Adaptive Beam Formation in Smart Antenna Using Tschebyscheff Distribution and Variants of Least Mean Square Algorithm

Anupama Senapati, Jibendu Sekhar Roy

Abstract - **Tschebyscheff distribution is used for adaptive beamforming for smart antenna with different variants of least mean square algorithm (LMS). First the adaptive beams are formed for linear antenna arrays using LMS algorithm and its variants, like, sign LMS (SLMS) and normalized LMS (NLMS). Then Tschebyscheff distribution with LMS (TDLMS), Tschebyscheff distribution with NLMS (TDNLMS) and Tschebyscheff distribution with SLMS (TDSLMS) are used for adaptive beam generation. The performances for accuracies of generated main beam direction, null direction and also side lobe level (SLL) are compared between LMS and TDLMS, NLMS and TDNLMS and SLMS and TDSLMS. The overall comparison of performances of all the algorithms is also presented.**

Keywords – **Adaptive beamforming, Tschebyscheff distribution, Variants of LMS algorithm, Side lobe level.**

I. INTRODUCTION

Smart antenna uses a number of antenna elements and signal received at each antenna element is adaptively combined to improve the overall performance in communication. Smart antennas can eliminate interference by producing null along the direction of undesired interferer and by producing only radiation beam along the direction of arrival (DOA) of signal [1-2]. The smart antenna technology can significantly improve wireless system performance by increasing signal quality, network capacity and coverage area. The digital beam forming method using smart antenna is shown in Fig 1. Signals are processed adaptively in order to exploit the spatial domain of the mobile radio channel. Usually the signals received at the different antennas are multiplied with complex weights and then adaptively weights are summed up. There are various types of algorithms for beam-forming, having their advantages and disadvantages [3-8]. In addition to various methods of DOA estimations, many iterative schemes applicable to adaptive beam-forming have been described in [3]. A sequential quadratic programming based algorithm is used for multi-lobe pattern and for adaptive nulling of the pattern in [5]. A beam-forming technique, for precise DOA estimation, based on hybridization of soft-computing methods is reported in [7]. Recently, a complex quaternion LMS algorithm is used [8] for beam-forming of polarization-sensitive electromagnetic vector-sensor. Side lobe reduction is another issue in adaptive smart antenna.

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Side lobes not only consume power but also cause of interference for other users. Therefore attention is also given for the reduction of side lobe levels in smart antennas [9-14].

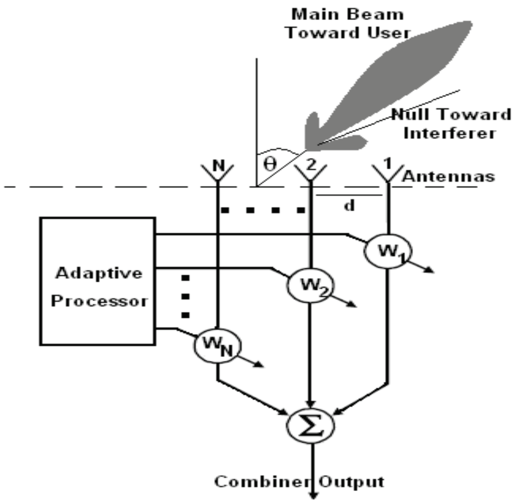

Fig. 1. Adaptive antenna array

Array synthesis methods reduce side lobe levels [9-11] for ordinary antenna arrays. But for adaptive antennas when beamforming algorithms are used in addition to array synthesis methods, due to weight updating side lobe level reduction like an ordinary array is not possible. Some of the reports are available where beamforming algorithms are used along with soft computing methods [12-14].

In this paper, Tschebyscheff distribution is used for adaptive beamforming for smart antenna of linear antenna arrays with different variants of least mean square (LMS) algorithm. Adaptive beams (null toward interferer and main beam toward desired user) are formed for linear antenna arrays using LMS algorithm and its variants, like, sign LMS (SLMS) and normalized LMS (NLMS). Then Tschebyscheff distribution with LMS (TDLMS), Tschebyscheff distribution with NLMS (TDNLMS) and Tschebyscheff distribution with SLMS (TDSLMS) are used for adaptive beam generation. A comparative study for beam generation and side lobe reduction using those hybrid algorithms are reported.

II. TSCHEBYSCHEFF DISTRIBUTION

Tschebyscheff distribution (TD) for current amplitude, fed in antenna, is used for antenna array synthesis [15, 16] to achieve narrowest beam for a given side lobe level or conversely to obtain lowest side lobe level for a given beam width. Tschebyscheff distribution, expressed in polynomial form as [16]

$$T_m(x) = \cos(m \cos^{-1} x) \quad -1< x< +1$$
$$= \cos(m \cosh^{-1} x); |x|>+1 \quad (1)$$

$$T_m(x_0) = b \quad (2)$$

It can be calculated by notifying that if b=coshρ, then

$$x_0 = \cosh(\rho/m) \quad (3)$$

Beamwidth is directly related to 'b' and 'x_0' is related to the position of main beam. Tchebyscheff distribution for odd value of 'm' is shown in Fig. 1*a*.

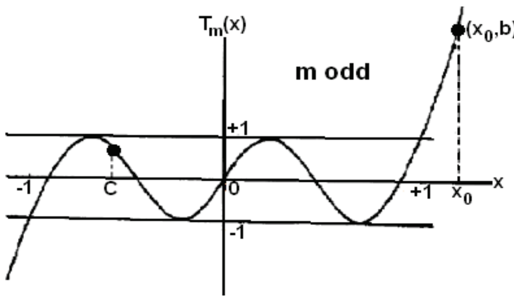

Fig. 1*a*. Tchebyscheff distribution for odd 'm'

The 'm' of $T_m(x)$ corresponds to the number of nulls of the array and also m=no. of antenna elements of the array (N) -1. The factor 'b' decides the beamwidth of the main beam and 'x_0' decides the position of the main beam. As the polynomial oscillates (Fig. 1a) between x=+1 and x=-1 with equal amplitude, in a synthesized antenna array using Tschebyscheff distribution only, the side lobes have equal peak.

III. Least Mean Square Algorithm and its Variants

A. Least Mean Square (LMS) Algorithm

To minimize error e(n) between desired signal d(n) and array output y(n) adaptive algorithm is used as [17-19]

$$e(n) = d(n) - y(n) \quad (4)$$

Output of adaptive beamformer, at time n, is given by a linear combination of the data at the N antenna elements, is expressed as:

$$y(n) = w^H x(n) \quad (5)$$

$$w = [w_1, w_2 \ldots, w_N]^H \quad (6)$$

where, *H* is Hermitian (complex conjugate) transpose. The weight vector *w* is a complex vector. Signal received by multiple antenna elements is

$$x(n) = [x_1(n), x_2(n), \ldots, x_N(n)] \quad (7)$$

In Least Mean Square (LMS) algorithm iterative procedure is used making successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. The weight vector updating equation for LMS algorithm is [17-19]

$$w(n+1) = w(n) + \mu x(n) e^*(n) \quad (8)$$

where, μ is the step size parameter and e(n) is the error between output and the desired signal.

B. Normalized Least Mean Square (NLMS) Algorithm

Normalized least mean square (NLMS) is used to achieve faster convergence and stability of the algorithm, where step size is divided by the norm of the input signal to avoid gradient noise amplification due to x(n). For normalized least mean square (NLMS) algorithm, the weight vector updating equation is [17-19]

$$w(n+1) = w(n) + \frac{\mu}{\|x(n)\|^2} x(n) e^*(n) \quad (9)$$

C. Sign Least Mean Square (SLMS) Algorithm

In Sign Least Mean Square (SLMS) algorithm only the polarity information of the error signal or data signal or both data and error signal is used for the error updating. In SLMS, adaptation equation is [17-19]

$$w(n+1) = w(n) + \mu x(n) e^*(n) . sgn[x(n)] \quad (10)$$

$$sgn[x(n)] = 1; \quad x(n) > 0$$
$$= 0; \quad x(n) = 0$$
$$= -1; \quad x(n) < 0$$

IV. Adaptive Beamforming

Adaptive beamforming is done for a linear array of uniform element spacing of 0.5λ and antenna arrays of 10 elements, 15 elements and 30 elements are considered.

Array factor (AF) for a linear array is given by [15, 16]

$$AF_L = \sum_{n=0}^{N-1} A_n e^{(jn(\frac{2\pi d}{\lambda}\cos\theta + \alpha))} \quad (11)$$

To generate main beam at wavelength λ, toward the desired beam direction θ^0 from the broadside direction, the progressive phase shift is given by

$$\propto = \frac{-2\pi d}{\lambda} \cos\theta_0 \quad (12)$$

And final array factor for Tschebyscheff array is

$$AF = AF_L . T_m(x) \quad (13)$$

Normalized array factor is

$$AF_{norm} = \frac{AF}{AF_{max}} \quad (14)$$

In Eq. (13), AF_{max} is the maximum value of array factor AF, given by Eq. (11).

The step-wise flow chart for the application of Tschebyscheff distribution with LMS algorithms (TDLMS) is shown in Fig 2.

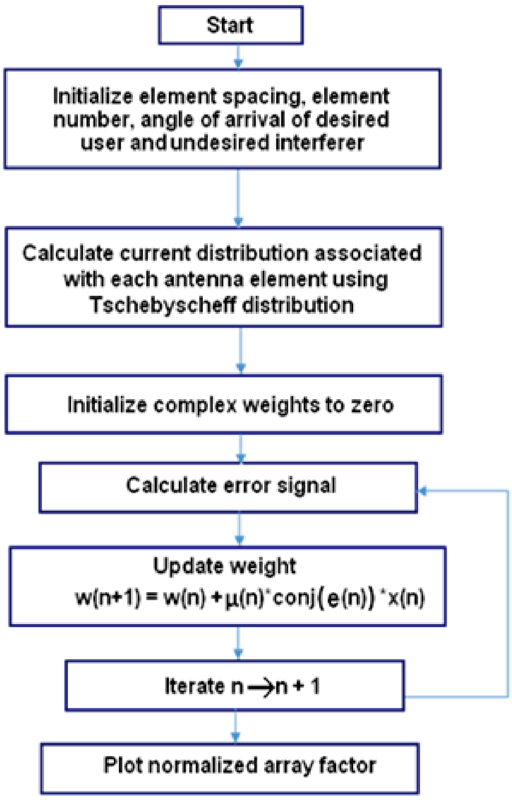

Fig. 2. Flow chart for TDLMS algorithm

For TDNLMS and TDSLMS same procedure, as mentioned in Fig. 2, is followed with different weight updating equations mentioned in Eq. (9) and in Eq. (10) respectively. Uniform linear antenna arrays of 10 elements, 15 elements and 30 elements are considered here. Comparisons of some results for LMS and TDLMS (Fig. 3 and Fig. 4), SLMS and TDSLMS (Fig. 5 and Fig. 6), NLMS and TDNLMS (Fig. 7 and Fig. 8) are shown below where in Tschebyscheff distribution SLL is fixed at -25 dB. Here, inter-element spacing d=0.5λ, desired angle of arrival 30^0 and angle of interferer (Null) 60^0. Number of Iterations of the program for each algorithm is 100.

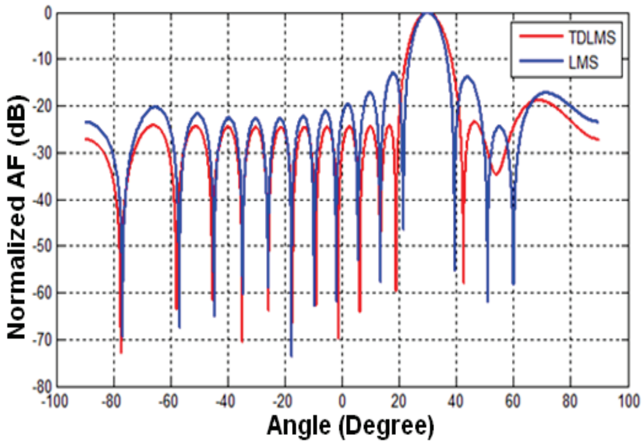

Fig. 3. Normalized array factor N=15 antenna elements

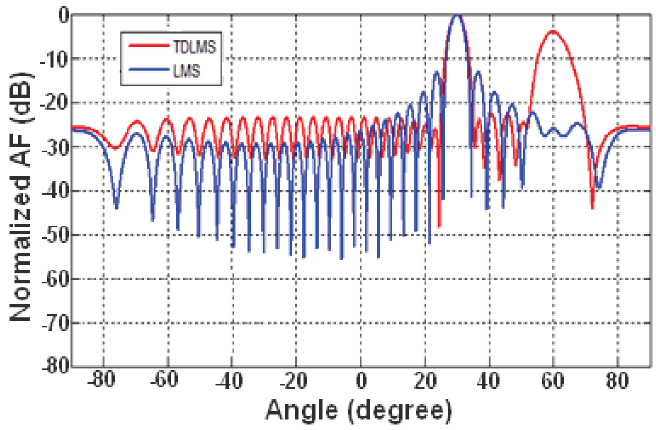

Fig. 4. Normalized array factor N=30 antenna elements

Like, phased array antenna with the angle of arrival grating lobe may appear, even after maintaining the maximum inter-element spacing [20]. Appearance of grating lobe in TDLMS is shown (at 60^0) in Fig. 4. Therefore this result is not acceptable for adaptive beamforming.

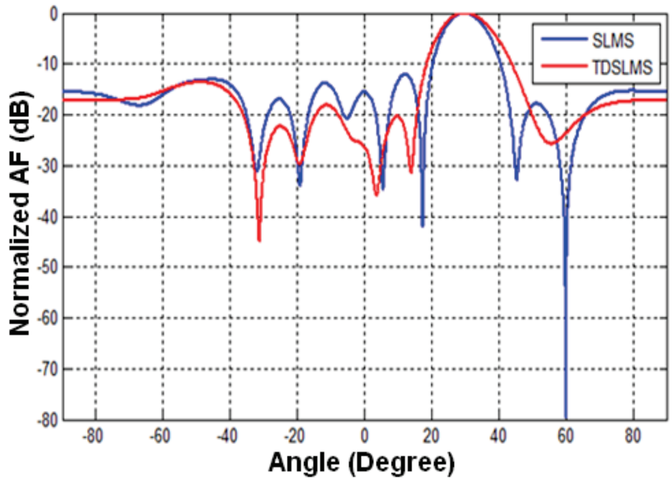

Fig. 5. Normalized array factor N=10 antenna elements

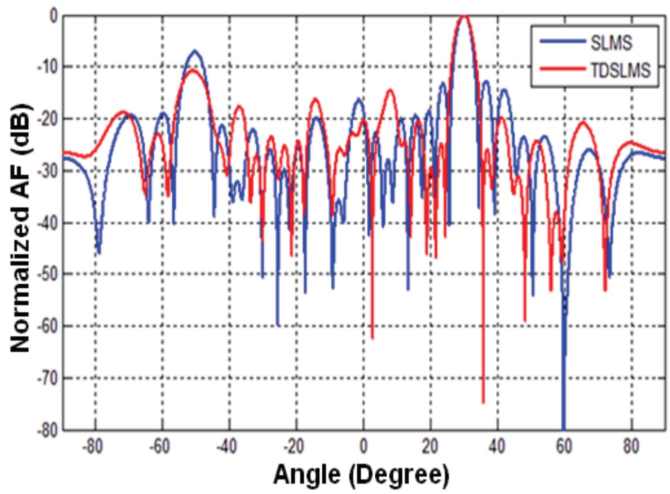

Fig. 6. Normalized array factor N=30 antenna elements

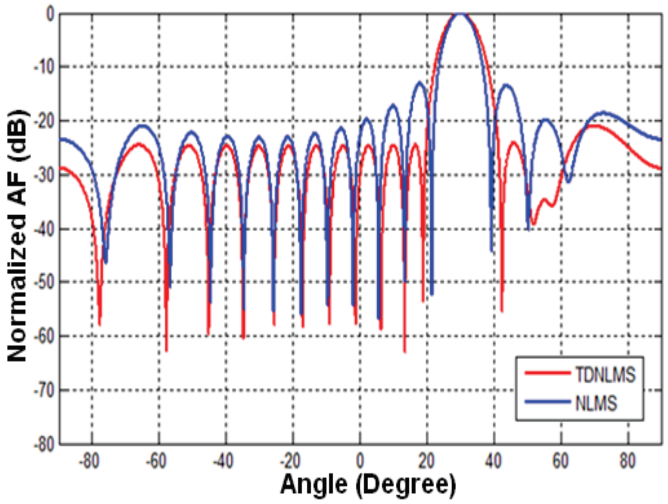

Fig. 7. Normalized array factor N=15 antenna elements

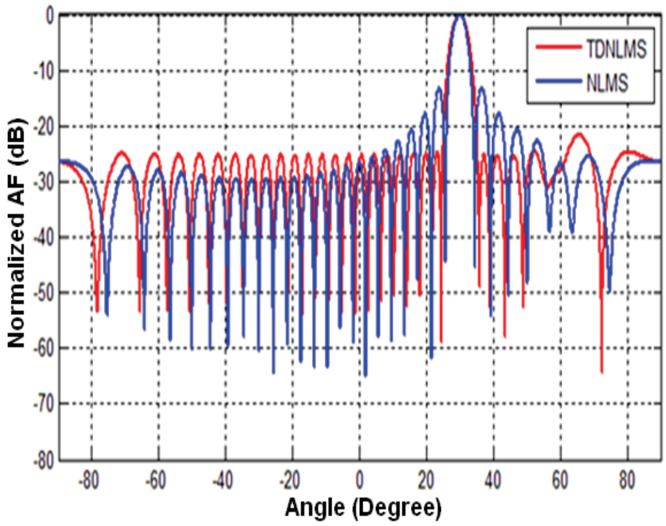

Fig. 8. Normalized array factor N=30 antenna elements

Comparisons of some results for LMS and TDLMS (Fig. 9 and Fig. 10), SLMS and TDSLMS (Fig. 11 and Fig. 12), NLMS and TDNLMS (Fig. 13 and Fig. 14) are shown below where in Tschebyscheff distribution SLL is fixed at -30 dB.

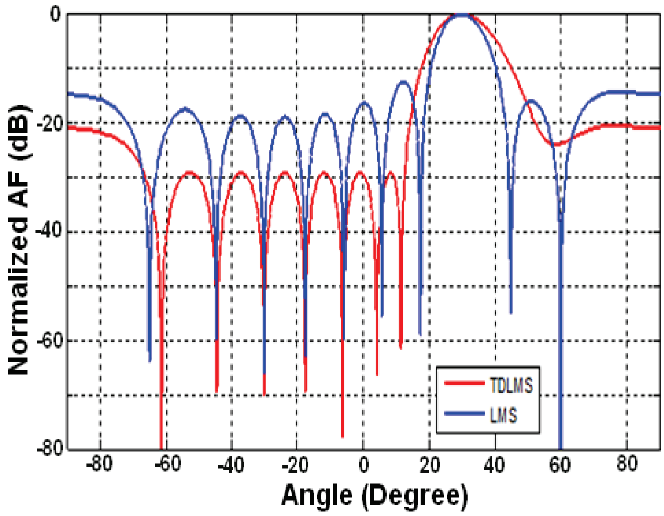

Fig. 9. Normalized array factor N=10 antenna elements

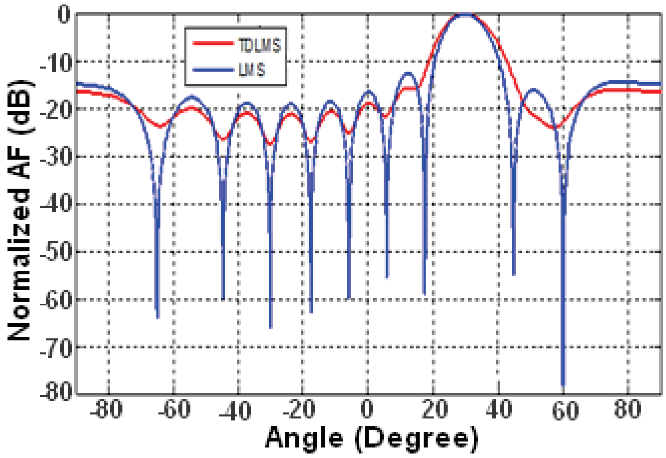

Fig. 10. Normalized array factor N=15 antenna elements

Performances of different algorithms for main beam direction, null direction and maximum side lobe level (SLL_{max}), used for adaptive beamforming, are compared in Table-1. Side lobe level (SLL) is fixed at -25 dB in Tschebyscheff distribution.

TABLE 1

COMPARISON OF RESULTS WHEN IN TSCHEBYSCHEFF DISTRIBUTION SLL=-25 dB

Algorithms	N=10			N=15			N=30		
	Main Beam (Deg)	Null (Deg)	SLL_{max} (dB)	Main Beam (Deg)	Null (Deg)	SLL_{max} (dB)	Main Beam (Deg)	Null (Deg)	SLL_{max} (dB)
LMS	30	60	-12.5	30	60	-12	Not working properly	Not working properly	Not working properly
TDLMS	30	57.5	-19	30	57	-19	Not working properly	Not working properly	Not working properly
NLMS	30	61	-12	30	62	-13.5	30	No precise null at 60	-13
TDNLMS	30	-58.5	-21	30	58	-21	30	No precise null at 60	-21
SLMS	30	60	-11.5	30	60	-10.5	30	60	-8
TDSLMS	30	57.5	-12.5	30	57.5	-12	30	59.8	-10.5

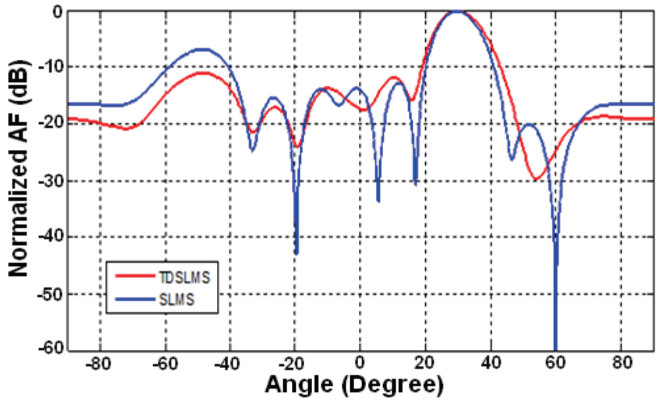

Fig. 11. Normalized array factor N=15 antenna elements

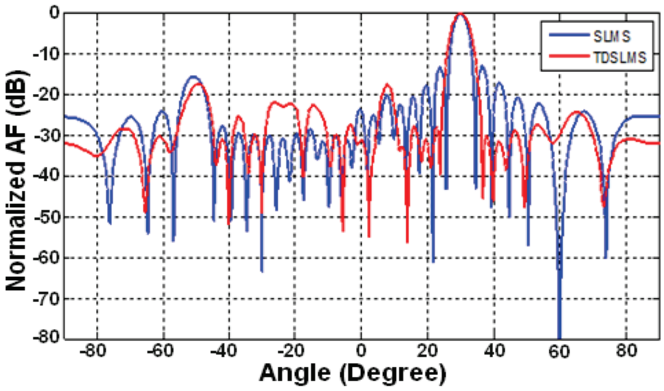

Fig. 12. Normalized array factor N=30 antenna elements

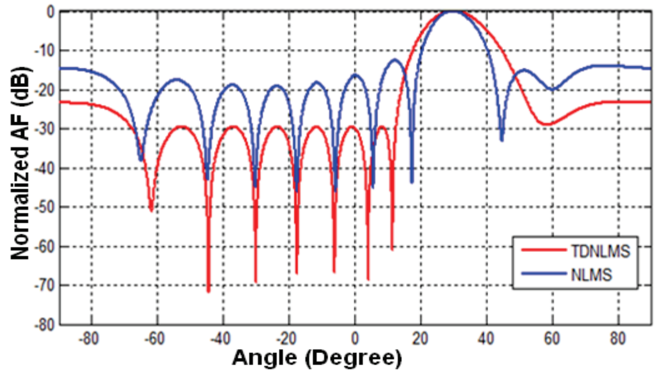

Fig.13. Normalized array factor N=10 antenna elements

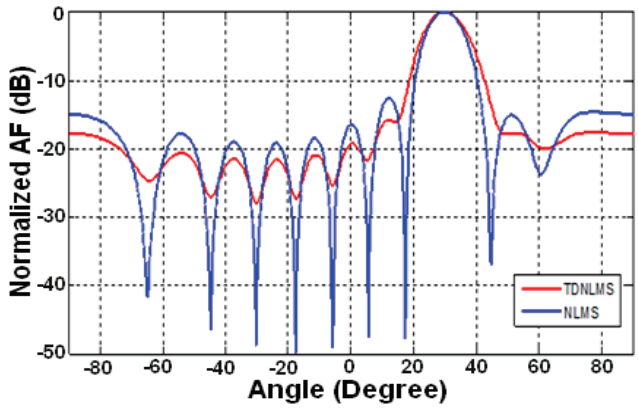

Fig. 14. Normalized array factor N=15 antenna elements

Performances of different algorithms for main beam direction, null direction and maximum side lobe level (SLL_{max}), used for adaptive beamforming, are compared in Table 2. Side lobe level (SLL) is fixed at -30 dB in Tschebyscheff distribution.

Square error plots for 10 antenna elements for LMS, NLMS and SLMS algorithms are shown in Fig. 15. This error is square error (SE) and this is square of the error e(n), mentioned in Eq. (4).

Error for LMS is more than the others but convergence of LMS is better.

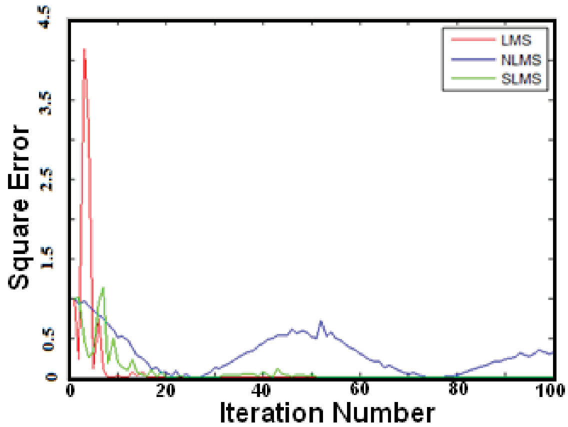

Fig. 15. Square error plot for 10 antenna elements with μ=0.02

TABLE 2
COMPARISON OF RESULTS WHEN IN TSCHEBYSCHEFF DISTRIBUTION SLL=-30 dB

Algorithms	N=10			N=15			N=30		
	Main Beam (Deg)	Null (Deg)	SLL_{max} (dB)	Main Beam (Deg)	Null (Deg)	SLL_{max} (dB)	Main Beam (Deg)	Null (Deg)	SLL_{max} (dB)
LMS	30	60	-12	30	60	-12.5	Not working properly	Not working properly	Not working properly
TDLMS	30	58	-20	30	57	-16	Not working properly	Not working properly	Not working properly
NLMS	30	60	-12	30	61	-12	30	No Precise Null	-13.5
TDNLMS	30	58.5	-23.5	30	62	-16	30	61	-28
SLMS	30	60	-11	30	60	-7	30	60	-13
TDSLMS	30	59	-13	30	55	-11	30	59	-17.5

V. Discussion and Conclusion

Performances of hybrid beamforming algorithms are compared in this paper. Remarkable side lobe level reduction is achieved. For all the cases step size of μ=0.02 is taken because extensive work on beamforming of adaptive antennas using LMS, NLMS and SLMS shows that for this value of μ, performances of all these algorithms are simultaneously good [21]. The performances of algorithms, tabulated in Tables 1 and 2, show that using TDLMS, TDSLMS and TDNLMS side lobe reduction is achieved but accuracies for interferer directions (nulls) have small deviation from the expected direction. But in those cases, at the null direction (at 60^0) radiation is equal to or below the -20dB which is not so effective radiation. Side lobe reduction using TDNLMS is better as compared to TDLMS and TDSLMS. For large antenna array LMS and NLMS, TDLMS and TDNLMS, proper adaptive beamforming is not achieved for step size of 0.02. It is found that performances of SLMS and TDSLMS are better for large antenna arrays than TDLMS.

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