Impact of Array Length on Power Radiated in Non-uniformly Spaced Arrays with Low Side Lobes

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Abstract — **This paper presents a relationship between the array length and power radiated in a non-uniformly spaced and uniformly excited antenna array with low side lobes. Using the Genetic Algorithm, a well-known evolutionary algorithm, several non-uniformly spaced arrays are created for low side lobe levels and for various array lengths. The normalized power radiated by each array created for N = 12, 16, 20, 25, 30 and 35 is calculated and a relationship is established between the array length and normalized power. A dipole antenna operating at 9.5 GHz is used as an array element to demonstrate the concept of the proposed array for one data set. From the radiation pattern of the antenna array normalized power is calculated and is compared with the result obtained using the proposed relationship. Also an interesting phenomenon observed beyond a certain length of the array has been reported.**

Keywords — **Antenna arrays, Antenna theory, Optimization methods, Antenna radiation patterns.**

I. INTRODUCTION

Modern radar systems employ antenna arrays with narrow beam widths and low side lobe levels. This avoids electromagnetic interference with other transmissions and hazardous emissions on transmit. Most commonly, for the specified side lobe level a threshold is set and the optimization is carried out to achieve the specified side lobe level. However these goals can be very well achieved by reducing the total power in the side lobes, rather than setting a threshold for the side lobe level. This is because of the fact that the radiation characteristic of an antenna array depends on its array factor and the array factor for a uniformly excited array depends on the inter-element spacing. Also the power radiated by such an array depends on the array factor. Therefore by minimizing the power radiated in the side lobes, one can effectively minimize the SLL in an effective manner. In the past Dolph-Chebyshev technique [1-2] or Taylor line source technique [3] were used to achieve arrays with low SLL. Recently several optimization techniques have been employed for this purpose [4-8]. In [9] impact of a number of elements on the array factor has been analyzed. In [10] the effect of higher order roots of the array factor polynomial on the beam width of the array has been studied. A relationship between SLL and beam width is presented in [11] plotted. In the proposed work the Genetic Algorithm is used to create non- uniformly spaced arrays with low SLL by minimizing the power in the SLL rather than the SLL itself. Several arrays are created using this technique and the SLL and array length in each case has been calculated. The normalized power, normalized with respect to the power radiated by each element of this array is calculated from the array factor and a relationship between the total power radiated and the array length is developed. The knowledge of the effect of array length on the power radiated would be beneficial in the design of such arrays. Also a dipole antenna operating at 9.5 GHz is designed using Method of Moments software IE3D. This antenna shows a near isotropic behaviour and is used to construct an array. The radiation pattern of the array is developed and the normalized power radiated by the array is calculated from the radiation pattern. This result is finally compared with the result obtained using the proposed relationship. Section II deals with the problem formulation, section III presents the simulation results and final conclusions are drawn in section IV.

II. PROBLEM FORMULATION

First an objective function is formulated for the array factor which calculates the total power in the side lobes and then the GA is used to minimize the power in the side lobes. The array is realized by assuming that the first element is placed at the origin and the distance of the other elements from this element is varied using the GA. In the process of optimization, the effect of the array length on the power radiated is observed.

The array factor is given as follows:

$$\mathrm{AF}(\theta) = 1 + \sum_{i=1}^{n-1} a_i e^{-j(\beta d_i \cos(\theta) + \phi_i)} \tag{1}$$

where n = number of elements; d_i = distance of the i^{th} element from the origin i.e. from the first antenna element; a_i = amplitude excitation of the i^{th} element; Φ_i = phase of the i^{th} element.

The absolute value of this array factor and its square are then calculated. Next its value is integrated over all the values of theta in the side lobes and the resultant function is optimized using GA, the parameter being the distance between the elements of the array of an element placed at the origin. A MATLAB program is developed for performing this optimization. The algorithm is presented as follows

Pseudo Code:

```
for theta= minor lobe range
    for d= the values of distance from origin
        $\mathrm{AF}(\theta) = 1 + \sum_{i=1}^{n-1} a_i e^{-j(\beta d_i \cos(\theta) + \phi_i)}$
    end
end
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The array length is calculated as follows

Array length = [max (d)-min (d)] if min (d) < 0

Array length = [max (d)] if min (d) ≥ 0

The objective function used for the purpose of optimization is given in equation (2)

$$\text{Objective function} = \text{Array length} + |AF(\theta)|^2 \qquad (2)$$

Each optimization is run with the following GA parameters:

No. of variables = N-1

Population size = 20

Crossover rate = 0.35

Maximum no. of generations =1000.

III. SIMULATION RESULTS

The elements of the array are uniformly excited. Relaxing the constraint of uniform spacing, the problem is formulated in terms of minimization of the side lobe level for different array length and the results are obtained for no. of elements N=12, 16, 20, 25, 30 and 35. No constraint is placed on the inter element spacing. The first element is always placed at origin. The distances of other elements are with respect to the origin (i.e. the first element) and normalized with respect to $\lambda/2$. All of the arrays generated have a proper radiation pattern and low SLL. Ten data sets obtained using GA for N=12 are shown in Table I. The SLL, power radiated and array length for various N are shown in Tables II, III, IV, V and VI. The radiated powers presented in these tables are represented in terms of normalized power, normalized, with respect to the power radiated by each element, where all the radiating elementsare assumed to be isotropic having identical power input. This is explained in Appendix.

Fig. 1 shows the radiation patterns for first three data sets and all of them have low side lobe levels. All the radiation patterns show a low side lobe level and are calculated for far field.

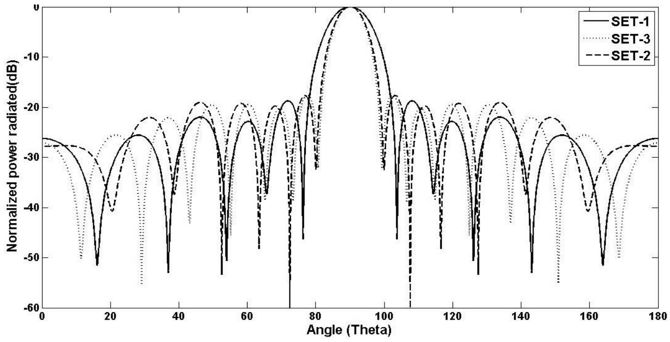

Fig. 1. Array pattern for data sets 1, 2 and 3

The following observations are made based upon the results:

- While dealing with the minimization of the power in the side lobes it is observed that just by varying the array length for a fixed number of elements, the output power radiated by the array increases with the decrease in the length of the antenna. The relationship between the total radiated power and array length shown in Fig. 2 and Fig. 3, which shows an inverse relationship.
- It is also observed that the SLL does not depend on the total power in the side lobes. Hence justifies the approach adopted in the proposed work. It is preferable to minimize the power radiated by the side lobes rather than setting a threshold for side lobe level since the purpose is to establish relationship between power radiated and array length.

Finally an empirical formula is obtained establishing a relationship between the normalized power radiated by small arrays such as N<50, the array length is given by equation (3). This relationship is obtained using the curve fitting Toolbox of MATLAB®. Moreover, the relationship holds good for low SLL, where low SLL indicates an SLL low in comparison to the uniform linear array for the same number of elements and almost the same beam width as that of a uniform linear array.

Normalized power radiated=

$$-18.56 * (\text{Array Length}) * N + 64.7 * N^{1.51} \qquad (3)$$

where N = number of elements in the array.

TABLE II

SLL AND ARRAY LENGTH FOR N=12

SET NO:	NORMALIZED POWER RADIATED	SLL (dB)	ARRAY LENGTH (λ)
1	1744.4174	-18.76	4.4275
9	1570.8691	-18.13	4.96
8	1550.585	-17.76	4.99
7	1534.1605	-18.2	5.086
4	1516.5664	-18.82	5.1609
10	1529.03	-21.14	5.24
5	1432.524	-16.71	5.3971
3	1337.933	-17.72	5.8853
2	1255.4552	-17.6	6.2526
6	1240.8227	-18.23	6.351

TABLE III

SLL AND ARRAY LENGTH FOR N=16

NORMALIZED POWER RADIATED	SLL (dB)	ARRAY LENGTH (λ)
2441.6	-19.26	5.953
2143.4	-19.15	6.808
2098.7	-19.4	6.967
2039	-18.51	7.086
1991.5	-18.99	7.229
1783.4	-19.85	8.226

TABLE I
TEN GA DATA SETS FOR DISTANCES IN TERMS OF Λ/2 FOR N=12

	SET1	SET2	SET3	SET4	SET5	SET6	SET7	SET8	SET9	SET 10
$2d_1/\lambda$	7.733	9.796	10.433	9.009	9.556	8.587	-1.423	8.592	8.663	8.947
$2d_2/\lambda$	5.654	7.024	7.079	6.212	6.683	7.416	-0.049	5.984	6.054	6.315
$2d_3/\lambda$	3.110	-1.307	3.401	3.504	3.225	1.399	5.071	3.279	3.316	3.337
$2d_4/\lambda$	1.952	1.957	2.494	2.134	2.246	-1.380	-3.049	1.963	1.931	2.175
$2d_5/\lambda$	4.496	4.842	5.246	5.035	4.953	5.351	3.664	4.614	4.677	5.105
$2d_6/\lambda$	8.855	8.408	11.771	10.322	10.794	11.322	-3.898	9.913	9.981	10.481
$2d_7/\lambda$	6.989	3.882	9.091	7.932	8.499	6.329	-1.845	7.571	7.652	7.912
$2d_8/\lambda$	3.112	2.890	4.394	3.333	4.020	3.471	-5.102	3.186	3.143	3.737
$2d_9/\lambda$	6.038	-2.710	8.087	6.876	7.536	9.962	1.497	6.661	6.672	6.939
$2d_{10}/\lambda$	4.452	5.903	6.185	4.812	5.733	4.367	1.599	4.831	4.843	5.033
$2d_{11}/\lambda$	1.220	0.939	1.249	1.269	1.221	2.495	2.768	1.186	1.192	1.283
ARRAY LENGTH (λ)	4.4275	6.2526	5.8853	5.1609	5.3971	6.351	5.086	4.99	4.96	5.24

TABLE IV
SLL AND ARRAY LENGTH FOR N=25

NORMALIZED POWER RADIATED	SLL (dB)	ARRAY LENGTH (λ)
2870.4	-13.46	12.1512
2815.6	-14.04	12.3594
2785.8	-14.15	12.5313
2779.3	-14.57	12.5843
2777.6	-14.44	12.5855
2675.4	-15.81	13.1616
2682.8	-15.42	13.1456
2805.4	-13.97	12.4089
2535.5	-16.96	13.9905
2471.5	-17.37	14.3619
2404.5	-16.88	14.7838
2347.6	-16.90	15.1355

TABLE V
SLL AND ARRAY LENGTH FOR N=30

NORMALIZED POWER RADIATED	SLL (dB)	ARRAY LENGTH (λ)
3421.3	-13.47	14.7252
3412	-13.60	14.7885
3396.9	-14.16	14.8750
3342.2	-14.46	15.1656
3245	-15.23	15.7466
3252.7	-15.12	15.6448
3155.4	-16.52	16.2330
3108.4	-16.81	16.5237
3040.8	-17.38	16.9608
3006.9	-17.33	17.1334

The power obtained using the above equation and the actual isotropic power radiated by uniform arrays for N=12, N=16 N=20 and N=25are compared and the comparison is shown in Table VII.

TABLE VI
SLL AND ARRAY LENGTH FOR N=35

NORMALIZED POWER RADIATED	SLL (dB)	ARRAY LENGTH (λ)
4011.1	-14.02	17.1869
3941.7	-13.97	17.4819
3937.6	-13.69	17.4943
3855.2	-14.64	17.9796
3797.8	-15.35	18.3210
3756	-16.02	18.6082
3703	-16.88	18.9410
3667.1	-17	19.1666

TABLE VII
COMPARISON OF VALUES FOR NORMALIZED RADIATED POWER

N	ARRAY LENGTH (λ)	NORMALIZED RADIATED POWER		ERROR (%)
		EMPERICAL RESULT	SIMULATION RESULT	
12	5.5	1532.24	1407.3	-8.87
16	7.5	2030	1866.8	-8.74
20	9.5	2436.5	2325.9	-4.75
25	12	2784.1	2911	+4.35

The practical applicability of the results obtained is demonstrated using a dipole antenna element which shows a near isotropic radiation pattern in the azimuth plane at the operating frequency of 9.5 GHz. The length of the dipole is 15 mm. Using this antenna an array is created using the data set 2 shown in Table I. This data set is selected because of sufficient inter element spacing which would prevent mutual coupling effects from effecting the radiating pattern. Each of the antenna elements is excited uniformly. The antenna is shown in Figure 4and the antenna array created using SET 2 is shown in Figure 5. The radiation pattern of single antenna and that of the array is shown in Figure6. Also the reflection coefficient characteristics are shown in Figure7. From the radiation characteristics the normalized power radiated by the array is calculated. This value obtained is 1254.1 as compared to the value of 1255.46obtained from Table II. Using equation (3) this value is obtained as1364.6. The difference in the simulation result and the result obtained from equation (3) is because of the difference in the ideal isotropic azimuth pattern of the dipole antenna and non-isotropic nature of the antenna pattern. Next, the equation (3) is used to predict the limit of the array length for a given number of array elements within which it is possible to achieve low side lobe levels and similarly the minimum number of array elements required for a given array length for which it is possible to achieve low side lobe levels.

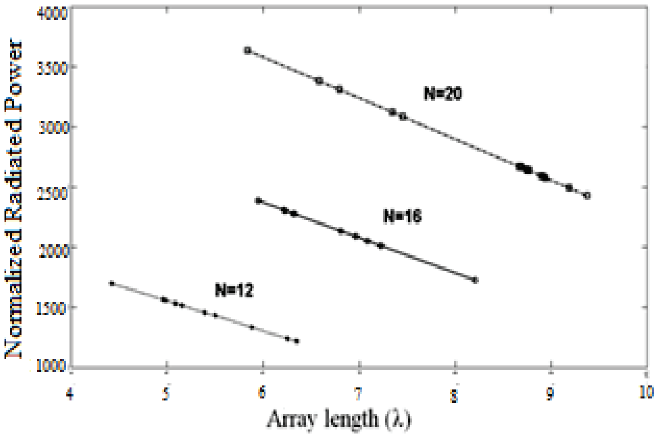

Fig. 2. Relationship between Array length and power radiated for N=12, 16 and 20

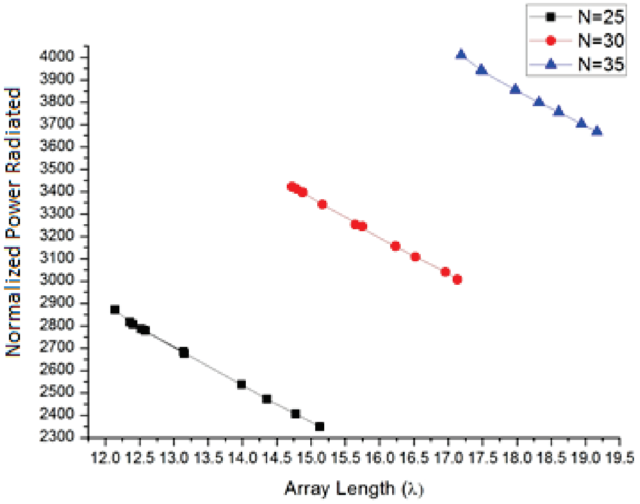

Fig. 3. Relationship between Array length and power radiated for N=25, 30 and 35

Since power radiated is always positive, equation (3) is always greater than zero. Using this inequality the following relationship is established between array length and the number of array elements for which it is possible to achieve low side lobe levels.

$$N > (0.0865)\,(\text{Array Length})^{1.961} \quad (4)$$

$$\text{Array Length} < (3.4860)\,(N)^{0.51} \quad (5)$$

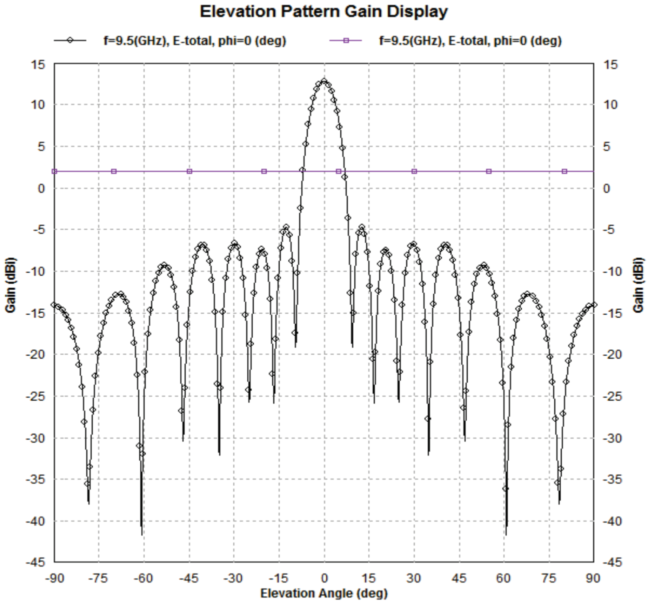

Fig. 6. Radiation pattern of the dipole antenna (blue) and radiation pattern of the dipole antenna array (black)

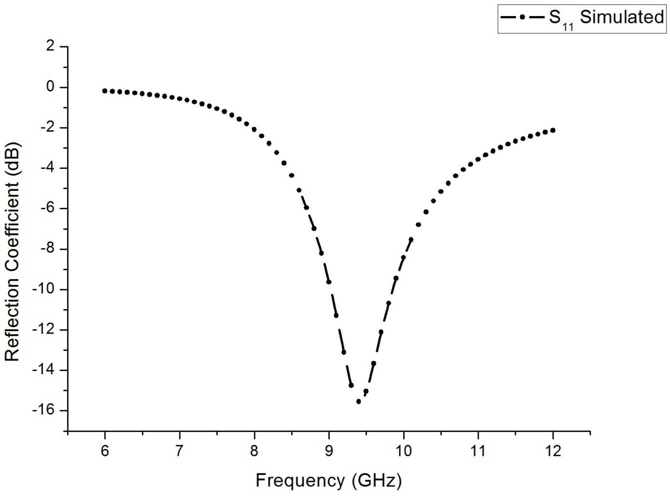

Fig. 7. Reflection coefficient of the dipole antenna

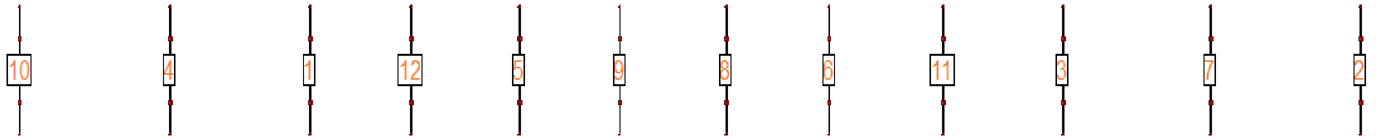

Fig. 5. Antenna Array created using SET 2

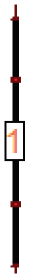

Fig. 4. Dipole antenna of length 15 mm

IV. CONCLUSION

An empirical formula showing a relationship between the normalized radiated power and the array length for a Non-Uniformly Space Linear Antenna array (NUSLA) with low side lobe level is developed. The GA optimization technique is employed for this purpose. The relationship holds good for arrays having low SLLs, which are lower than the SLLs of the uniform array formed with the same number of elements and having more or less same beam width. It is observed that by decreasing the length of the array the power radiated by the array increases while maintaining a low SLL. Therefore by suitably selecting a small array, one can concentrate maximum power into the main beam mostly suitable for radar systems. From the empirical formula, it can be understood that there exists a critical length beyond which it is not possible to generate arrays having low side lobe levels for a fixed number of radiating elements. This relationship is very important in predicting the power radiated by compact arrays having low SLL without going into the details of the analysis. The effect of mutual coupling between the elements can be considered in future work.

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APPENDIX

Let ‘E’ be the electric field strength of an isotropic antenna. The electric field strength of the array formed using this element would be:

$$E_{ARRAY} = E \times \left(AF\ (\theta)\right)$$

The Power radiated by the antenna array is given as:

$$P_{ARRAY} = |E|^2 \times |AF\ (\theta)|^2$$

In order to find the total power radiated by the array, we have to integrate the power radiated by the array over the range of θ, from 0° to 180°.

$$P_{TOTAL} = \int_0^{\pi} P_{ARRAY}\, d\theta$$

$$P_{TOTAL} = \int_0^{\pi} |E|^2 \times |AF\ (\theta)|^2 d\theta$$

$$P_{TOTAL} = |E|^2 \times \int_0^{\pi} |AF\ (\theta)|^2 d\theta$$

The normalized power radiated can be obtained by dividing the above expression with the power radiated by an isotropic array element. The expression for the Normalized Radiated Power is:

$$P_{NORMALIZED} = \int_0^{\pi} |AF\ (\theta)|^2 d\theta$$

This is just a number and is unitless.