Fuzzy Control for Chaotic Response Improvement in MEMS Resonators

Joan Barceló[1], Zlatica Marinković[2], Vera Marković[2], Jaume Verd[1]

Abstract – **Starting with the design and biasing conditions to enable cross-well chaotic behaviour in a nonlinear MEMS resonator, this paper presents for the first time a control method based on fuzzy logic to maximize such a chaotic response, providing a design way for its potential use in secure communication applications. Theoretical analysis and simulations performed on a cc-beam MEMS resonator model designed in a CMOS technology verify the effectiveness of the proposed control method.**

Keywords – **Fuzzy logic control, MEMS resonators, nonlinear and chaotic dynamics.**

I. INTRODUCTION

Nonlinearity is an inherent feature of the MEMS oscillators, and it is displayed both in mechanical and electrical domains [1].Through such nonlinearity, under certain design and operating conditions, a chaotic behaviour may arise as experimentally observed in [2]-[3]. In these works, a robust chaotic motion is achieved in electrostatically actuated non-overlapping and capacitive readout comb MEMS devices, operating at a natural frequency in the range of KHz. The deterministic and apparently noise-like behavior of the chaotic regime, together with its capability of auto-synchronization (demonstrated by Pecora and Carrol for the Lorenz system in [4] and experimentally tested by Cuomo and Oppenheim in [5]) make it a potential candidate in cryptographic applications through either masking or modulation chaotic encryptions [6].

The use of hardware based real time encryption schemes meets the need of high speed in data communication systems over traditional software cryptographic protocols. To fulfil the requirements for this purpose, an improvement of the dynamic features (in terms of frequency and chaotic response) regarding to the models reported in literature, is demanded, and at the same time keeping the benefits that they provide. Designs with complex geometries, like non-overlapping comb-drives reported in [2] - [3] present more constraints in terms of fabrication and relative low natural frequencies. Contrary, the use of simple resonant structures as a clamped-clamped beam (cc-beam) resonator, provides significant benefits in terms of integration feasibility and scalability to nanometer dimensions (NEMS resonators) enabling both high sensitivity and high frequency operation [7]. In addition the use of electrostatic actuation and capacitive readout in MEMS resonators enables an easy and low cost integration of such electromechanical devices with electronics into a single CMOS die (monolithic integration) [8] - [9]. The development of chaotic MEMS oscillators with better performance (for instance, in terms of resonance frequency and miniaturization capabilities) comparing to the devices reported in literature may imply a boost for a new generation of MEMS devices for competing with optical devices in secure communication applications.

Fuzzy logic has been revealed to be a powerful method to control different kind of plants. Particularly useful is its application in the control of nonlinear, subject to noise, or partially unknown systems. The applications of fuzzy-logic controllers to chaotic systems reported in literature are focused on control and suppression of the chaotic response, which is considered to be undesirable. Thus, in [10] a fuzzy controller obtained from a knowledge base formed by fuzzy sets and inference matrix is used to perturb a variable of the Chua circuit and stabilize the chaotic output. Meanwhile, in [11] Gaussian membership based fuzzy models are used to estimate the unknown parameters and functions needed to build a controller that is proved, by means of the Lyapunov stability theory, to be useful to control a Duffing system. Concerning to the dynamics of MEMS, fuzzy systems have been designed in [12]-[14] to mimic ideal controllers based on the Sliding Mode Control, i.e. feedback controllers which cause the state trajectory to reach a sliding surface and behaving as a stable trajectory. In such references, the fuzzy control is used to convert chaotic oscillations into desired regular ones, exhibiting a periodic behavior.

Even when the conditions to achieve robust and extensive chaotic response in nonlinear MEMS oscillators are verified, the attainment of the proper parameters for chaotic signal generation is not immediate. Instead of a large amount of time consuming and approximate numerical simulations, in this paper a new control method is proposed to achieve the desired chaotic response. For the first time a fuzzy-logic based control is used to maximize the chaotic response of a nonlinear MEMS system.

The paper is organized as follows. In Section II, the cc-beam MEMS considered in this work as the chaotic signal generator is mathematically modelled, and the limits for its chaotic operation are stated. In Section III, the fuzzy control is proposed and discussed, the convergence to the desired goal is mathematical studied and simulation results are presented. Finally the main concluding remarks are given in Section IV.

[1]J. Barceló and J. Verd are with the University of Balearic Islands Electronic Systems Group. Dept of Physics,, 07122 Palma, Illes Balears, Spain. E-mails: j.barcelo@uib.es, jaume.verd@uib,es

[2]Zlatica Marinković and Vera Marković are with the University of Nis, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia. E-mails: zlatica.marinkovic@elfak.ni.ac.rs, vera.markovic@elfak.ni.rs

II. MATHEMATICAL MODEL OF CC-BEAM RESONATOR

The nonlinear resonant cc-beam CMOS-MEMS under consideration (depicted in Fig. 1) is constituted by a microbeam placed symmetrically and in-plane between two electrodes, in which a sinusoidal excitation with amplitude V_{ac} and frequency $\omega/2\pi$ (excitation electrode) and capacitive detection (readout electrode) are respectively performed. A biasing voltage V_{dc} is applied to the bridge to generate a motional current due to the resonator oscillation that is detected by the readout electrode [9]. The motional current can be measured and converted to a voltage signal by a transimpedance amplifier. The use of this topology minimizes the parasitic feedthrough current in comparison with other topologies. The system can be conceived as a lumped mass-spring model, assuming that the mass is located at the middle point of the cc-beam [15]. Eq. (1) describes the mechanical resonator dynamics in one dimension (x-axis)

$$m_{eff}\ddot{x} + k_1 x + k_3 x^3 + \gamma\dot{x} = F_{exc}, \qquad (1)$$

where $\dot{x}$ and $\ddot{x}$ represent, respectively the first and second time derivative of the position variable x, m_{eff} is the resonator effective mass, γ is the damping coefficient, k_1 and k_3 are the linear and cubic mechanical stiffness respectively, and F_{exc} is the net excitation force.

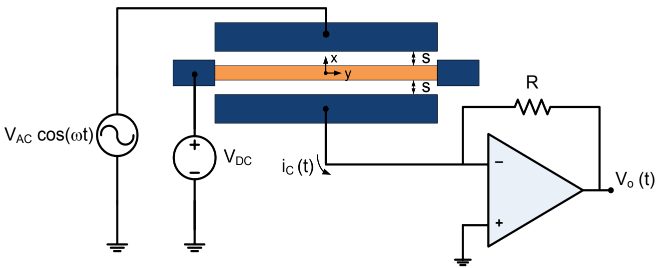

Fig. 1. Electrical scheme of a cc-beam resonator with electrostatic actuation and capacitive readout

Following the scheme depicted in Fig. 1, the dynamics of the MEMS resonator is driven by the excitation force given by

$$F_{exc} = \frac{C_o s}{2}\left(\frac{(V_{ac}\cos(\omega t) - V_{dc})^2}{(s-x)^2} - \frac{(V_{dc})^2}{(s+x)^2}\right), \qquad (2)$$

where, C_0 is the capacitance of the resonator-driver at rest, and s is the initial gap width. The parameters on Eqs. (1) and (2) are given in Table I as functions of the cc-beam resonator dimensions (l: length, w: width and t: thickness), where ω_0 is the elastic natural frequency given by $\omega_0 = \sqrt{k_1/m_{eff}}$, E and ρ are the Young modulus and the mass density of the resonator material respectively, and k_n is the eigenvalue for the fundamental vibration mode of a cc-beam. The bias voltage can be used to tune the effective linear and nonlinear effective stiffness, [3] and [12]. Mainly, the chaotic behaviour

TABLE 1
MAIN PARAMETERS OF IN-PLANE RESONANT CC-BEAMS [7] AND THEIR VALUES FOR POLYSILICON AMS-C35 TECHNOLOGY, WITH A 1MHZ NATURAL FREQUENCY

Parameter	Symbol (units)	Expression	Value
Linear stiffness	k_1 (N/m)	$\frac{16Etw^3}{l^3}$	0.19
Cubic stiffness	k_3 (N/m^3)	$\frac{12{,}272Etw}{l^3}$	$1.19\cdot10^{12}$
Effective mass	m_{eff} (kg)	$\frac{192\rho wlt}{k_n^4}$	$4.82\cdot10^{-15}$
Coupling capacitance at rest	C_0 (F)	$C_0 \frac{\varepsilon tl}{s}$	$1.62\cdot10^{-16}$
Damping factor	γ (N·s/m)	$\frac{m_{eff}\omega_0}{Q}$	$1.83\cdot10^{-10}$

arises in a stationary, robust and showing wide frequency spectrum in the form of cross-well motion [16], namely for a potential distribution of two minima. Cross-well chaos is defined by the classical chaotic Duffing attractor and can be obtained in a double-well Duffing system for a wide set of system parameters. In this sense, in order to achieve robust and stationary chaotic behavior, a double-well potential distribution is desired. From Eqs. (1) and (2), the potential function for a cc-beam, biased by a DC voltage and placed symmetrically between the two driving electrodes, can be expressed as

$$U(x,V_{dc}) = U_{elastic}(x) + U_{electric}(x,V_{dc}) =$$
$$= \frac{1}{2}k_1 x^2 + \frac{1}{4}k_3 x^4 - \frac{sV_{dc}^2 C_0}{2}\left(\frac{1}{(s-x)} + \frac{1}{(s+x)}\right), \qquad (3)$$

which implies a typical Pitchfork bifurcation of the potential function. By solving $\frac{\partial U(x,V_{dc})}{\partial x} = 0$, $\frac{\partial^2 U(x,V_{dc})}{\partial x^2} = 0$ the

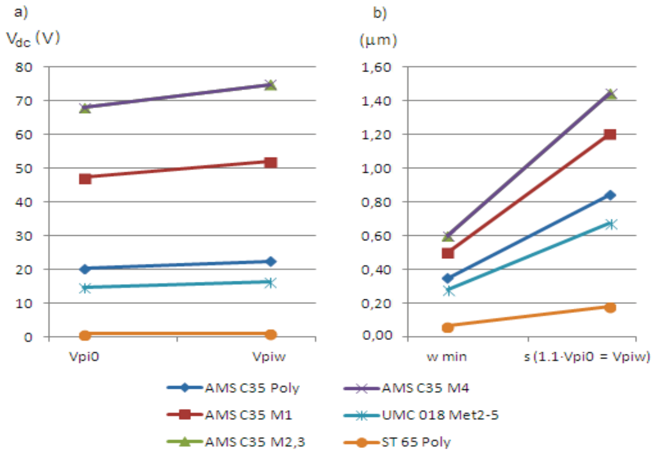

Fig. 2. Comparative representation amongst different commercial CMOS technologies a) for a 1MHz natural frequency, the bias margin between boundary values to achieve two potential well, and b) dimensional ratio (minimum width available versus gap size) to allow 10% of bias voltage margin

singular points are found to be $\left(x_{pi0}, V_{pi0}\right)=\left(0,\sqrt{\frac{k_1 s^2}{2C_0}}\right)$, and $\left(x_{piw}, V_{piw}\right)=\left(\pm\sqrt{\frac{k_3 s^2-2k_1}{3k_3}},\sqrt{\frac{2\left(k_3 s^2+k_1\right)^3}{27C_0 s^2 k_3^2}}\right)$, where $\left(x_{piw}, V_{piw}\right)$ only exists if $s>\sqrt{2\frac{k_1}{k_3}}$, and for the expressions given in Table 1, this inequality implies the design condition $s > 1.615w$. The two-well potential distribution will be achieved for V_{dc} values between V_{pi0} and V_{piw}. To ensure a 10% of bias voltage margin between these bias boundary voltage values, by solving $1.1 \cdot V_{pi0}=V_{piw}$ the value of the gap size *s* is obtained [7]. Fig. 2 shows a graphical comparison of the values of theses parameters for different available commercial CMOS technologies.

The choice of the natural frequency of the resonator determines the DC bias boundary values needed to obtain the two-well potential distribution From the definitions given in Table I, and the expression of the natural angular frequency (namely $\omega_0=\sqrt{k_1/m_{eff}}$), the lower bias boundary value is given as a function of the natural frequency and the width *w* of the beam. V_{pi0} grows linearly with the frequency, and with the square root of the beam width.

$$V_{pi0}\left(\omega_0, w\right)=\omega_0\sqrt{\frac{6\rho w s^3}{\varepsilon k_n^4}} \tag{4}$$

Thus, an increase of the resonator working frequency requires the operation at higher bias voltages. Eq. (4) indicates that such requirement is minimized by designing the resonator with the minimum width available for the MEMS fabrication technology.

Applying last considerations when using the AMS-C35 technology polysilicon layer as the resonator structural layer, and given a 1MHz cc-beam resonator, the parameter values for the Eqs. (1) and (2) are listed in Table 1. In this case the value of the gap parameter is found to be s = 842 nm. Considering now a new time variable $\tau=t\omega_0$, the dimensionless equation of the system can be obtained [12]:

$$z''+\beta z+\alpha z^3+\delta z'=\mu_1\left(\frac{1}{(1-z)^2}-\frac{1}{(1+z)^2}\right)-\mu_2\frac{\cos(\Omega\tau)}{(1-z)^2}+\mu_3\frac{(\cos(\Omega\tau))^2}{(1-z)^2} \tag{5}$$

where *z* is the dimensionless position variable given by *x/s*, the derivative operator (') denotes the derivative with respect to τ, and all the dimensionless parameters are defined as

$$\beta=\frac{k_1}{m_{eff}\omega_0^2};\ \alpha=\frac{k_3 s^2}{m_{eff}\omega_0^2};\ \delta=\frac{\gamma}{m_{eff}\omega};\ \Omega=\frac{\omega}{\omega_0} \tag{6}$$

By substituting the parameter definitions given in Table I into the expressions of Eq. (6), the dimensionless parameters are found to be either constant or dependent only on the ratio *s/w*. The other parameters of the equation are defined as

$$\mu_1=\mu V_{dc}^2;\ \mu_2=\mu V_{dc}V_{ac};\ \mu_3=\mu V_{ac}^2; \tag{7}$$

with $\mu=\frac{C_0}{2m_{eff}\omega_0^2 s^2}$. The potential distribution along the position *z* of the Eq. (5) shows two minima (as it is required in order to achieve the cross-well motion) for a bias voltage range between V_{pi0} and V_{piw} with the boundary values range of the corresponding μ_1 parameter denoted as $\mu_{1\min}$ and $\mu_{1\max}$. By substituting the definitions of the voltage boundary values into the μ_1 definition, it is found that $\mu_{1\min}$ is constant ($\mu_{1\min}$=1/4), and $\mu_{1\max}$ depends only on *s/w* ratio (and only valid for $s > 1.1615w$):

$$\mu_{1\max}=\left(12{,}272\left(\frac{s}{w}\right)^2+16\right)^3\cdot\left(27\cdot(12.272)^2\cdot 16\left(\frac{s}{w}\right)^4\right)^{-1} \tag{8}$$

In this way, Eq. (5) allows to characterize dynamically cc-beam resonators, designed with different dimensions and technologies, for a given ratio between *s* and *w.* Since the natural frequency has been used in the process to obtain Eq. (5), the natural frequency has no influence in the dimensionless equation. Moreover, since all the parameters and the position variable are dimensionless, such equation is simpler for numerical simulation than the dimensional Eq. (1). In this work we consider a 1-MHz polysilicon resonator with a gap value that allows a 10% of margin between V_{pi0} and V_{piw}, and a quality factor *Q=165.* The values of the dimensionless parameters result in this case α = 4.44; β =1 (fixed value) ; δ =0.6061; $\mu_{1\min}$ = 0.25 (fixed value); and $\mu_{1\max}$ = 0.3025.

III. Fuzzy Inference-Based Chaotic Improvement

Although the biasing conditions to enable a two-well potential distribution can be directly found in the way indicated in previous section, the conditions required for chaotic dynamic behavior are not straightforward, since it is highly sensitive to parameter variations. In this sense, the need for a robust chaotic behavior, like needed in secure communication systems, requires a large amount of numerical simulations that results in a highly time consuming process.

The new approach presented in this work consists in the application of a fuzzy controller (Fig. 3) for the MEMS system that performs a search for the conditions of maximum richness in the chaotic behavior of the plant system (MEMS) within the design dimensions and biasing conditions that assure a two-well potential distribution. This search is executed by means of iterations that involve the change of an operating parameter and the measure of dynamic features in the output signal on a time window. The spectrum bandwidth parameter of the MEMS dynamics time-series is taken as a metric of the nonlinear and chaotic behavior of the system. In order to enable its application in an experimental setting, the bandwidth is measured from the number of points, *n*, in the periodogram with a power higher than a boundary value (for instance 10% of the maximum power), which can be directly obtained from a spectrum analyzer.

The control surface, obtained by fuzzy reasoning, relates the input bandwidth parameter n_k, obtained from a spectrum analyzer (S.A.), of the nonlinear signal generated by the MEMS system and the value of the μ_1 parameter (namely the dimensionless value of the actual value of the DC voltage in the system) to the output control value a_k, ranging from -1 to 1. The subscript k denotes the number of the iteration. The control output value a_k is adjusted to the dimensional value by an amplification block (K) to be applied directly to the MEMS system as a variation of the DC voltage value. This amplification block has the parameters corresponding to the MEMS system technology, the iteration number k and the initial DC value applied. The control surface is designed to be stable and useful to maximize the chaotic response, knowing the high variance of the dynamical behavior with small changes in the parameters (that can be caused by fabrication tolerances). Since the control surface is obtained from the normalized parameters proper to the dimensionless equation of the MEMS system, Eq. (5), the same control surface is valid for any cc-beams resonator dimensions, and for different fabrication technologies, while having the same *s/w* ratio. In this sense, to apply the control system to cc-beam devices built with different dimensions, or in different technologies, only the amplification block K has to be modified.

To build the fuzzy-based control surface, several steps in fuzzy reasoning are performed [17]. First of all, state and control variables are identified and the membership functions of the several linguistic labels for each variable are constructed. In the present case, the bandwidth quantification (n) and the μ_l parameter are identified as state variables and the normalized output (u), as control variable. After that, an implication matrix between the linguistic labels is defined. The singleton function of the actual state variable values as fuzzification and the Mamdami function as fuzzy implication function are used. For each pair of inputs, the union (maximum function) of the intersections between the output membership function and the minimum of the input singletons is obtained. The defuzzified value is obtained as the centroid of this area. For each pair of inputs, a single defuzzified value is obtained and the whole set of defuzzified values can be represented as the mesh of the control surface.

It is preferable to control the normalized bias input parameter rather than the normalized excitation amplitude, because the margin between the bias boundary values (given by V_{pi0} and V_{piw}) is wider than the margin of the AC amplitude (bounded by the dynamic pull-in effect). DC values upper than the pull-in limit are avoided, so the fuzzy implication matrix between linguistic variables ensures a negative increment of the control variable for the linguistic variable corresponding to values of μ_1 close to the upper limit.

Furthermore, to avoid the creation of infinite loops, the control surface have a non symmetric transition from positive to negative output sign.

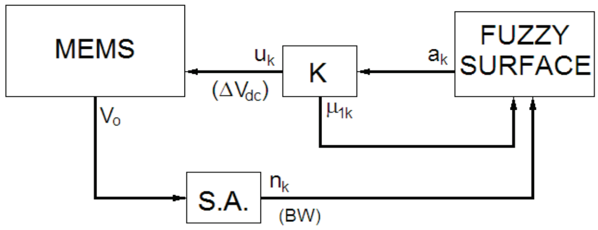

Fig. 3. Scheme of the proposed fuzzy control system

A. Analytical Study of the Control Convergence

The MEMS system Eq. (1) can be expressed as the function $\dot{\bar{x}} = f(\bar{x}, t)$, and moreover, considering the definition of the driving force Eq. (2), with V_{ac} constant and V_{dc} parameter to be the control variable, it can be expressed as the function $\dot{\bar{x}} = f(\bar{x}, t, V_{dc})$. The number of points in the periodogram with a significant power, n_k , is $n_k = g(\bar{x}, V_{dc} + u_{k-1})$, with u_k the output of the whole control system (both n_k and u_k, values of the k-th iteration), and g an, a-priori, unknown function. N_d is considered the minimum acceptable measure value for the bandwidth, i.e. the lower bound of the iteration procedure goal. If the error function of each iteration is defined as $e_k = H[N_d - n_k - 1]$, where $H[x]$ is the Heavyside function, and a Lyapunov candidate function that verifies the conditions of the Lyapunov stability for discrete-time systems [18] is found, we will be able to assure the convergence to zero of the error function. A discrete-time Lyapunov candidate function has been found to be

$$V_k = e_k u_k^{\,2} \,. \tag{9}$$

Ensuring that u_k can only be zero if e_k is zero, the value of the function V_k is zero if and only if e_k is zero; otherwise V_k is always positive. The variation between iterations of this Lyapunov candidate function is

$$V_k - V_{k-1} = e_k u_k^{\,2} - e_{k-1} u_{k-1}^{\,2} \,. \tag{10}$$

Following with the statement that if e_k is zero then u_k is zero, then e_{k-1}=0 implies e_k = u_k = 0, and in this case $V_k - V_{k-1} = 0$. For e_{k-1}> 0, e_k can be any of these options

- e_k = 0 and thus $V_k - V_{k-1} = -u_{k-1}^{\,2} < 0$
- e_k > 0 and thus $V_k - V_{k-1} = u_k^{\,2} - u_{k-1}^{\,2}$. In consequence the only condition needed to be ensured is $|u_k| < |u_{k-1}|$.

Finally, the sets G_o and B_θ are defined to be respectively $G_0 = \{e \in \Re : V(e) = 0\}$, and $B_\theta = \{x \in \Re : |x| < \theta\}$, with θ an arbitrary real positive number, and h^+ is defined to be the set of functions $h^+ = \{h^p, p \in N\}$ with h, the iteration function ($h(e_k) = e_{k+1}$). The last condition to be fulfilled is that e = 0 must be G_0-assimptotically stable, i.e., $\forall \varepsilon > 0 \exists \delta > 0 : h^+(B_\delta \cap G_0) \subset B_\varepsilon$, and there have to exist a $\delta > 0$ such that $\lim_{p \to \infty} h^p(e) = 0 \forall e \in (G_0 \cap B_\delta)$. It is straightforward to see that, since $G_0 = \{0\}$, these latter two conditions are fully satisfied and, in consequence the zero error is stable in terms of Lyapunov criteria. In summary, in order to verify the analytical conditions that assure the error function convergence to zero, the control system output must be zero only once the procedure goal is reached, and its absolute value must decrease in each iteration.

B. Results and Simulations

After defining the fuzzy sets of the linguistic variables, and the fuzzy implication matrix, the control surface is obtained. The membership functions, and in consequence the control surface may be optimized to allow a faster convergence. The condition of zero output control value for all the bandwidth inputs higher than the boundary value N_d is imposed by the control surfaces. The condition $|u_k| < |u_{k-1}|$ is imposed by means of a cycle divider, in the block K, that performs an amplification given by a q^{-k} factor with k the iteration number and q an integer value higher than the ratio max/min nonzero output value of the control surface. From this condition a compromise solution is needed: the control surface must be smooth enough to be able to be obtained by the fuzzy inference procedure, but on the other hand a small ratio of the max/min nonzero output of the control surface is required because otherwise the output value of the control system would decrease quickly and an increasing number of iterations would be needed to reach the goal of the control system.

Numerical simulations with a 1MHz natural frequency polysilicon cc-beam resonator (designed in AMS-C35 technology with the parameters listed in Section II) were performed, using the control system designed to verify the convergence conditions given in previous section, with the aim to corroborate its capability in leading to a desired chaotic response. The control surface used is shown in Fig. 4. In each

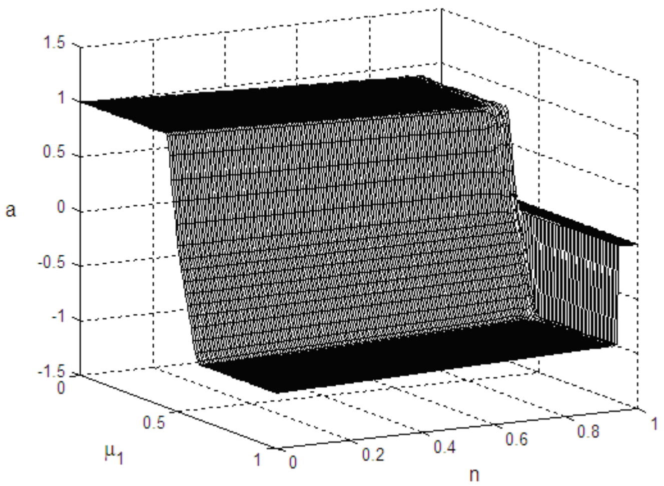

Fig. 4. Control surface designed to verify the convergence conditions given in the Section IIIA. The response is only zero for the normalized bandwidth value equal or greater than the desired boundary, and the ratio between the maximum and minimum nonzero response is 1.11. The z axis is the normalized response of the control systems, while x-axis is the normalized actual μ_1 value and y-axis , the normalized actual bandwidth measure

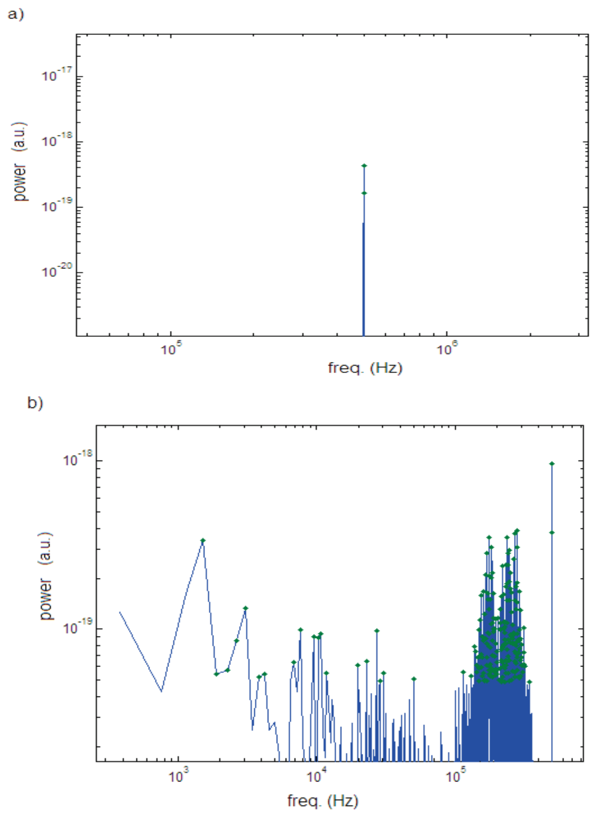

Fig. 5. a) Bandwidth response in the first iteration b) Bandwidth response after 3 iterations applying the control surface given in Fig. 4 and the division of the output amplitude by the factor 1.112 in each iteration

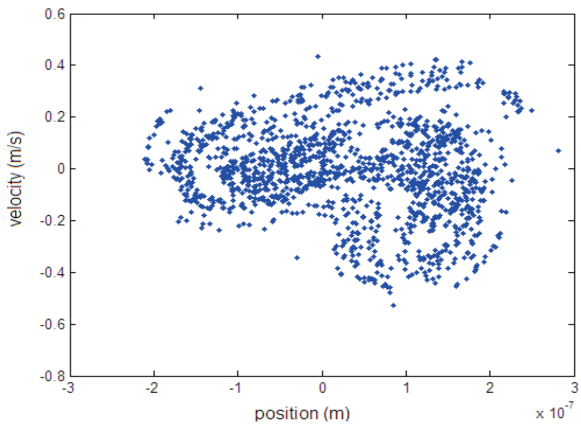

Fig. 6. Poincare map of the dynamical time-series obtained by the MEMS system in last iteration

iteration, the control system output was applied to the input parameters of the next simulation. The spectral representation of the MEMS system dynamic response time series shows a significant increase of the bandwidth after few iterations, as it is shown in Fig. 5. A further analysis of the position time-series of the MEMS system confirms that, while the response after the first iteration corresponds to a linear behavior, the dynamics of the system after the third iteration of the control system corresponds to a robust cross-well chaotic behavior, as can be deduced from the typical chaotic shape of the Poincare map, depicted in Fig. 6. Furthermore, a positive value of the maximal Lyapunov exponent is obtained for such time-series response. An analysis of the numerical simulations indicates that the stability conditions obtained in section III A are sufficient but not necessary to achieve the system convergence to the desired rich and complex chaotic stationary response. However, the implementation of a control system that verifies such conditions guarantees mathematically the convergence.

IV. Conclusions

In order to obtain a robust, stationary and extensive chaotic response, with potential application in cryptographic schemes, a control method has been developed to be applied to nonlinear submicrometric electromechanical resonator. This approach proposes, for the first time, a fuzzy controlled increase of the chaotic response. The stability requirements and the conditions for the convergence to the desired chaotic dynamics have been analytically determined, and applied to simulate the response of a cc-beam resonator designed in a CMOS technology with the aim to prove the proposal. Beside the intrinsic advantages of fuzzy controllers, the control system method proposed in this work performs an automatic search for the optimal operating parameters, which provide the desired chaotic response. Thus, the exact parameters of the resonator system are not required, and in consequence the control system can be applied in practical applications, in which, due to the hypersensitivity of chaotic behaviour, small parameter variations due to fabrication tolerances may invalidate the results obtained from numerical simulations.

Acknowledgement

The research presented in this work has been partially supported by the Spanish Ministry of Economy and Competitiveness (under the project TEC2014-52878-R), and has been performed within the Erasmus Mundus Action 2 Euroweb Scholarship program.

References

[1] V. Kaajakari, T. Mattila, et al. “Nonlinear Limits for Single-Crystal Silicon Microresonator”, *Journal of Microelectromechanical Systems,* vol. 13, no. 5, pp. 715-724, 2004.

[2] Y. C. Wang, S. G. Adams, J. S. Thorp, N. C. MacDonald, P. Hartwell, and F. Bertsch, “Chaos in MEMS, Parameter Estimation and its Potential Application”, *IEEE Trans. Circuits Syst. I Fundam. Theory Appl.*, vol. 45, no. 10, pp. 1013–1020, 1998.

[3] B. E. DeMartini, H. E. Butterfield, J. Moehlis and K. L. Turner, “Chaos for a Microelectromechanical Oscillator Governed by the Nonlinear Mathieu Equation”, *J Microelectromech Syst*, vol. 16, pp. 1314-1323, 2007.

[4] L. M. Pecora, and T. L. Carroll, “Synchronization in Chaotic Systems”, *Physical Review Letters*, vol. 64, no. 8, pp. 821-824, 1990.

[5] K. M. Cuomo et al., “Synchronization of Lorentz-based Chaotic Circuits with Applications to Communications”, *IEEE Transactions on Circuits and Systems II*, vol. 40, no. 10, pp. 626-633, 1993.

[6] B. Jovic, “Synchronization Techniques for Chaotic Communication Systems”, *Signals and Communication Technology,* Springer, Verlag, Berlin Heidelberg, 2011.

[7] J. Barceló, J. L. Rosselló, S. Bota, J. Segura, J. Verd, “Electrostatically Actuated Microbeam Resonators as Chaotic Signal Generators: a Practical Perspective”, *Commun Nonlinear Sci Number Simulat*, Article in press, 2015.

[8] C. Lo. F. Chen, and G. K. Fedder, “Integrated HF CMOS-MEMS Square-Frame Resonators with On-Chip Electronics and Electrothermal Narrow Gap Mechanism“, *Tech. Digest of IEEE Transducers*, vol. 2, pp. 2074- 2077, 2005.

[9] J. Verd, A. Uranga, G. Abadal, J. L. Teva, F. Torres, J. L. Lopez, E. Perez-Murano, J. Esteve, and N. Barniol, “Monolithic CMOS MEMS Oscillator Circuit for Sensing in the Attogram Range“, *IEEE Electron Device Letters*, vol. 29, pp. 146-148, 2008.

[10] O. Calvo, J. Cartwritght, “Fuzzy Control of Chaos”, *International Journal of Bifurcation and Chaos*, vol. 8, pp. 1743-1747, 1998.

[11] L. Chen, G. Chen, and Y. Lee, “Fuzzy Modeling and Adaptive Control of Uncertain Chaotic Systems”, *Information. Science*, vol. 121, pp. 27–37, 1999.

[12] H. Haghighi, and A. Markazi, “Chaos Prediction and Control in MEMS Resonators”, *Commun Nonlinear Sci Numer Simulat* vol. 10, no. 15, pp. 3091-3099, 2010.

[13] A. Poursamad, and A. Markazi, “Robust Adaptive Fuzzy Control of Unknown Chaotic Systems”, *Applied Soft Computing*, pp. 970-976, 2009.

[14] S. Zhankui, and K. Sun, “ Nonlinear and Chaos Control of a Micro-electro-mechanical System by Using Second Order Fast Terminal Sliding Mode Control”, *Commun Nonlinear Sci Number Simulat*, vol. 18, pp. 2540-2548, 2013.

[15] R. Mestrom, R. Fey. et al. “Modelling the Dynamics of a MEMS Resonator: Simulation and Experiments”, *Sensors and Actuators A.*, vol. 42, pp. 306-315, 2008.

[16] S. Lenci, and G. Rega, “Forced Harmonic Vibration in a Duffing Oscillator with Negative Linear Stiffness and Linear Viscous Damping”, I. Kovacic, M. Brenan, *The Duffing Equation*, John Wiley & Sons, 2011.

[17] J.-S. R. Jang, and C.-T. Sun, “Neuro-Fuzzy Modeling and Control”, *Proceedings of the IEEE,* vol. 83, no. 3, pp. 378-406, 1995.

[18] A. Iggidr, and M. Bensoubaya, “Stability of Discrete-time Systems: New Criteria and Applications to Control Problems”, *Institut National de Recherche en Informatique et en Automatique*, 1996.