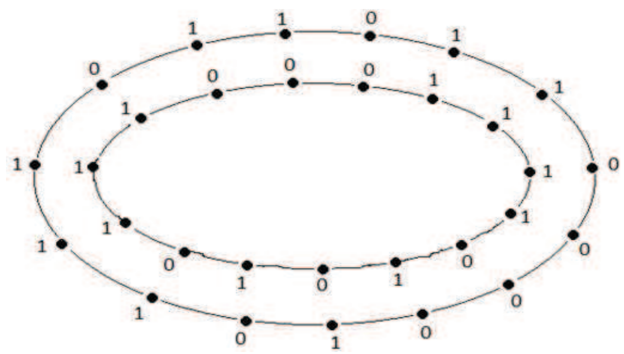

Fig. 14. Geometry of optimized thinned array

In all the above cases, both for elliptical and concentric elliptical arrays, k=6.75 m^{-1}, that is, frequency is 322.45 MHz.

V. Discussion and Conclusion

When Inter-element spacing between the antenna elements is greater than or equal to λ, unwanted lobes with maximum amplitudes may appear which are known as grating lobes. To avoid appearance of grating lobes, in array design, Inter-element spacing is kept below λ [22]. In thinned array antenna design, the case is different; when some of the elements are "off" overall Inter-element spacing of the array becomes non uniform. Inter-element spacing between antenna elements in single elliptical arrays and concentric elliptical arrays is not increased more than 0.7λ. This is because of the fact that in optimized results it is found that if spacing between the antenna elements is about 0.8λ, then sometimes unwanted grating lobes appear. In PSO optimization, when eccentricity is varied, Fig. 3, Fig. 5 and Fig. 7 change according to the corresponding array factor. Also, due to change of eccentricity, 'on' and 'off' states of Fig. 4, Fig.6 and Fig.8 (that is, the arrangement and filling) become different. Optimization of elliptical array shows that lower side lobe level can be achieved by varying eccentricity of the ellipse. In this optimization good SLLs are achieved, but attention has not been paid to maximize the aperture efficiency of the thinned array. The ratio of 'on' element to total number of elements is known as aperture efficiency of a thinned array and only to switch off the maximum number of elements in the array, to obtain lowest SLL, is not sufficient in many cases.

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