Genetic Algorithm using Novel Objective Function for Realizing Compact Uniform Linear Array with Non-Uniform Element Spacing

Appasani Bhargav, Nisha Gupta

Abstract – **This paper presents a technique to realize a compact uniform linear array with non-uniform spacing. The well known soft computing technique, the Genetic Algorithm using a novel objective function based on minimizing the side lobes power is employed for this purpose. A set of genetically produced arrays are created for each case and the array satisfying the minimal coupling criterion is selected from the set. The simulation is performed on several *n* element array and results are compared with the results obtained using other soft computing techniques such as Particle Swarm optimization (PSO), Genetic Algorithm–Conjugate Gradient (GA-CG), Ant Colony Optimization (ACO) and Fire-fly algorithms (FA) etc. It is found that the Genetic Algorithm using proposed objective function outperforms all other algorithms both in terms of offering minimum total length of the array and minimum side lobe levels.**

Keywords – **Uniform linear array, Genetic Algorithm, Particle Swarm optimization, Fire fly algorithm.**

I. INTRODUCTION

Antenna arrays have become an important component of modern communication systems. Modern communication systems demand compact array antennas with high directivity and low side lobe levels. The radiation characteristics of the antenna array depends on relative magnitude and phase of the excitation current of each radiating element, the separation distance between the array elements and the geometrical configuration of the array. By controlling these parameters an antenna array can be designed to produce almost any arbitrary desired pattern. However, increasing the number of radiating elements to increase the directivity may not be always feasible in small handheld devices as increasing the number of elements increases the aperture of the array antenna. The desired low side lobe levels can be achieved by optimizing the amplitudes and phases of the excited array elements with uniform spacing. In amplitude tapering approach different window functions such as Kaiser or Dolph Chebyshev are used [1] and this technique requires a complicated feed system design.

Another technique though less popular for side lobe level reduction is by appropriately changing the inter-element spacing [2-4].

Appasani Bhargav and Nisha Gupta are with the Department of Electronics and Communication, Birla Institute of Technology, Mesra, Ranchi, 835215, India. E-mails: appybarkas@gmail.com, ngupta@bitmesra.ac.in

The NUSLA was first proposed [5] for side lobe reduction employing perturbation methods. In [6], an iterative method was proposed for this purpose. The NUSLA has also been designed using Fourier transform and window techniques [7]. Apart from these techniques, several optimization algorithms such as Genetic Algorithm (GA) [8-9], Particle swarm optimization (PSO) [10], Ant colony optimization (ACO) [11], Firefly algorithm (FA) [12] have also been used to synthesize the NUSLA. In all these earlier work the researchers have chosen the objective function to minimize the side lobes levels without giving attention towards minimizing the array length. In the present paper, the NUSLA problem is formulated as a GA optimization problem where the objective function is based on minimization of the side lobes power instead of side lobes level along with realizing the minimum array length. It is found that the results obtained using the present approach outperforms all other results in terms of total length of the array and side lobe levels.

II. PROBLEM FORMULATION

The motivation behind the proposed work is to realize the compact array antenna structures by reducing the array length. Therefore, an attempt is made to reduce the array length while keeping the side lobe levels to the minimum. The side lobe level minimization is based on the minimization of side lobes power. First an objective function is formulated for array factor which calculates the total power in the side lobes and then the GA is used to minimise the power in the side lobes. In all other previous reported works, the objective function is formulated based on minimising the ratio of power in the side lobes to the power in the main lobes. In the process of optimisation an interesting phenomenon is noticed showing the affect of the array length on the power radiated. The array is synthesized by assuming that the first element is placed at the origin and the distance of the other elements from this element is varied using the GA. The objective function is formulated for the array factor. The power is also calculated for the array factor so that it can be applied for different types of antenna. The array factor is given in (1)

$$AF(\theta) = 1 + \sum_{i=1}^{n-1} a_i e^{-j(\beta d_i \cos(\theta) + \phi_i)} \qquad (1)$$

where n = number of elements, d_i = distance of the i^{th} element from origin.

The absolute value of this array factor and its square are then calculated. Next its value is integrated over all the values of theta in the side lobe and the resultant function is optimised using GA the parameter being the distance between the elements of the array with respect to an element placed at origin. A MATLAB program is developed for performing this optimisation. The algorithm is presented next.

Algorithm:

Function optimisation:

for theta= *(beam width)/2 to* 90°
for d= the *values of distance from origin*
Sum= $1+\sum_{i=1}^{n-1} e^{-j(\beta d_i \cos\theta)}$
end
end
Fitness value = abs(Sum)

The array length is calculated as follows:

Function array length:
Array length= max(d)-min(d)

Fitness value = abs(Sum) +Array length*1000000

III. PROBLEM FORMULATION

First of all the proposed algorithm is tested for the case of uniform linear array with uniform spacing as shown in Fig. 1. The objective function is formulated for the array factor which is independent of the type of the antenna element. Each element is assumed to be fed uniformly. The results obtained are compared with the results available in the literature and are found to be in close agreement. The GA parameters are chosen as
No. of variables: 19,
Population size: 20,
Crossover rate: 0.35
Mutation: Gaussian
Maximum no. of generations: 10000.

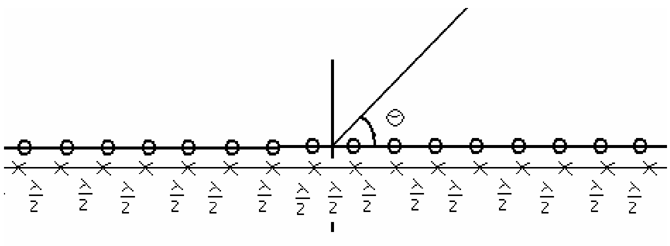

Fig. 1. Uniform Linear array

The compact array design involves three steps.

A. Design Step I

In the first step without imposing the condition of minimum spacing, the SLL is minimised and the results are obtained for no. of elements N=12, N=16, N=20 and N=32 in terms of SLL, array length and HPBW as shown in Table 1. In the previous works the optimization procedure takes into account only one half of the length of the array considering the array to be symmetric with respect to origin. However, in the present work the optimization is performed over the total length of the array. The convergence in the fitness value is shown in Fig.2. The computational time was around 5-10 minutes for each data set and convergence was obtained after 2000 iterations for most of the data sets.

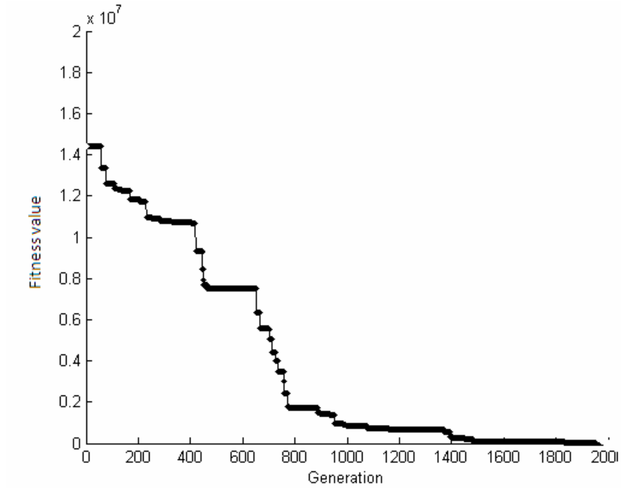

Fig. 2. Convergence in fitness value

The results obtained for no. of elements N=20 tabulated in Table 2 show the results for 10 GA data sets obtained. The data represents the distance of each element with respect to reference element placed at origin in terms of λ/2. It is seen that each set offers a different length of the array with different inter element spacing. The SLL, HPBW and array length for each of this data set for the case of N=20 are tabulated in Table 3. It is seen that the first data set offers both minimum SLL and minimum length of the array but the HPBW is not minimum. However, based on the design requirements the designer can pick up the particular data set for the design.

TABLE 1
ARRAY PARAMETERS

No. of Elements	SLL (dB)	Array length (λ)	HPBW (degrees)
12	13.06	5.5	8.7
16	13.15	7.5	6.5
20	13.19	9.5	5.1
32	13.24	15.5	3.3

TABLE 2
TEN GA DATA SETS FOR DISTANCES IN TERMS OF λ/2

SET1	SET2	SET3	SET4	SET5	SET6	SET7	SET8	SET9	SET 10
1.346217	-3.9698	-3.8426	-3.835	-3.8653	-3.8756	-3.8509	-3.8484	-3.8501	-3.9383
2.015257	-2.417	-2.3105	-2.3012	-2.295	-2.354	-2.34	-2.3404	-2.3512	-2.3412
3.137935	-1.0115	-0.8276	-0.8102	-0.7969	-0.9196	-0.8996	-0.902	-0.9241	-0.9382
3.209753	1.0292	1.2177	1.2535	1.2816	1.0913	1.0621	1.0618	1.0545	1.0627
3.955638	1.6707	1.4851	1.4438	1.4592	1.5924	1.6272	1.6463	1.6885	1.7052
4.461687	2.7914	2.8766	2.8744	2.8481	2.581	2.6133	2.6251	2.65	2.6551
5.199844	3.1404	2.8944	2.8815	2.9728	3.1746	3.1909	3.2131	3.2615	3.284
5.202819	4.3527	3.8744	3.8365	3.8714	3.8935	3.9179	3.9316	3.9801	3.9708
5.593789	4.6431	4.6949	4.7751	4.8347	4.7566	4.767	4.7845	4.8339	4.854
6.445214	5.8184	5.0218	4.9104	4.9666	5.2603	5.246	5.253	5.2947	5.29
6.783158	6.2895	5.8506	5.8544	5.9271	6.1371	6.1279	6.1356	6.186	6.1644
6.951448	7.3286	6.8289	6.8024	6.8392	6.842	6.8034	6.8078	6.8359	6.8666
7.428532	7.9836	6.8301	6.8077	6.9439	7.5007	7.4769	7.4758	7.5218	7.476
8.288286	8.9717	8.2565	8.2737	8.3517	8.4455	8.3961	8.3965	8.4267	8.4332
8.642228	9.847	8.4577	8.3875	8.513	9.0938	9.068	9.0639	9.1005	9.0612
8.789714	10.9168	9.7045	9.6779	9.8028	10.1454	10.1074	10.1012	10.1323	10.1214
10.12016	11.9459	10.5142	10.4705	10.5948	11.0534	11.0364	11.0355	11.0668	11.0981
10.20567	13.279	11.9989	11.963	12.0937	12.4546	12.4519	12.4558	12.4828	12.5551
11.6871	14.7807	13.5366	13.501	13.6652	13.9987	13.946	13.9499	13.9727	14.1752

In GA-CG [13] technique, it is stated that for a narrow beam width and small N, low SLL is not realisable. It is also stated that compact arrays with smaller spacing and low SLL are impossible. The results obtained for N=20 SET 1 outperforms the results obtained in [13] and also offers the minimum array length. The array pattern for SET 1 is shown in Fig. 3. There are other data sets as well which offers low SLL, beam width and array length such as set 3 to set 10.

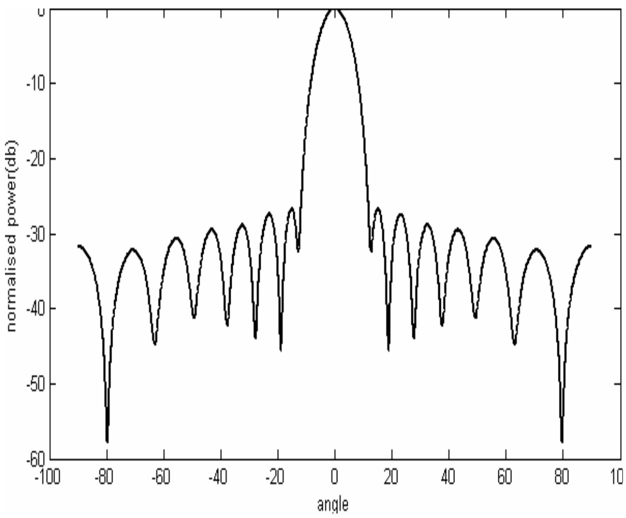

Fig. 3. Array Pattern for Data Set 1 for Array length 5.8435 λ

TABLE 3
ARRAY PARAMETERS FOR EACH DATA SET

No.	SLL (dB)	Array length (λ)	HPBW
SET1	-26.73	5.8435	9.5
SET2	-20.8	9.3752	5.7
SET3	-23.14	8.6896	6.4
SET4	-23.05	8.6680	6.5
SET5	-23.31	8.7653	6.4
SET6	-23.0	8.9371	6.1
SET7	-23.42	8.8985	6.05
SET8	-23.61	8.8991	6.05
SET9	-23.69	8.9114	6.15
SET10	-24.5	9.0568	8.4

B. Design Step II

This step imposes the minimum spacing criteria on the array element spacing. The minimum distance between the elements is taken to be 0.35λ [12] to satisfy minimum coupling criteria. Next, based on this value; the data sets satisfying this condition are selected for each case. The best case for N=20 is shown in Fig. 4. This corresponds to SET 10 which outperforms the parameters of array obtained using FA [12] in terms of SLL and array length but beam width is increased by 1 degree. The resultant pattern is a Chebyshev pattern. The data set 8 and 9 also satisfy the minimum spacing criteria with much lower SLL and smaller array lengths than shown in [12].

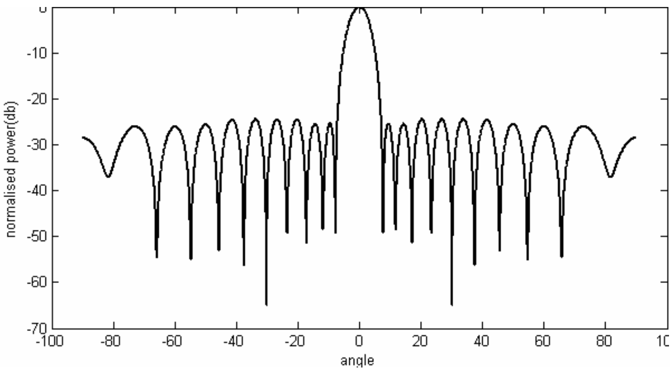

Fig. 4. Array pattern for Data Set 10 for Array length 9.0568 λ

C. Design Step III

In this step both the conditions of satisfying minimum array length as well as minimum spacing criteria are taken into consideration. The array length for SET 10 is found to be slightly lower than the array length obtained using FA [12]. The next step in the design is to use SET 1 which has the smallest array length but does not satisfy the minimum spacing criterion and replace the closely spaced elements with a single element but with double excitation. The GA data set obtained for N=20 is shown in Table 4 and the array is shown in Fig. 5. The SLL is obtained as -24.88dB and HPBW as 5 degrees for this particular data set which outperforms the results obtained in [12] in terms of all array length and SLL.

Next data sets are created for N=32 for the purpose of comparing the results with the results available in the literature for PSO [10] and ACO [11]. The best result obtained after design step 3 is shown in Table 5. The array pattern obtained is shown in Fig. 6. The results thus obtained are compared with that obtained by PSO and Ant Colony Optimisation [11]. The SLL achieved using proposed algorithm is -23.4dB and beam width as 7.4° whereas the results obtained using PSO offers SLL = -20dB and using ACO, SLL = -15dB and beam width is 7.7°. The first null obtained in the proposed work is almost 48dB lower where as that obtained using PSO [10] is lowered by just 30dB clearly showing an improvement using proposed GA algorithm. Finally the results of the proposed algorithm are compared with the results for GA-CG, FA, PSO and ACO and are tabulated in Table 6.

TABLE 4
DISTANCE OF EACH ELEMENT FROM ORIGIN FOR N=20

Distances (d_i)	λ/2	Distances (d_i)	λ/2
d_1	1.3462	d_{11}	6.9514
d_2	2.0153	d_{12}	6.9514
d_3	3.1379	d_{13}	7.4285
d_4	3.1379	d_{14}	8.2883
d_5	3.9556	d_{15}	8.6422
d_6	4.4617	d_{16}	8.6422
d_7	5.1998	d_{17}	10.1202
d_8	5.1998	d_{18}	10.1202
d_9	5.5938	d_{19}	11.6871
d_{10}	6.4452		

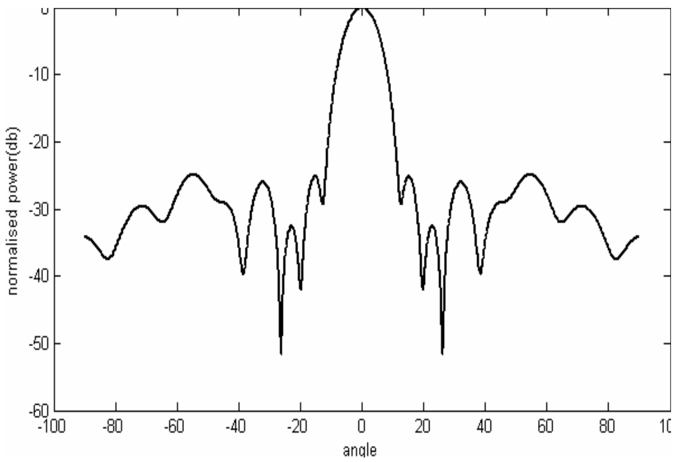

Fig. 5. Array pattern for N = 20

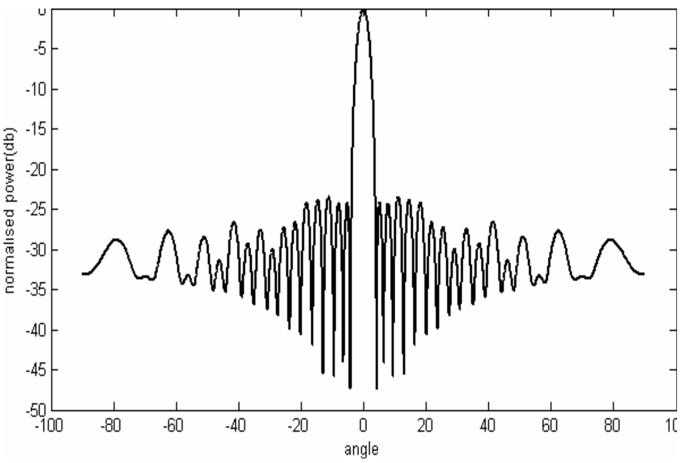

Fig. 6. Array Pattern for N = 32

TABLE 5
DISTANCE OF EACH ELEMENT FROM ORIGIN FOR N=32

Dist (d_i)	λ/2	Dist (d_i)	λ/2	Dist (d_i)	λ/2
d_1	1.517	d_{11}	11.649	d_{21}	19.726
d_2	1.708	d_{12}	12.302	d_{22}	20.573
d_3	3.364	d_{13}	13.428	d_{23}	21.454
d_4	4.758	d_{14}	13.791	d_{24}	22.427
d_5	5.895	d_{15}	15.184	d_{25}	23.383
d_6	7.010	d_{16}	15.550	d_{26}	24.490
d_7	7.985	d_{17}	16.670	d_{27}	25.614
d_8	8.961	d_{18}	17.018	d_{28}	27.016
d_9	9.860	d_{19}	18.150	d_{29}	28.686
d_{10}	10.71	d_{20}	18.796	d_{30}	30.401
				d_{31}	31.922

TABLE 6
COMPARISON OF VARIOUS METHODS

	PROPOSED METHOD				PSO [10]	GA-CG [13]		ACO[11]	FA [12]
	STEP 1	STEP 2	STEP 3	STEP 3		STEP 1	STEP 2		STEP 2
N	20	20	20	32	32	20	20	32	20
SLL (dB)	-26.76	-24.5	-24.88	-23.4	-22.24	-24.87	-22.6	-17.5	-23.38
Beam Width (degrees)	25	15.6	25	7.4	not specified	16	14.6	7.7	14.6
Array length (λ)	5.843	9.056	5.843	16.8	16	8.74	10.12	16.3	9.26

IV. CONCLUSION

A systematic approach of a GA technique employing a novel objective function based on calculation of side lobes power is presented for a uniform linear array with non uniform spacing in order to realize a compact array. It is found that the proposed GA technique outperforms all other techniques in terms of length of the array and side lobes level. The data set generated also offers flexibility in selecting the desired parameters for the specific design goals.

REFERENCES

[1] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beamwidth and side-lobe level", *Proc. IRE*, vol. 34, no. 6, pp. 335-348, 1946.

[2] R. Harrington, "Sidelobe reduction by nonuniform element spacing", *IRE Transactions on Antennas and Propagation*, vol. 9, no. 2, pp. 187–192, 1961.

[3] B. P. Kumar and G. R. Branner, "Design of unequally spaced arrays for performance improvement", *IEEE Transactions on Antennas and Propagation,* Special Issue on Phased Arrays, vol. 47, no. 3, pp. 511–523, 1949.

[4] B. P. Kumar and G. R. Branner, "Generalized analytical technique for the synthesis of unequally spaced arrays with linear, planar, cylindrical or spherical geometry", *IEEE Transactions on Antennas and Propagation*, vol. 53, no.2, pp. 621–634, 2005.

[5] M. I. Skolnik, G. Nemhauser, and J. W. Sherman, III, "Dynamic programming applied to unequally spaced arrays", *IEEE Transactions on Antennas and Propagation*, vol. 12, no. 1, pp. 35–43, 1964.

[6] F. Hodjat and S. A. Hovanessian, "Nonuniformly spaced linear and planar array antennas for sidelobe reduction", *IEEE Transactions on Antennas and Propagation.*, vol. 26, no. 2, pp. 198–204, 1978.

[7] X. F. Ren, J. A. Azevedo, and A. M. Casimiro, "Synthesis of non-uniformly spaced arrays using the Fourier transform and window techniques", *IET Microwaves, Antennas and Propagation,* vol. 3, no. 8, pp. 1245–1253, 2009.

[8] D. G. Kurup, M. Himdi, and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm", *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 9, pp. 2210–2217, 2003.

[9] K. Chen, Z. He, and C. Han, "A modified real GA for the sparse linear array synthesis with multiple constraints", *IEEE Transactions on Antennas and Propagation*, vol. 54, no. 7, pp. 2169–2173, 2006.

[10] M. M. Khodier, and C. G. Christodoulou, "Linear array geometry synthesis with minimum sidelobe level and null control using particle swarm optimization", *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 8, pp. 2674–2679, 2005.

[11] E. Rajo-lglesias and Q. Quevedo-Teruel, "Linear array synthesis using an ant-colony-optimization-based algorithm", *IEEE Antennas and Propagation Magazine*, vol. 49, no. 2, pp. 70–79, 2007.

[12] M. A. Zaman and M. A. Matin, "Nonuniformly spaced linear antenna array design using Firefly Algorithm", *International Journal of Microwave Science and Technology*, pp. 1-8, 2012.

[13] H. Oraizi and M. Fallahpour, "Non-uniformly spaced linear array design for the specified beam-width/sidelobe level or specified directivity/sidelobe level with coupling considerations", *Progress in Electromagnetic Research M,* vol. 4, pp. 185-209, 2008.