Fuzzy Control for Chaotic Response Improvement in MEMS Resonators

Joan Barceló¹, Zlatica Marinković², Vera Marković², Jaume Verd¹

Abstract – Starting with the design and biasing conditions to enable cross-well chaotic behaviour in a nonlinear MEMS resonator, this paper presents for the first time a control method based on fuzzy logic to maximize such a chaotic response, providing a design way for its potential use in secure communication applications. Theoretical analysis and simulations performed on a cc-beam MEMS resonator model designed in a CMOS technology verify the effectiveness of the proposed control method.

Keywords – Fuzzy logic control, MEMS resonators, nonlinear and chaotic dynamics.

I. INTRODUCTION

Nonlinearity is an inherent feature of the MEMS oscillators, and it is displayed both in mechanical and electrical domains [1]. Through such nonlinearity, under certain design and operating conditions, a chaotic behaviour may arise as experimentally observed in [2]-[3]. In these works, a robust chaotic motion is achieved in electrostatically actuated non-overlapping and capacitive readout comb MEMS devices, operating at a natural frequency in the range of KHz. The deterministic and apparently noise-like behavior of the chaotic regime, together with its capability of auto-synchronization (demonstrated by Pecora and Carrol for the Lorenz system in [4] and experimentally tested by Cuomo and Oppenheim in [5]) make it a potential candidate in cryptographic applications through either masking or modulation chaotic encryptions [6].

The use of hardware based real time encryption schemes meets the need of high speed in data communication systems over traditional software cryptographic protocols. To fulfill the requirements for this purpose, an improvement of the dynamic features (in terms of frequency and chaotic response) regarding to the models reported in literature, is demanded, and at the same time keeping the benefits that they provide. Designs with complex geometries, like non-overlapping comb-drives reported in [2] - [3] present more constraints in terms of fabrication and relative low natural frequencies. Contrary, the use of simple resonant structures as a clamped-clamped beam (cc-beam) resonator, provides significant benefits in terms of integration feasibility and scalability to nanometer dimensions (MEMS resonators) enabling both high sensitivity and high frequency operation [7]. In addition the use of electrostatic actuation and capacitive readout in MEMS resonators enables an easy and low cost integration of such electromechanical devices with electronics into a single CMOS die (monolithic integration) [8] - [9]. The development of chaotic MEMS oscillators with better performance (for instance, in terms of resonance frequency and miniaturization capabilities) comparing to the devices reported in literature may imply a boost for a new generation of MEMS devices for competing with optical devices in secure communication applications.

Fuzzy logic has been revealed to be a powerful method to control different kind of plants. Particularly useful is its application in the control of nonlinear, subject to noise, or partially unknown systems. The applications of fuzzy-logic controllers to chaotic systems reported in literature are focused on control and suppression of the chaotic response, which is considered to be undesirable. Thus, in [10] a fuzzy controller obtained from a knowledge base formed by fuzzy sets and inference matrix is used to perturb a variable of the Chua circuit and stabilize the chaotic output. Meanwhile, in [11] Gaussian membership based fuzzy models are used to estimate the unknown parameters and functions needed to build a controller that is proved, by means of the Lyapunov stability theory, to be useful to control a Duffing system. Concerning to the dynamics of MEMS, fuzzy systems have been designed in [12]-[14] to mimic ideal controllers based on the Sliding Mode Control, i.e. feedback controllers which cause the state trajectory to reach a sliding surface and behaving as a stable trajectory. In such references, the fuzzy control is used to convert chaotic oscillations into desired regular ones, exhibiting a periodic behavior.

Even when the conditions to achieve robust and extensive chaotic response in nonlinear MEMS oscillators are verified, the attainment of the proper parameters for chaotic signal generation is not immediate. Instead of a large amount of time consuming and approximate numerical simulations, in this paper a new control method is proposed to achieve the desired chaotic response. For the first time a fuzzy-logic based control is used to maximize the chaotic response of a nonlinear MEMS system.

The paper is organized as follows. In Section II, the cc-beam MEMS considered in this work as the chaotic signal generator is mathematically modelled, and the limits for its chaotic operation are stated. In Section III, the fuzzy control is proposed and discussed, the convergence to the desired goal is mathematical studied and simulation results are presented. Finally the main concluding remarks are given in Section IV.
II. MATHEMATICAL MODEL OF CC-BEAM RESONATOR

The nonlinear resonant cc-beam CMOS-MEMS under consideration (depicted in Fig. 1) is constituted by a microbeam placed symmetrically and in-plane between two electrodes, in which a sinusoidal excitation with amplitude $V_{\text{ac}}$ and frequency $\omega/2\pi$ (excitation electrode) and capacitive detection (readout electrode) are respectively performed. A biasing voltage $V_{\text{dc}}$ is applied to the bridge to generate a motional current due to the resonator oscillation that is detected by the readout electrode [9]. The motional current can be measured and converted to a voltage signal by a transimpedance amplifier. The use of this topology minimizes the parasitic feedthrough current in comparison with other topologies. The system can be conceived as a lumped mass-spring model, assuming that the mass is located at the middle point of the cc-beam [15]. Eq. (1) describes the mechanical resonator dynamics in one dimension (x-axis)

$$m_{\text{eff}} \ddot{x} + k_1 x + k_3 x^3 + \gamma \dot{x} = F_{\text{exc}},$$

(1)

where $\dot{x}$ and $\ddot{x}$ represent, respectively the first and second time derivative of the position variable $x$, $m_{\text{eff}}$ is the resonator effective mass, $\gamma$ is the damping coefficient, $k_1$ and $k_3$ are the linear and cubic mechanical stiffness respectively, and $F_{\text{exc}}$ is the net excitation force.

Following the scheme depicted in Fig. 1, the dynamics of the MEMS resonator is driven by the excitation force given by

$$F_{\text{exc}} = C_c x \left( \frac{V_{\text{ac}} \cos(\omega t) - V_{\text{dc}}}{s - x} \right)^2 - \frac{V_{\text{dc}}^2}{(s + x)^2},$$

(2)

where, $C_c$ is the capacitance of the resonator-driver at rest, and $s$ is the initial gap width. The parameters on Eqs. (1) and (2) are given in Table I as functions of the cc-beam resonator dimensions ($l$: length, $w$: width and $t$: thickness), where $\omega_0$ is the elastic natural frequency given by $\omega_0 = \sqrt{k_1/m_{\text{eff}}}$, $E$ and $\rho$ are the Young modulus and the mass density of the resonator material respectively, and $k_n$ is the eigenvalue for the fundamental vibration mode of a cc-beam. The bias voltage can be used to tune the effective linear and nonlinear effective stiffness, [3] and [12]. Mainly, the chaotic behaviour arises in a stationary, robust and showing wide frequency spectrum in the form of cross-well motion [16], namely for a potential distribution of two minima. Cross-well chaos is defined by the classical chaotic Duffing attractor and can be obtained in a double-well Duffing system for a wide set of system parameters. In this sense, in order to achieve robust and stationary chaotic behavior, a double-well potential distribution is desired. From Eqs. (1) and (2), the potential function for a cc-beam, biased by a DC voltage and placed symmetrically between the two driving electrodes, can be expressed as

$$U(x, V_{\text{dc}}) = U_{\text{elastic}}(x) + U_{\text{electric}}(x, V_{\text{dc}}) = \frac{1}{2} k_1 x^2 + \frac{1}{4} k_3 x^4 - \frac{1}{2} \frac{C_c V_{\text{dc}}^2}{s} \left( \frac{1}{(s - x)^2} + \frac{1}{(s + x)^2} \right),$$

(3)

which implies a typical Pitchfork bifurcation of the potential function. By solving $\partial U(x, V_{\text{dc}})/\partial x = 0$, $\partial^2 U(x, V_{\text{dc}})/\partial x^2 = 0$ the

![Fig. 1. Electrical scheme of a cc-beam resonator with electrostatic actuation and capacitive readout](image)

![Fig. 2. Comparative representation amongst different commercial CMOS technologies a) for a 1MHz natural frequency, the bias margin between boundary values to achieve two potential well, and b) dimensional ratio (minimum width available versus gap size) to allow 10% of bias voltage margin](image)

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>MAIN PARAMETERS OF IN-PLANE RESONANT CC-BEAMS [7] AND THEIR VALUES FOR POLYSILICON AMS-C35 TECHNOLOGY, WITH A 1MHZ NATURAL FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Symbol (units)</td>
</tr>
<tr>
<td>Linear stiffness</td>
<td>$k_1$ (N/m)</td>
</tr>
<tr>
<td>Cubic stiffness</td>
<td>$k_3$ (N/m^3)</td>
</tr>
<tr>
<td>Effective mass</td>
<td>$m_{\text{eff}}$ (kg)</td>
</tr>
<tr>
<td>Coupling capacitance at rest</td>
<td>$C_o$ (F)</td>
</tr>
<tr>
<td>Damping factor</td>
<td>$\gamma$ (N·s/m)</td>
</tr>
</tbody>
</table>
singular points are found to be \( (x_{p0}, V_{p0}) = \left(0, \sqrt{\frac{k_1 s^2}{2C_0}} \right) \), and 

\[
(x_{p\nu}, V_{p\nu}) = \left( \pm \sqrt{\frac{k_3 s^2 - 2k_1}{3k_3}}, \sqrt{\frac{2(k_3 s^2 + k_1)}{27C_0 s^2 2k_3}} \right),
\]

where \( (x_{p\nu}, V_{p\nu}) \) only exists if \( s > \frac{2k_1}{k_3} \), and for the expressions given in Table 1, this inequality implies the design condition \( s > 1.615w \). The two-well potential distribution will be achieved for \( V_{dc} \) values between \( V_{p0} \) and \( V_{p\nu} \). To ensure a 10% of bias voltage margin between these bias boundary voltage values, by solving \( 1.1V_{p\nu} = V_{p\nu} \) the value of the gap size \( s \) is obtained [7]. Fig. 2 shows a graphical comparison of the values of theses parameters for different available commercial CMOS technologies.

The choice of the natural frequency of the resonator determines the DC bias value boundaries needed to obtain the two-well potential distribution. From the definitions given in Table 1, and the expression of the natural angular frequency (namely \( \omega_0 = \sqrt{\frac{k_1}{m_{eff}}} \)), the lower bias boundary value is given as a function of the natural frequency and the width \( w \) of the beam. \( V_{p0} \) grows linearly with the frequency, and with the square root of the beam width. 

\[
V_{p0}(\omega_0, w) = \omega_0 \sqrt{\frac{6\rho w s^5}{c k_n^4}} \quad (4)
\]

Thus, an increase of the resonator working frequency requires the operation at higher bias voltages. Eq. (4) indicates that such requirement is minimized by designing the resonator with the minimum width available for the MEMS fabrication technology.

Applying last considerations when using the AMS-C35 technology polysilicon layer as the resonator structural layer, and given a 1MHz cc-beam resonator, the parameter values for the Eqs. (1) and (2) are listed in Table 1. In this case the value of the gap parameter is found to be \( s = 842 \) nm. Considering now a new time variable \( \tau = \omega_0 t \), the dimensionless equation of the system can be obtained [12]:

\[
z' + \beta z + \alpha z^3 + \delta e' = \mu_1 \left( \frac{1}{(1-z)^2} - \frac{1}{(1+z)^2} \right) - \mu_2 \frac{\cos(\Omega r)}{(1-z)^2} + \mu_3 \frac{\cos(\Omega r)}{(1-z)^2} \quad (5)
\]

where \( z \) is the dimensionless position variable given by \( x/s \), the derivative operator \( \cdot \) denotes the derivative with respect to \( \tau \), and all the dimensionless parameters are defined as

\[
\beta = \frac{k_1}{m_{eff} \omega_0^2}; \quad \alpha = \frac{k_3 s^2}{m_{eff} \omega_0^2}; \quad \delta = \gamma; \quad \Omega = \frac{\omega}{\omega_0} \quad (6)
\]

By substituting the parameter definitions given in Table 1 into the expressions of Eq. (6), the dimensionless parameters are found to be either constant or dependent only on the ratio \( s/w \). The other parameters of the equation are defined as

\[
\mu_1 = \frac{\mu V_{dc}}{V_{ac}}; \quad \mu_2 = \frac{\mu V_{dc} V_{ac}}{V_{ac}}; \quad \mu_3 = \frac{\mu V_{ac}^2}{V_{ac}} \quad (7)
\]

with \( \mu = \frac{C_0}{2m_{eff} \alpha_0^2 s^2} \). The potential distribution along the position \( z \) of the Eq. (5) shows two minima (as it is required in order to achieve the cross-well motion) for a bias voltage range between \( V_{p0} \) and \( V_{p\nu} \) with the boundary values range of the corresponding \( \mu_1 \) parameter denoted as \( \mu_{min} \) and \( \mu_{max} \). By substituting the definitions of the voltage boundary values into the \( \mu_1 \) definition, it is found that \( \mu_{min} \) is constant \( (\mu_{max}=1/4) \), and \( \mu_{max} \) depends only on \( s/w \) ratio (and only valid for \( s > 1.1615w \)):

\[
\mu_{max} = \left(12.272 \left( \frac{s}{w} \right)^2 + 16 \right)^{1/2} \left(27(12.272)^2 + 16 \left( \frac{s}{w} \right)^4 \right)^{-11/2} \quad (8)
\]

In this way, Eq. (5) allows to characterize dynamically cc-beam resonators, designed with different dimensions and technologies, for a given ratio between \( s \) and \( w \). Since the natural frequency has been used in the process to obtain Eq. (5), the natural frequency has no influence in the dimensionless equation. Moreover, since all the parameters and the position variable are dimensionless, such equation is simpler for numerical simulation than the dimensional Eq. (1). In this work we consider a 1-MHz polysilicon resonator with a gap value that allows a 10% of margin between \( V_{p0} \) and \( V_{p\nu} \), and a quality factor \( Q=165 \). The values of the dimensionless parameters result in this case \( \alpha = 4.44; \beta = 1 \) (fixed value); \( \delta = 0.6061; \mu_{min} = 0.25 \) (fixed value); and \( \mu_{max} = 0.3025 \).

### III. FUZZY INFERENCE-BASED CHAOTIC IMPROVEMENT

Although the biasing conditions to enable a two-well potential distribution can be directly found in the way indicated in previous section, the conditions required for chaotic dynamic behavior are not straightforward, since it is highly sensitive to parameter variations. In this sense, the need for a robust chaotic behavior, like needed in secure communication systems, requires a large amount of numerical simulations that results in a highly time consuming process.

The new approach presented in this work consists in the application of a fuzzy controller (Fig. 3) for the MEMS system that performs a search for the conditions of maximum richness in the chaotic behavior of the plant system (MEMS) within the design dimensions and biasing conditions that assure a two-well potential distribution. This search is executed by means of iterations that involve the change of an operating parameter and the measure of dynamic features in the output signal on a time window. The spectrum bandwidth parameter of the MEMS dynamics time-series is taken as a metric of the nonlinear and chaotic behavior of the system. In order to enable its application in an experimental setting, the bandwidth is measured from the number of points, \( n \), in the periodogram with a power higher than a boundary value (for...
instance 10% of the maximum power), which can be directly obtained from a spectrum analyzer.

The control surface, obtained by fuzzy reasoning, relates the input bandwidth parameter \( n_k \), obtained from a spectrum analyzer \((S.A.,)\), of the nonlinear signal generated by the MEMS system and the value of the \( \mu_i \) parameter (namely the dimensionless value of the actual value of the DC voltage in the system) to the output control value \( a_k \), ranging from -1 to 1. The subscript \( k \) denotes the number of the iteration. The control output value \( a_k \) is adjusted to the dimensionless value by an amplification block \((K)\) to be applied directly to the MEMS system as a variation of the DC voltage value. This amplification block has the parameters corresponding to the MEMS system technology, the iteration number \( k \) and the initial DC value applied. The control surface is designed to be stable and useful to maximize the chaotic response, knowing the high variance of the dynamical behavior with small changes in the parameters (that can be caused by fabrication tolerances). Since the control surface is obtained from the normalized parameters proper to the dimensionless equation of the MEMS system, Eq. \((5)\), the same control surface is valid for any cc-beams resonator dimensions, and for different fabrication technologies, while having the same \( s/w \) ratio. In this sense, to apply the control system to cc-beam devices built with different dimensions, or in different technologies, only the amplification block \( K \) has to be modified.

To build the fuzzy-based control surface, several steps in fuzzy reasoning are performed [17]. First of all, state and control variables are identified and the membership functions of the several linguistic labels for each variable are constructed. In the present case, the bandwidth quantification \((n)\) and the \( \mu_i \) parameter are identified as state variables and the normalized output \((u)\), as control variable. After that, an implication matrix between the linguistic labels is defined. The singleton function of the actual state variable values as fuzzification and the Mamdami function as fuzzy implication function are used. For each pair of inputs, the union (maximum function) of the intersections between the output membership function and the minimum of the input singlet ons is obtained. The defuzzied value is obtained as the centroid of this area. For each pair of inputs, a single defuzzified value is obtained and the whole set of defuzzified values can be represented as the mesh of the control surface.

It is preferable to control the normalized bias input parameter rather than the normalized excitation amplitude, because the margin between the bias boundary values (given by \( V_{p0} \) and \( V_{p1,0} \)) is wider than the margin of the AC amplitude (bounded by the dynamic pull-in effect). DC values upper than the pull-in limit are avoided, so the fuzzy implication matrix between linguistic variables ensures a negative increment of the control variable for the linguistic variable corresponding to values of \( \mu_i \) close to the upper limit.

Furthermore, to avoid the creation of infinite loops, the control surface have a non symmetric transition from positive to negative output sign.

\[ V_k = e_k u_k^2. \] (9)

Ensuring that \( u_k \) can only be zero if \( e_k \) is zero, the value of the function \( V_k \) is zero if and only if \( e_k \) is zero; otherwise \( V_k \) is always positive. The variation between iterations of this Lyapunov candidate function is

\[ V_k - V_{k-1} = e_k u_k^2 - e_{k-1} u_{k-1}^2. \] (10)

Following with the statement that if \( e_k \) is zero then \( u_k \) is zero, then \( e_{k+1} = 0 \) implies \( e_k = u_k = 0 \), and in this case \( V_k - V_{k-1} = 0 \). For \( e_k \geq 0 \), \( e_k \) can be any of these options

- \( e_k = 0 \) and thus \( V_k - V_{k-1} = -u_{k-1}^2 < 0 \)
- \( e_k > 0 \) and thus \( V_k - V_{k-1} = u_k^2 - u_{k-1}^2 \). In consequence the only condition needed to be ensured is \( |u_k| < |u_{k-1}| \).

Finally, the sets \( G_0 \) and \( B_\theta \) are defined to be respectively \( G_0 = \{e \in \mathbb{R} : V(e) = 0\} \), and \( B_\theta = \{x \in \mathbb{R} : |x| < \theta \} \), with \( \theta \) an arbitrary real positive number, and \( h^* \) is defined to be the set of functions \( h^* = \{h^p : p \in N\} \) with \( h \), the iteration function (\( h(e_k) = e_{k+1} \)). The last condition to be fulfilled is that \( e = 0 \) must be \( G_0 \)-assymptotically stable, i.e.,

\[ \forall e > 0 \exists \delta > 0 : h^*(B_\delta \cap G_0) \subset B_{\varepsilon}, \] and there have to exist a \( \delta > 0 \) such that \( \lim_{p \to \infty} h^p(e) = 0 \forall e \in (G_0 \cap B_\delta) \). It is straightforward to see that, since \( G_0 = \{0\} \), these latter two
conditions are fully satisfied and, in consequence the zero error is stable in terms of Lyapunov criteria. In summary, in order to verify the analytical conditions that assure the error function convergence to zero, the control system output must be zero only once the procedure goal is reached, and its absolute value must decrease in each iteration.

B. Results and Simulations

After defining the fuzzy sets of the linguistic variables, and the fuzzy implication matrix, the control surface is obtained. The membership functions, and in consequence the control surface may be optimized to allow a faster convergence. The condition of zero output control value for all the bandwidth inputs higher than the boundary value \( N_d \) is imposed by the control surfaces. The condition \( |\mu_k| < |\mu_{k-1}| \) is imposed by means of a cycle divider, in the block K, that performs an amplification given by a \( q^k \) factor with \( k \) the iteration number and \( q \) an integer value higher than the ratio max/min nonzero output value of the control surface. From this condition a compromise solution is needed: the control surface must be smooth enough to be able to be obtained by the fuzzy inference procedure, but on the other hand a small ratio of the max/min nonzero output of the control surface is required because otherwise the output value of the control system would decrease quickly and an increasing number of iterations would be needed to reach the goal of the control system.

Numerical simulations with a 1MHz natural frequency polysilicon cc-beam resonator (designed in AMS-C35 technology with the parameters listed in Section II) were performed, using the control system designed to verify the convergence conditions given in previous section, with the aim to corroborate its capability in leading to a desired chaotic response. The control surface used is shown in Fig. 4. In each iteration, the control system output was applied to the input parameters of the next simulation. The spectral representation of the MEMS system dynamic response time series shows a significant increase of the bandwidth after few iterations, as it is shown in Fig. 5. A further analysis of the position time-series of the MEMS system confirms that, while the response after the first iteration corresponds to a linear behavior, the dynamics of the system after the third iteration of the control system corresponds to a robust cross-well chaotic behavior, as can be deduced from the typical chaotic shape of the Poincare map, depicted in Fig. 6. Furthermore, a positive value of the
maximal Lyapunov exponent is obtained for such time-series response. An analysis of the numerical simulations indicates that the stability conditions obtained in section III A are sufficient but not necessary to achieve the system convergence to the desired rich and complex chaotic stationary response. However, the implementation of a control system that verifies such conditions guarantees mathematically the convergence.

IV. CONCLUSIONS

In order to obtain a robust, stationary and extensive chaotic response, with potential application in cryptographic schemes, a control method has been developed to be applied to nonlinear submicrometric electromechanical resonator. This approach proposes, for the first time, a fuzzy controlled increase of the chaotic response. The stability requirements and the conditions for the convergence to the desired chaotic dynamics have been analytically determined, and applied to simulate the response of a cc-beam resonator designed in a CMOS technology with the aim to prove the proposal. Beside the intrinsic advantages of fuzzy controllers, the control system method proposed in this work performs an automatic search for the optimal operating parameters, which provide the desired chaotic response. Thus, the exact parameters of the resonator system are not required, and in consequence the control system can be applied in practical applications, in which, due to the hypersensitivity of chaotic behaviour, small parameter variations due to fabrication tolerances may invalidate the results obtained from numerical simulations.

ACKNOWLEDGEMENT

The research presented in this work has been partially supported by the Spanish Ministry of Economy and Competitiveness (under the project TEC2014-52878-R), and has been performed within the Erasmus Mundus Action 2 Euroweb Scholarship program.

REFERENCES


