Iterative MoM-Based Technique for Evaluation of Bistatic RCS of Electrically Large Scatterers

Miodrag S. Tasic[1], Branko M. Kolundzija[2]

Abstract – **A simple technique for evaluation of bistatic radar cross-section (RCS) is presented. The technique is purely algebraic, and it is applied to a system of equations obtained by the Method of moments (MoM). Since the MoM matrix need not to be inverted and stored in this technique, as with the standard MoM approach, one can expect lowering computational costs. Numerical examples are given, with the comparison of bistatic RCS results obtained by the presented technique and by the standard MoM approach.**

Keywords – **Electromagnetic analysis, Method of moments, Radar Cross Section (RCS), Scatterers.**

I. INTRODUCTION

Method of moments (MoM) [1], applied to the surface integral equations (SIEs) in frequency domain, is a versatile tool for calculation of radar cross section (RCS) of arbitrary shaped objects. MoM transforms SIEs into a system of linear equations, which N unknowns are weighting coefficients of adopted basis functions (BFs). Direct solution of the system is not suitable when N is large, because of memory requirement $O(N^2)$, and calculation complexity $O(N^3)$. Multi level fast multipole algorithm (MLFMA) [2] reduces both the memory requirement and the computational complexity to $O(N \log N)$, but since it uses iterative solvers, the system of equations must be preconditioned, and a substantial number of iterations is needed for a convergence. A few years ago, we introduced new iterative technique for analysis of the perfectly conducting (PEC) scatterers [3]. Here we present improved technique, which works with both PEC and dielectric structures. The main idea of [3] is unchanged—in each iteration we add new macro basis functions (MBFs), and adjust their weighting coefficients to minimize some criterion function. The difference is in a way MBFs are created. For the proposed technique, the memory requirement is $O(N)$, and its calculation complexity per iteration is $O(N^2)$. The technique is relatively simple and pure algebraic—it operates only with calculated elements of MoM system of equations. Furthermore, acceptable results are obtained in a few iterations. The technique, which will be presented in the following section, is suitable for evaluation of bistatic RCS only (monostatic RCS would require a new analysis for each direction), and is not completely optimized for speed and efficiency. Numerical examples are given in the third section.

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II. ITERATIVE MOM-BASED TECHNIQUE

Conventional MoM solution of the surface integral equations, $\mathbf{f}_\mathrm{a}$, is expressed as a linear combination of N known vector BFs, $\mathbf{f}_k$, multiplied by unknown complex scalar coefficients, a_k,

$$\mathbf{f}_\mathrm{a} = \sum_{k=1}^{N} a_k \mathbf{f}_k \,. \tag{1}$$

Coefficients a_k are obtained as a solution of the MoM system of equations [4]:

$$\sum_{k=1}^{N} z_{jk} a_k = v_j, \quad j = 1,..,N \,, \tag{2}$$

where z_{jk} and v_j are known complex scalars. In the matrix form, the system (2) can be expressed as:

$$\mathbf{z}\,\mathbf{a} = \mathbf{v} \,, \tag{3}$$

where $\mathbf{z}$ is $N \times N$ (MoM) matrix of the system, $\mathbf{v}$ is $N \times 1$ excitation vector, and $\mathbf{a}$ is $N \times 1$ solution vector, with elements z_{jk}, v_j, and a_k from (2), respectively.

In the proposed technique, coefficients a_k are obtained in iterative manner. First, the set of MoM BFs, $\{\mathbf{f}_1,...,\mathbf{f}_k,...,\mathbf{f}_N\}$, indexed from 1 to N, is split into M subsets—groups:

$$\{\boldsymbol{f}_{k_{l1}},..,\boldsymbol{f}_{k_{lj}},..,\boldsymbol{f}_{k_{lN_l}}\}, \quad l = 1,..,M \,, \tag{4}$$

where N_l is the number of BFs in the lth group, and k_{lj} is index of jth BF in lth group. Each BF must belong to at least one group. We introduce the lth group matrix, $\mathbf{G}_l$, with N_l rows and N columns. Each row j has only one nonzero element, corresponding to the index of lth group jth BF, i.e. elements of matrix $\mathbf{G}_l$ are:

$$G_l[j,k] = \begin{cases} 1, k = k_{lj} & j = 1,..,N_l \\ 0, \text{otherwise} & k = 1,..,N \end{cases}, \tag{5}$$

where j represents a row, and k represents a column. Next we define the lth group partial MoM system of equations:

$$\mathbf{G}_l\mathbf{z}\mathbf{G}_l^{\mathrm{T}}\mathbf{b}_l^{(i)} = \mathbf{G}_l\mathbf{e}^{(i)}, l = 1,..,M\ , \tag{6}$$

where $\mathbf{G}_l\mathbf{z}\mathbf{G}_l^{\mathrm{T}}$ is $N_l \times N_l$ matrix of the partial system, $\mathbf{G}_l\mathbf{e}^{(i)}$ is $N_l \times 1$ excitation vector in the *i*th iteration, and $\mathbf{b}_l^{(i)}$ is $N_l \times 1$ vector obtained as a solution of the system (6). Essentially, the system (6) is obtained by extraction of self-coupling and mutual-coupling terms, corresponding to BFs of the *l*th group, from the system (3).

The groups (4) are bases for creation of MBFs in the *i*th iteration, $\mathbf{F}_l^{(i)}$, with the elements of the vector $\mathbf{b}_l^{(i)}$, $b_{lk}^{(i)}$, as weighting coefficients:

$$\boldsymbol{F}_l^{(i)} = \sum_{j=1}^{N_l} b_{lk}^{(i)} \boldsymbol{f}_{k_{lj}}, \quad l = 1,..,M\ . \tag{7}$$

The sum in (7) is for all BFs of the *l*th group. However, for the sake of mathematical clarity, it is more suitable to have the formal sum for all MoM BFs. To do so, we formally (in program realization this is unnecessary) use transformation:

$$\mathbf{a}_l^{(i)} = \mathbf{G}_l^{\mathrm{T}}\mathbf{b}_l^{(i)}, \tag{8}$$

which "returns" elements of the vector $\mathbf{b}_l^{(i)}$ to its "original" positions in the $N \times 1$ vector $\mathbf{a}_l^{(i)}$, with elements $a_{lk}^{(i)}$. Now we can replace (7) with:

$$\boldsymbol{F}_l^{(i)} = \sum_{k=1}^{N} a_{lk}^{(i)} \boldsymbol{f}_k, \quad l = 1,..,M\ . \tag{9}$$

Let us suppose, for the moment, that we determined $\mathbf{a}_l^{(i)}$ for all groups, $l = 1,..,M$, and all iterations, $i = 0,..,n$. The approximate solution in the *n*th iteration is adopted as

$$\mathbf{f}_{\mathrm{a}}^{(n)} = \sum_{i=0}^{n}\sum_{l=1}^{M} c_{il}^{(n)} \mathbf{F}_l^{(i)}\ , \tag{10}$$

i.e. as linear combinations of all MBFs created in all iterations, where $c_{il}^{(n)}$ are unknown coefficients that should be determined. By substituting expression for $\mathbf{F}_l^{(i)}$ from (9) into (10), after some rearrangements, (10) can be expressed as

$$\boldsymbol{f}_{\mathrm{a}}^{(n)} = \sum_{k=1}^{N}\left(\sum_{i=0}^{n}\sum_{l=1}^{M} c_{il}^{(n)} a_{lk}^{(i)}\right)\boldsymbol{f}_k = \sum_{k=1}^{N} a_k^{(n)} \boldsymbol{f}_k\ . \tag{11}$$

Expression (11) is formally the same as (1), except that coefficients $a_k^{(n)}$ do not satisfy the MoM system (2). Instead, for each of the equations in (2) we can define a residual error

$$R_j^{(n)} = v_j - \sum_{k=1}^{N} z_{jk} a_k^{(n)},\ j = 1,..,N\ , \tag{12}$$

and a mean square value for all residual errors—residuum:

$$R^{(n)} = \frac{1}{N}\sum_{j=1}^{N}\left|R_j^{(n)}\right|^2 = \frac{1}{N}\left|v_j - \sum_{i=0}^{n}\sum_{l=1}^{M} c_{il}^{(n)} Z_{jl}^{(i)}\right|^2\ , \tag{13}$$

where we introduced auxiliary "matrix"

$$Z_{jl}^{(i)} = \sum_{k=1}^{N} z_{jk} a_{lk}^{(i)}\ . \tag{14}$$

Coefficients $c_{il}^{(n)}$ are determined in a way to minimize residuum (13). By imposing condition

$$\frac{\partial R^{(n)}}{\partial c_{km}^{(n)}} = 0, \quad k = 0,..,n \quad m = 1,..,M\ , \tag{15}$$

we obtain a system of equations

$$\sum_{i=0}^{n}\sum_{l=1}^{M} c_{il}^{(n)}\left(\sum_{j=1}^{N} Z_{jl}^{(i)} Z_{jm}^{(k)*}\right) = \sum_{j=1}^{N} v_j Z_{jm}^{(k)*}\ . \tag{16}$$
$$k = 0,..,n, \quad m = 1,..,M$$

As a solution of the system (16) we obtain coefficients $c_{il}^{(n)}$, i.e. solution (10).

As we already seen, coefficients for creation of MBFs, given by vector $\mathbf{b}_l^{(i)}$ (i.e. its formally extended version $\mathbf{a}_l^{(i)}$), are solution of the system (6). For the initial solution (referenced as 0th iteration) we use $\mathbf{e}^{(0)} = \mathbf{v}$, i.e. original excitation of the system (3). For the iterations $i > 0$ we use $\mathbf{e}^{(i)} = \mathbf{R}^{(i-1)}$, i.e. excitation is performed with residual error vector (12) from previous iteration.

To obtain the solution (10), in each iteration, we need to create and solve the systems (6)—for MBFs, and the system (16)—for MBFs weighting coefficients. Each of the systems (6) gives us coupling between BFs of the group, whereas the system (16), basically, gives us coupling between the groups. If we choose the number of the groups, *M,* to be the square root of the number of MoM unknowns, *N*, and if the number of iterations, *n*, is relatively small (e.g. less than 10), then the dominant number of operations is $O(N^2)$, for calculation of the MoM matrix $\mathbf{z}$. This is reduction compared to $O(N^3)$ for matrix inversion in standard MoM. For efficient calculation of the matrix of the system (16), given by the expression in the brackets, we do not need to know the entire MoM matrix, but only a single row at once. Hence, if the MoM matrix is large, instead of calculating once and store it, we can calculate it in each iteration, one row at a time, thus decreasing memory requirement from $O(N^2)$ to $O(N)$.

III. NUMERICAL EXAMPLES

We will analyze two canonical shapes—a sphere and a cube, both in two versions: PEC and dielectric, and one real life example—PEC dron. Geometrical modeling is performed by bilinear plates (quadrilaterals), and current expansion is modeled with higher order polynomial BFs. These elements, together with the MoM matrix, are taken from the WIPL-D Pro electromagnetic solver [5]. All models are excited by an electromagnetic plane wave. Equivalent surface currents are determined by using standard MoM (3), and by using proposed iterative technique (6)–(16). Using these currents, bistatic RCS (marked as σ) is calculated, and the results for the relative RCS (σ/λ^2), for both MoM and the proposed technique, are compared (results for the proposed technique are depicted with the letter "i", followed by the order number of iteration).

As a measure of convergence (toward the MoM solution) of the proposed method, we use a normalized residuum:

$$R_{\text{norm}}^{(n)} = \frac{R^{(n)}}{\frac{1}{N}\sum_{j=1}^{N}\left|v_j\right|^2}, \tag{17}$$

which is 0 for MoM solution, and is 1 if all coefficients a_k in (1) are zero.

We perform grouping of BFs using two different procedures, both based on grouping of plates. In the first procedure, named "box", we enclose the model by a parallelepiped, divide the parallelepipeds volume into a number of elemental parallelepipeds—boxes, and then group plates which centers belong to the same box. The number of groups of plates is equal to the number of nonempty boxes. In the second procedure, named "uniform", we split plates of the model in two groups, with equal numbers of plates (or these numbers are different by one), along one of three orthogonal axes (we choose the axis with the largest span by the model). Then we, in the same manner, split two groups in four, four groups in eight, and so on. At the end of this iterative procedure, the number of groups of plates is equal to 2^m, where m is the number of iterations. Finally, for both procedures, we add neighboring plates to each group, so that we have some overlapping between the groups. These procedures are easy for implementation and efficient for CPU execution.

For each group of plates, we create a group of BFs, which are defined over the plates of the group. Since uniform procedure provides groups with virtually identical number of plates, the number of BFs in different groups will be similar. This is important for balanced operations number when calculating (6). However, for some models uniform procedure can group physically distant plates. Partial systems (6) for such groups have less physical meaning, so we expect slower convergence of the proposed iterative solution. Box procedure generally has good "localization" of plates in a group, but number of plates (and, consequently, BFs) can vary substantially between groups.

A. Canonical Examples

All canonical models are excited by electromagnetic circularly polarized plane wave (with free-space wavelength λ), coming from a direction defined by angles $\phi = 45°$ and $\theta = 45°$ (in the spherical coordinate system; θ is zero in the *xy* plane). Bistatic RCS is calculated in the plane defined by ϕ–cuts $45°/225°$ (which includes maximal RCS). BFs were divided into 64 groups using uniform procedure—the box procedure of grouping for these examples gives very similar results. The system (16) has $64(i+1)$ unknowns in *i*th iteration. The maximal polynomial order for the PEC cube is three, and for all other examples is five.

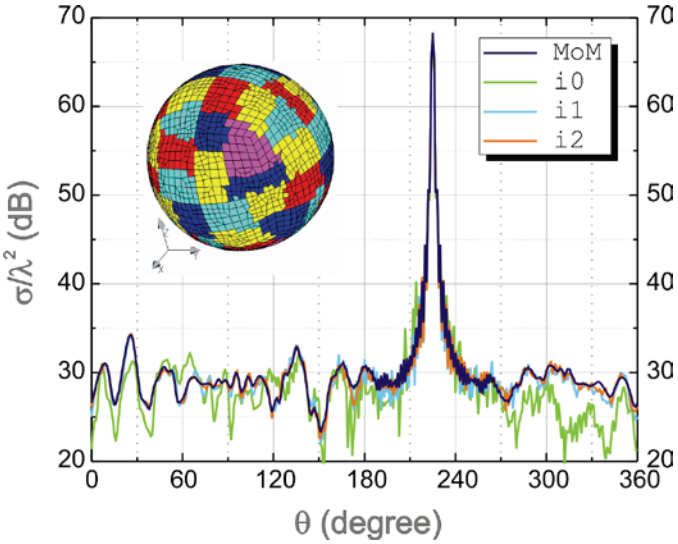

Fig. 1. RCS of the PEC sphere, 103220 BFs, 64 groups

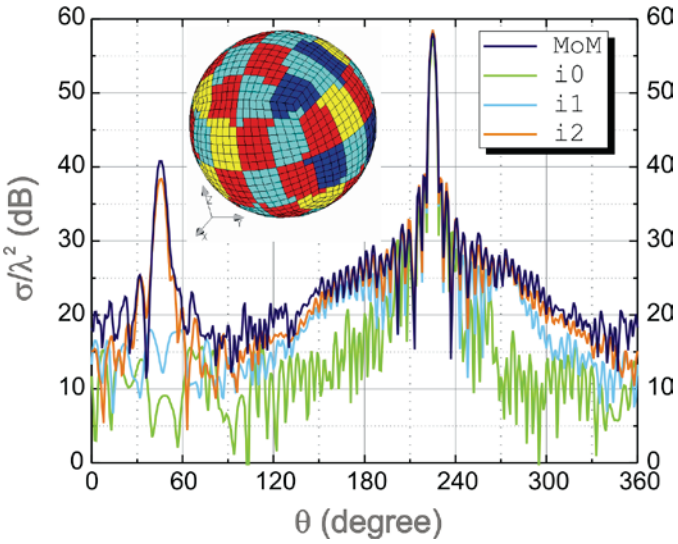

Fig. 2. RCS of the dielectric sphere, 98304 BFs, 64 groups

RCS for the PEC sphere, with a diameter of 30.7 wavelengths and 103220 BFs, is shown in Fig. 1. The proposed technique converge very fast to the MoM solution—the difference for RCS is almost non existing after the second iteration.

RCS for the dielectric sphere, with a diameter of 33.7 wavelengths (in the dielectric) and 98304 BFs, is shown in Fig. 2. Though not quite a match to that of the PEC sphere, the convergence of the proposed technique is still excellent—again, two iterations are enough for acceptable matching with RCS obtained by MoM.

Note that both spheres are initially modeled with 384 bilinear plates, which are then subdivided into smaller ones. Because of that, the sphere curvature is not perfectly modeled, which affects RCS results. However, our focus is comparison of the proposed technique with MoM, for the same model.

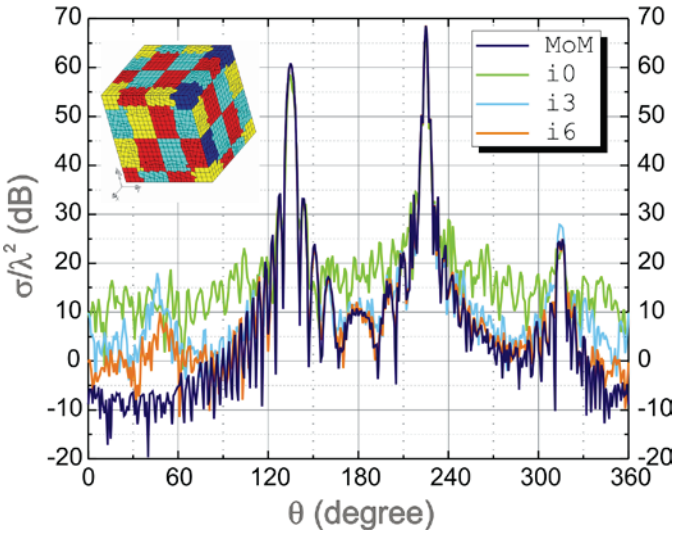

Fig. 3. RCS of the PEC cube, 94872 BFs, 64 groups

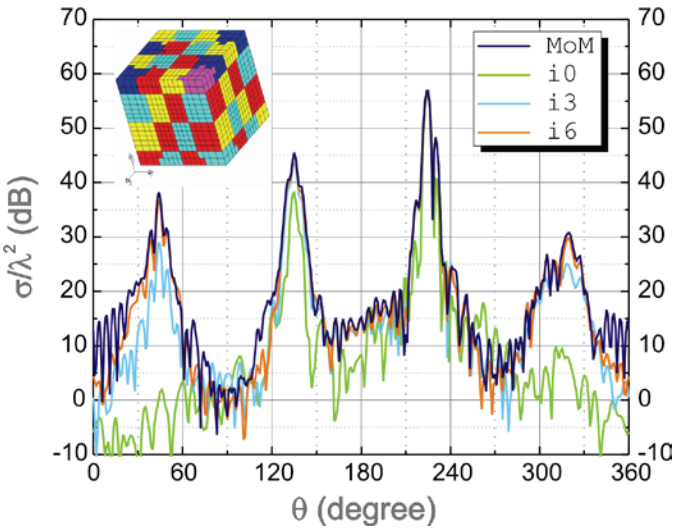

Fig. 4. RCS of the dielectric cube, 98304 BFs, 64 groups

RCS for the PEC cube, with a diameter of 21 wavelengths and 94872 BFs, is shown in Fig. 3. The convergence rate of the proposed technique for the cube is lower than for the sphere, so we depicted iterations 0, 3, and 6. We can see that RCS obtained in the sixth iteration is very similar to one obtained by MoM. Noticeable disagreement exists only for the angles between 30 and 60 degrees—this is corrected in the higher iterations.

RCS for the dielectric cube, with a diameter of 22.67 wavelengths (in the dielectric) and 98304 BFs, is shown in Fig. 4. The convergence is similar as for the PEC cube. We didn't notice significant disagreements between the MoM result and the higher iterations of the proposed techniques.

Normalized residuum, for all models, is shown in Fig. 5, as a function of the number of iterations. The residuum decreases trough the iterations, and the convergence speed decreases also. Correlation between the residuum and the RCS is dependent of the model, but for the given examples we can say that a residuum of 10^{-2} provides acceptable RCS, from engineering point of view, whereas a residuum of 10^{-3} provides very good to excellent matching with RCS obtained by the MoM.

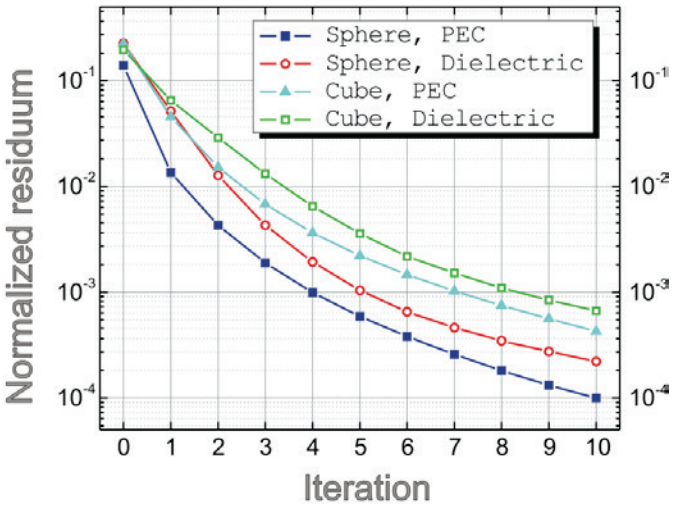

Fig. 5. Normalized residuum for canonical examples

CPU times are shown in Fig. 6. Dielectric sphere is analyzed using Intel Core 2 Quad Q9400 2.67 GHz PC with 4 GB RAM, whereas all other examples are analyzed using Intel Core i7–2600 3.4 GHz PC with 16 GB RAM. CPU times for MoM analysis, shown in the brackets, are much longer than times of the proposed technique (for 1E-3 residuum). The memory for the proposed technique is neglecting.

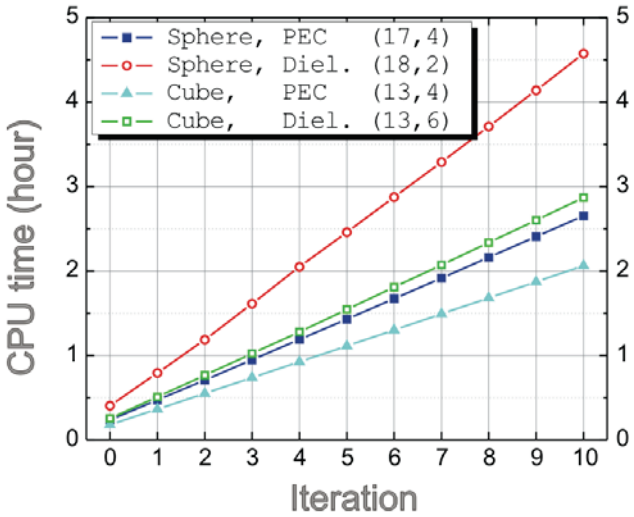

Fig. 6. CPU time for canonical examples

B. Real Life Example

We consider PEC dron, about 55 wavelengths long, which wings span is about 97 wavelengths, and with 132926 BFs. We will examine ten models, obtained with both grouping procedures, and with different number of groups. Two particular models, with the same number (32) of groups, obtained by different grouping procedures, are shown in Fig. 7 (box procedure), and Fig. 8 (uniform procedure). All models are excited by electromagnetic linearly polarized ($\theta-$component) plane wave coming from the direction defined by angles $\phi=-45°$ and $\theta=-45°$. The maximal polynomial order for all examples was seven.

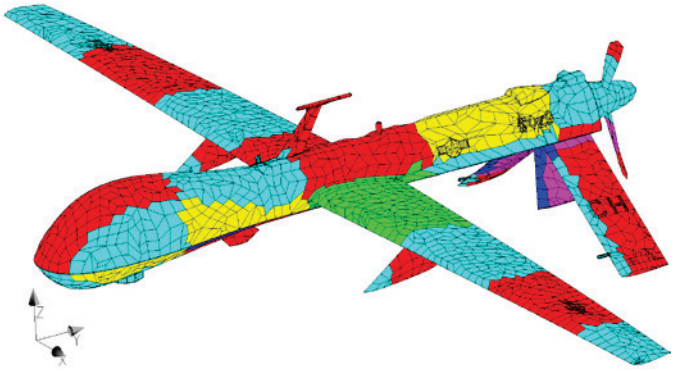

Fig. 7. Dron with 32 groups of plates created by box procedure

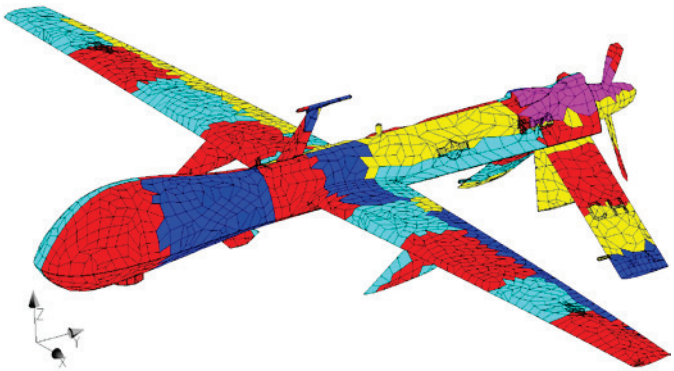

Fig. 8. Dron with 32 groups of plates created by uniform procedure

Normalized residuum throughout ten iterations, for different number of groups created by box procedure, is shown in Fig. 9. A similar graph, but for the uniform procedure, is shown in Fig. 10. The number of groups, for different grouping procedures, are almost identical. We can see that number of groups have little influence to the speed of convergence. However, box grouping provide slightly better convergence (smaller normalized residuum). One can notice that the vertical antenna radome (between wings, red part in Fig. 7) belongs to the single group when box procedure is used. In the case of uniform procedure, a part of the radome is assigned to a group which is not connected with the radom (blue part in Fig. 8). As we mentioned earlier, such groups are more frequently created with the uniform procedure. We believe that unconnected groups (i.e. partial systems based on them) have less physical meaning, leading toward slower convergence throughout iteration.

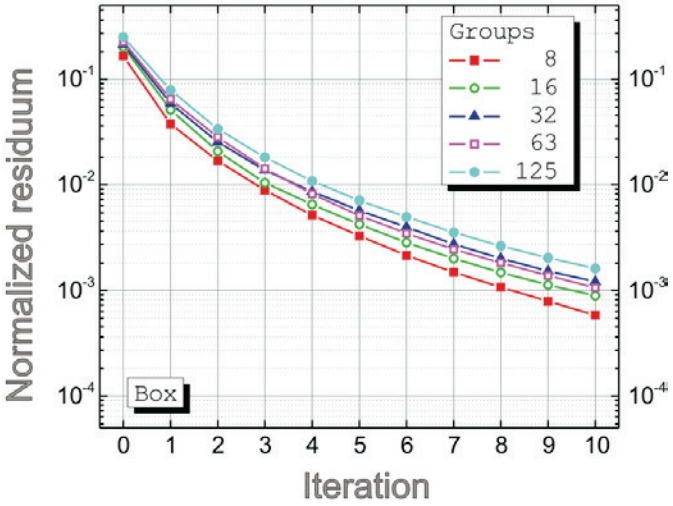

Fig. 9. Dron, normalized residuum, box procedure

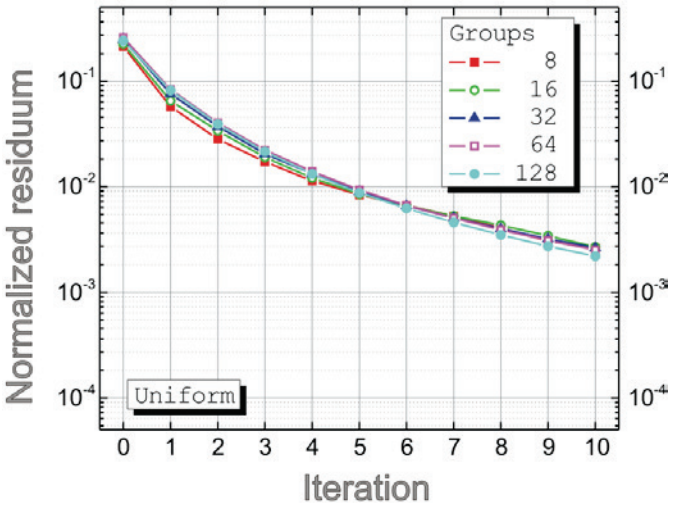

Fig. 10. Dron, normalized residuum, uniform procedure

CPU times, throughout ten iterations, are shown in Fig. 11 for box grouping, and in Fig. 12 for uniform grouping. Analyses are performed using Intel Xeon E3–1231 v3 3.40 GHz PC with 16 GB RAM. (CPU time for MoM analysis is 9.5 hours.) CPU time for corresponding number of groups is larger for box grouping, and this difference culminates for eight groups. Since box grouping doesn't have explicit care for evenness of plates number in different groups, two of eight groups are much larger than the others, in this particular box grouping. Solutions of partial systems (6) for these groups become the dominant in CPU time. This rarely happens with uniform grouping.

Based on surface currents obtained by the MoM and by the proposed technique, relative bistatic RCS (σ/λ^2) is calculated in the plane defined by $\phi-$cuts $-45°/+135°$ (which includes maximal RCS). MoM result is compared with box grouping results in Fig. 13, and with uniform grouping results in Fig. 14. Iterations 0, 1, and 5 are chosen because they are decisive to RCS shape. By fifth iteration RCS matching to MoM is very good. Normalized residuum for fifth iteration is about 10^{-2}.

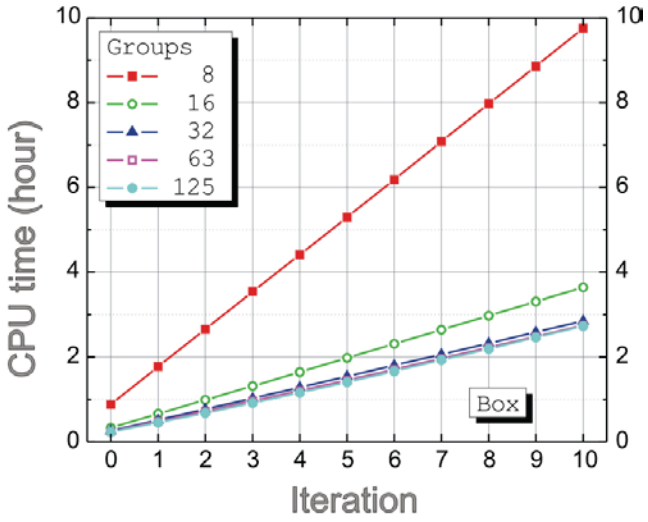

Fig. 11. Dron, CPU time, box procedure

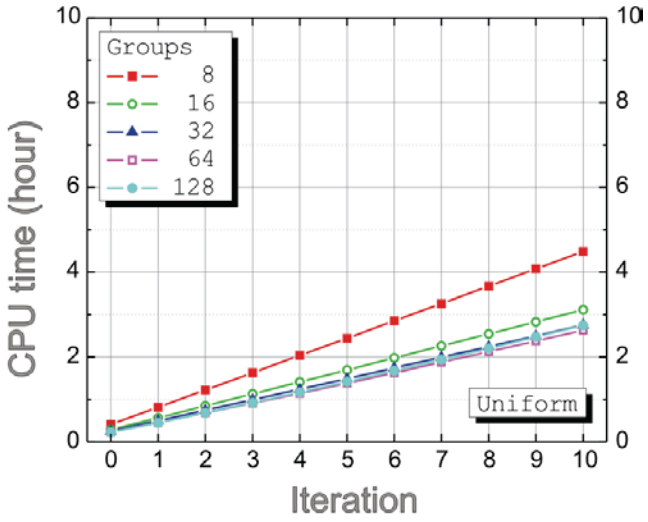

Fig. 12. Dron, CPU time, box procedure

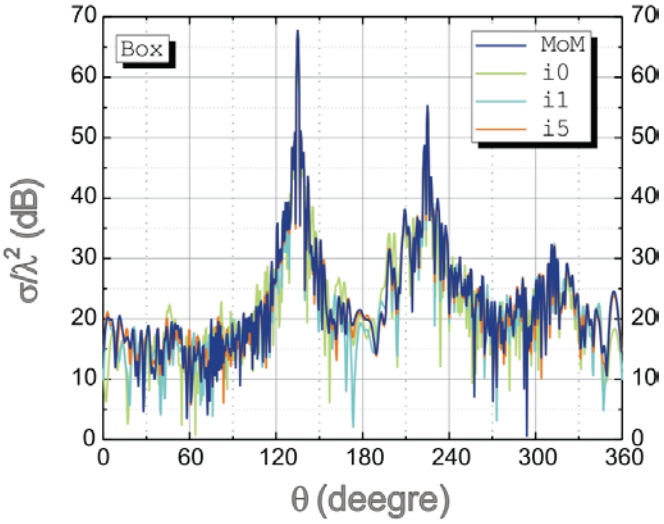

Fig. 13. Dron, 32 groups, RCS, box procedure

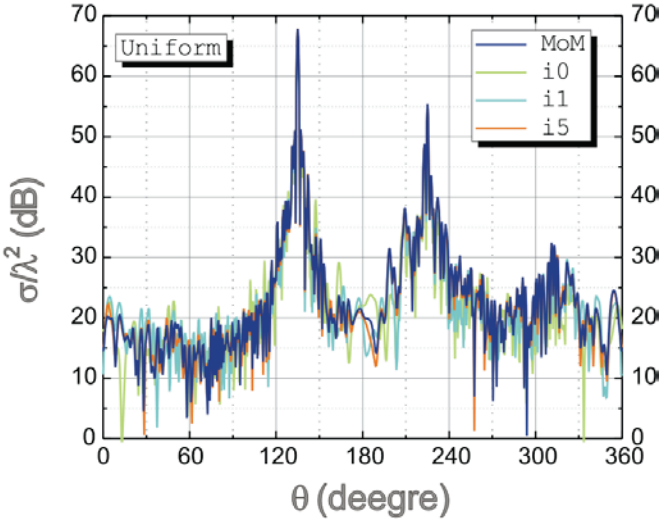

Fig. 14. Dron, 32 groups, RCS, uniform procedure

IV. Conclusion

We presented a simple iterative technique for evaluation of bistatic RCS of arbitrary shaped scatterers of homogenous material. Numerical example shows good agreement with the MoM solution, but with less memory and CPU time. The focus in the future work will be on the optimization of a speed and an efficiency of the technique, particularly for calculations of monostatic RCS.

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